An Implication of “Gravity as the Weakest Force”

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Abstract

The negative specific heat of a radiating black hole is indicative of a cataclysmic endpoint to the evaporation process. In this letter, we suggest a simple mechanism for circumventing such a dramatic outcome. The basis for our argument is a conjecture that was recently proposed by Arkani-Hamed and collaborators. To put it another way, we use their notion of “Gravity as the Weakest Force” as a means of inhibiting the process of black hole evaporation.
It has long been realized that black holes\footnote{We will be working — for simplicity — with Schwarzschild black holes in a four-dimensional spacetime. Nonetheless, the discussion easily generalizes (given a healthy dose of algebra) into other dimensionalities and many classes of black holes.} are thermodynamic entities with a readily identifiable temperature

\[
T = \frac{\hbar}{8\pi GM}
\]

(1)

and entropy

\[
S = \frac{4\pi GM^2}{\hbar};
\]

(2)

also known as the Hawking temperature and the Bekenstein–Hawking entropy [1,2]. Here, \(M\) is the mass of the black hole (or, alternatively, its Schwarzschild radius divided by \(2G\)) and all non-explicit fundamental constants have conveniently been set to unity.

As an immediate consequence of this captivating framework, black holes possess a negative specific heat

\[
\frac{dM}{dT} = -\frac{8\pi GM^2}{\hbar} < 0
\]

(3)

— meaning that the black hole must become progressively hotter as its mass diminishes. If one extrapolates this trend to zero mass, the temperature diverges and the ultimate outcome would appear to be an apocalyptic-like explosion.

This last conclusion is, however, rather naive. Semi-classical intuitions can only be extended so far, and one expects quantum-gravitational effects to intervene at somewhere around the Planck scale (\(i.e.,\) near lengths of order \(L_p = \sqrt{\hbar G}\) or masses of order \(M_p = \sqrt{\hbar/G}\)). On the other hand, it is hard to envision how quantum gravity can step in at (close to) the proverbial last moment and suddenly halt what is essentially a runaway process. Which is to say, one might anticipate some type of semi-classical “braking” mechanism to be in order. This would presumably act to (at least) slow down the rate of black hole radiation; at a time well before a Planck-order mass has been attained. Finally, upon reaching the Planck scale, the slowed-down rate of evaporation would allow for a smooth transition from the semi-classical to the quantum-gravitational realms. In this way, quantum gravity can gracefully take over and do whatever it does to avoid a final singularity.\footnote{Let us suppose that there is no braking mechanism; then there would have to be an abrupt phase transition between the two realms. This is, of course, not unfeasible but would be aesthetically undesirable.}

So let us restrict ourselves to semi-classical physics and then make the natural query: what braking mechanism?! For this question, there is no convincing nor definitive answer.\footnote{The idea that black hole radiation could stop prematurely does have a history (\(e.g.,\) [3,4]); with the primary motivation being its potential to resolve the black hole information loss paradox [5, 6]. We would argue, however, that a precise account of just how this could happen is still lacking. Moreover, we will contend (near the end of the letter) that, in deciding between a fully stopped and a slowed-down evaporation, the latter scenario is actually the preferable one.} But we can help fill in this gap by way of an observation that follows from a
recent conjecture put forward by Arkani-Hamed, Motl, Nicolis and Vafa (henceforth, to be known as AMNV) [7]. Before detailing our observation, let us briefly discuss the salient points of the cited work.

AMNV actually make two conjectures of relevance, with the second (the one of current interest) being a direct consequence of the first:

*Conjecture 1:* Given a U(1) gauge coupling with a coupling strength of $g$, there must exist a “sufficiently light” charged particle such that the mass ($m$) of this particle satisfies

$$m < g \cdot M_p.$$  \hspace{1cm} (4)

*Motivation for 1:* The existence of such a particle ensures that extremal black holes are able to decay and thus avoids the problems that are inherent to the stability of highly entropic objects. To elaborate, if such black holes were indeed stable, there would be an extremely large entropy [$\ln(1/g)$ with typically $g << 1$] associated with a near Planck-sized object. In this event, one would expect virtual extremal black holes to dominate every conceivable scattering process, and we would have a catastrophe that is tantamount to the so-called black hole remnant problem [3,14]. [In the conventional remnant problem, the highly entropic but stable objects would be the conjectural entities that survive after black hole evaporation has (prematurely) terminated.]

*Conjecture 2:* An effective field theory that is relevant to the coupling $g$ must break down at (or below) an energy scale $\Lambda$ such that

$$\Lambda < g \cdot M_p.$$  \hspace{1cm} (5)

*Motivation for 2:* Conjecture 2 follows from conjecture 1 by way of the following heuristic argument. For a magnetic monopole (or an analogue thereof), equation (4) may be recast as

$$m < \frac{1}{g} \cdot M_p.$$  \hspace{1cm} (6)

Meanwhile, if the field theory has an ultraviolet cutoff of $\Lambda$, then the (otherwise divergent) monopole mass is expected to be of the order

$$m \sim \frac{\Lambda}{g^2}.$$  \hspace{1cm} (7)

Combining equations (6) and (7), one finds that the bound of equation (5) trivially follows. Let us also mention here that the AMNV notion of “gravity as the weakest force”

\footnote{For related discussion, the reader may also refer to [8–12] and, especially, [13].}

\footnote{As carefully stipulated in [7].}

\footnote{That is, maximally charged black holes for which $M = QM_p$ (with $Q$ being the gauge charge of the black hole in units of $g$).}

\footnote{The current author finds the second conjecture to be somewhat less compelling than the first. Nevertheless, the AMNV paper has stirred sufficient interest to merit a treatment such as ours, irrespective of any given author’s or reader’s personal opinion.}
follows from the gravity force (with “coupling” \( m \)) being overwhelmed by the gauge force when expressed in Planck units (\( m < g \)).

We will now, as advertised, proceed to exploit the AMNV conjectures (in particular, the second one) in the context of black hole evaporation.\(^8\) Let us remind the reader that the discussion is to be kept at the level of semi-classical physics, so that conventional quantum-mechanical reasoning and effective field-theoretic descriptions should both apply.

First of all, a necessary prerequisite for a particle to be radiated by a black hole is the capability of “fitting inside”. To elaborate, let us suppose that some particle spatially extends far outside the black hole (and we will appropriately use its Compton wavelength as a measure of this extent), while a second particle is localized entirely inside the horizon. Then, by simple probability arguments, the former (delocalized) particle has much less chance of interacting with the black hole gravitational field. (Obviously, such an interaction much precede the process of radiation.) It thus follows that the Compton length of a radiated particle should be (roughly) bounded by the Schwarzschild radius of the black hole,\(^9\) or\(^10\)

\[
\frac{\hbar}{m} < 2GM. \tag{8}
\]

Next, applying \( m < \Lambda \) (as must be the case given an effective field-theoretical description) and \( M_p^2 = \hbar/G \), we have

\[
\frac{1}{\Lambda} < \frac{M}{M_p^2}. \tag{9}
\]

We will now bring the gravity-as-the-weakest-force conjecture in the guise of equation \((5)\) into play. The inversion \( 1/gM_p < 1/\Lambda \) allows us to rewrite equation \((9)\) in the following way:

\[
\frac{1}{g} < \frac{M}{M_p}. \tag{10}
\]

or, more succinctly, in Planck units,

\[
\frac{1}{g} < M. \tag{11}
\]

\(^8\)By agreeing to work with the Schwarzschild model, we are really insisting that the black hole is (essentially) neutral with respect to all gauge charges that are implied by this discussion. Fortunately, from a realist’s perspective, such neutrality would be the normal state of affairs.

\(^9\)Alternatively, this is just a restatement of the following fact \([2]\): the major contribution to Hawking radiation comes from particles with wavelengths that are peaked around the inverse of the Hawking temperature — that is, peaked about the Schwarzschild radius \([\text{cf. equation}(1)\]).

\(^10\)One might be concerned over the values inserted into equation \((8)\), given that length (energy) scales are extremely red (blue) shifted in the proximity of a black hole horizon. For our discussion, however, the relevant observer is asymptotically situated, as this is the type of observer who would detect thermal emissions from a black hole. Significantly, such an observer would attribute the lengths in question with the values as given in equation \((8)\).
We have finally arrived at the crux of the matter. As the black hole radiates away particles, $M$ obviously becomes progressively smaller. Hence, as the evaporation proceeds, the bound on $1/g$ becomes tighter and tighter. (Recall that, typically, $g << 1$, so that $1/g >> 1$.) Meaning, the number of species of particles that the black hole can emit is monotonically decreasing throughout the evaporation. To put it another way, light-massed black holes will only be able to emit the most strongly interacting (and, presumably, rarest) species of particles. Hence, we have a natural mechanism that — not only inhibits the black hole radiative process but — works at direct cross purposes to the destabilizing effect of the negative specific heat. Let us suppose that this species-drop-off effect is sufficiently competitive with the (otherwise) accelerating evaporation; sufficient enough so that the black hole can remain relatively stable upon its approach to the Planck scale. Then we have identified precisely the type of braking mechanism that was advocated earlier in the letter.

One might wonder if our rigorous restriction to a semi-classical framework is too stringent a condition. After all, the immense gravitational attraction of a black hole coupled with the intrinsically quantum process of it radiating is strongly suggestive of a quantum-gravitational realm. But here is the vital point: The process of black hole radiation is, as we best understand it, based solely on semi-classical reasoning. Most notably, Hawking’s original derivation [2] in the context of (“standard”) quantum field theory in curved spacetime [17]; although other semi-classical treatments have deduced black hole thermality with the same value for the temperature (e.g., [18–22]). Hence, once quantum gravity is explicitly needed in the discussion, we have no compelling reason to believe that the Hawking radiative process survives in any recognizable form.

To summarize, we have argued that the hypothesis of “gravity as the weakest force” (as conjectured by Arkani-Hamed et. al. [7]) implies a mechanism that competes directly with the negative specific heat of a shrinking black hole. This mechanism — which is based on limiting the number of particle species available for black hole emission — should be able to slow down an otherwise runaway evaporation process. Assuming this braking is strong enough, we have anticipated a smooth transition from the semi-classical to the quantum-gravitational regimes as the black hole mass descends towards the Planck scale.

Even more ambitiously, our conjectured mechanism could halt the process of black hole evaporation altogether; thus resolving the conundrum of what happens to the information that was trapped inside of a black hole after it evaporates — the so-called information loss paradox [3, 5, 6]. On the other hand, because of the previously mentioned remnant problem (i.e., the existence of small stable objects with extremely large entropy would wreak havoc on low-energy physics), it is not clear that this is a favorable resolution. In fact, given the context of the current discussion, it is clearly not favorable. That is, if the evaporation stopped completely, then the AMNV conjecture would simply be trading off

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11 Having said that, we should acknowledge the progress made in understanding black hole radiation in the context of string theory [15] and loop quantum gravity [16]. These are, however, highly model-specific methodologies.

12 Interestingly, the same basic rationale has been used to argue generically against black hole radiation [23]. The quantum-gravitational sticking point is, in this case, the trans-Planckian energies that unavoidably arise when one traces the outgoing particles back to the horizon.
one remnant-type problem (extremal black holes) for another (Schwarzschild remnants).

As a final note, let us point out that such a braking mechanism provides the potential for a experimentally verifiable signature of the gravity-as-the-weakest-force paradigm.

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References


