Bound Electron Screening Corrections to Reactions in Hydrogen Burning Processes

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Abstract

We estimate the bare astrophysical S-factor ($S_b(E)$) of reactions in PP-chains, through a polynomial expression with the adiabatic enhancement factor due to the electron screening. The obtained $S_b$ is significantly different from the simple extrapolation from high energy data, however $S_b$ at zero incident energy is in agreement with the results of a recent R-matrix analysis.

Key words: Astrophysical S-factor; Electron screening in the laboratories

1 Introduction

The series of reactions which convert hydrogen into helium on stellar site is known as the proton-proton chains. It is a key to understand the evolution of the stars. These reactions are measured at laboratory energies and are then extrapolated to thermal energies\cite{1}, because of their small cross sections at such low energies. This extrapolation is done by introducing the astrophysical S-factor:

$$S(E) = \sigma(E)Ee^{2\pi\eta(E)},$$

(1)

where $\sigma(E)$ is the reaction cross section at the incident center-of-mass energy $E$ and $\eta(E) = Z_TZ_P\alpha\sqrt{\frac{mc^2}{2E}}$, $Z_T$, $Z_P$, $\mu$ denoting the atomic numbers and the reduced mass of the target and the projectile. $\alpha$ and $c$ are the fine-structure constant and the speed of light, respectively. The exponential term in the equation represents the Coulomb barrier penetrability. Since one has factored out the strong energy dependence of $\sigma(E)$ due to the barrier penetrability, the S-factor could be approximated by a polynomial expression in the absence of low-energy resonance. In laboratory experiments, the targets are usually in
gas or solid state. There, in the low energy region, the S-factors obtained from experiments show large enhancement to the extrapolation from high energy data for various reactions [2]. This enhancement is, usually, attributed to the screening by the bound electrons around the target. In contrast, in the stellar nucleosynthesis, nuclei are almost fully ionized and are surrounded by the plasma electrons. The nuclear reactions in such a circumstance are affected by a different mechanism of the plasma electron screening. Hence the screening effect of the bound electrons should be removed from the S-factor data, in order to assess the reaction rate in the stellar site correctly. The enhancement by the bound electrons is discussed in terms of a constant potential shift (screening potential $U_e$). However, a puzzle had been that the experimentally observed enhancements are systematically and significantly larger than the adiabatic limit, which is given by the difference of the binding energies of the target atom and the united atom and is thought to give the upper limit of the screening enhancement theoretically.

On this issue, the dynamical effect to this problem has been studied by Caltech group [3]. They followed the time evolution of the atomic wave function in the classical allowed region by solving the time dependent Hartree-Fock equation and evaluated the screening potential. Their results suggest that the screening potential approaches the adiabatic limit as the incident energy becomes lower.

The influence of the tunneling phenomenon to this problem has been studied as well [4]. And, there, the screening potential could go over the above-mentioned adiabatic limit, only in the case where the electronic wave-function has some excited state components at the classical turning point of the inter-nuclear motion. This excess compared with the adiabatic limit is, nevertheless, too small to explain the large discrepancies of all the reactions. We have examined the problem using molecular dynamics approach with constraints [5, 6], to see the effect of the fluctuations in our previous studies [7, 8]. The obtained average enhancement factors, again, do not exceed the adiabatic limit, there are events which give larger enhancement factors than that in the adiabatic limit. There are other attempts [2, 9, 10] to explain the mechanism to get such a large enhancement, over the adiabatic limit, however none of them is affirmative to the screening potential which goes over the adiabatic limit.

In this connection, we mention that the difficulty lies in the determination of the bare S-factor. Recall that the bare S-factor is usually determined by extrapolations from high energy data and then the screening potential is determined by taking the ratio of the data and extrapolated S-factor. Instead, Barker [11] performed the fit including the whole data using either a polynomial (quadratic or cubic) or R-matrix determining the parameters simultaneously and he obtained more consistent values of screening potentials to the adiabatic limit for some reactions. Experimentally, the Catania group tried to extract the bare cross section using the Trojan Horse method (THM) [12, 13, 14].
In this paper, we determine the bare $S$-factors ($S_b(E)$) of the reactions, especially in hydrogen burning process, through the fitting of experimental data making use of the polynomial expression with the screening enhancement in the adiabatic limit. The results are compared with the extrapolated $S_b(E)$ by polynomial expressions from high energy data. The obtained $S_b$ at zero incident energy are compared with the results of the R-matrix analyses [15].

This paper is organized as follows, In Sec. 2 we describe the enhancement factor by the bound electrons within the adiabatic limit briefly. We list up the reactions in PP-chains in Sec. 3 and explain how we incorporate the adiabatic limit into the fitting procedure. Some reactions are analyzed in this section. We summarize the paper in Sec. 4.

2 Enhancement factor in the adiabatic limit

In order to discuss the enhancement quantitatively, we determine the enhancement factor:

$$f_e = \frac{\sigma(E)}{\sigma_0(E)},$$

in terms of the real cross section $\sigma(E)$ and the bare cross section $\sigma_0(E)$. If one assumes that the effect of the electron screening can be represented by the constant shift $U_e$(screening potential) of the potential barrier, the enhancement factor is approximated by,

$$U_e \sim \frac{E}{\pi \eta(E)} \log f_e.$$  

The $U_e$ can be estimated easily in two limiting cases. One is the case where the inter-nuclear velocity is much higher than that of electrons velocity, i.e. at the sudden limit. Within this limit the electron wave function is frozen during the reaction. In the opposite case where the inter-nuclear motion is much slower than electrons motion, the bound electrons follow the motion of the nuclei adiabatically. Within this adiabatic limit the screening potential is expressed by the difference of the binding energies between the initial target atom($BE_T$) and the united atom($BE_{UA}$) which is formed during the reaction.

$$U_e^{(AD)} = BE_T - BE_{UA}$$

The screening potential within this limit gives the upper limit.
Table 1
Reactions in PP-chains, their minimum incident energy in the center-of-mass system measured so far and the enhancement factor within the adiabatic limit at the minimum energy.

<table>
<thead>
<tr>
<th>reactions</th>
<th>$E_{\text{min}}$ [keV]</th>
<th>$f_e^{(AD)}(E_{\text{min}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{H}(p, \beta^+\nu_e)D$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{D}(p, \gamma)^3\text{He}$</td>
<td>2.52</td>
<td>1.07</td>
</tr>
<tr>
<td>$^3\text{He}(^3\text{He},2p)^4\text{He}$</td>
<td>20.76</td>
<td>1.22</td>
</tr>
<tr>
<td>$^3\text{He}(\alpha,\gamma)^7\text{Be}$</td>
<td>107.2#,, 127.*</td>
<td>1.02</td>
</tr>
<tr>
<td>$^7\text{Be}(e^-,\nu_e)^7\text{Li}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^7\text{Li}(p, \alpha)^4\text{He}$</td>
<td>12.7, 10.**</td>
<td>1.18</td>
</tr>
<tr>
<td>$^7\text{Be}(p, \gamma)^8\text{B}$</td>
<td>115.6</td>
<td>1.01</td>
</tr>
</tbody>
</table>

# prompt-$\gamma$ method   *activation method   **THM

3 Bare S-factors of PP-chain reactions

A list of reactions in PP-chains are shown in Table 1.

The first reaction $\text{H}(p, \beta^+\nu_e)D$ involves the $\beta$-decay and has too small cross section to be measured experimentally. Its S-factor is calculated from first principle [16]. We, therefore, concentrate on the other 5 reactions except the electron capture reaction $^7\text{Be}(e^-,\nu_e)^7\text{Li}$. In the table the minimum incident energies, measured so-far, for each reaction are also shown. For three reactions $\text{D}(p, \gamma)^3\text{He}$, $^3\text{He}(^3\text{He},2p)^4\text{He}$ and $^7\text{Li}(p, \alpha)^4\text{He}$ cross sections have been measured already including the low energy region. The other two reactions are proton or $\alpha$ capture reactions which have even smaller cross sections. The S-factor of the reaction $^3\text{He}(\alpha,\gamma)^7\text{Be}$ has been re-determined with high precision recently by detecting $\gamma$-ray from $^7\text{Be}$ [17]. Its S-factor in the low energy region is extrapolated from high energy data by R-matrix fitting. The reaction $^7\text{Be}(p, \gamma)^8\text{B}$ involves unstable nuclei. The S-factor of this reaction has been determined by means of the direct capture reaction [18, 19] and the Coulomb dissociation method [20] and it is one of questions under discussion that there is a discrepancy between the results by two methods [19, 21].

The previous studies of the electron screening effect suggests that the enhancement factor cannot be over the adiabatic limit. We, therefore, adopt the enhancement factor within the adiabatic limit:

$$f_e^{(AD)} = e^{\pi\eta(E)}\frac{\eta^{(AD)}(E)}{\eta(E)}$$  (5)
Fig. 1. S-factor for the reaction $^7\text{Li}(p, \alpha)^4\text{He}$ as a function of the incident center-of-mass energy. The experimental points are from [22] (Cassagnou62), from [23] (Rolfs86), from [24, 25] (Engstler92) and from [13] (Lattuada01).

and determine the bare S-factor by fitting the experimental data of the reactions. The fit of the experimental data is performed by assuming a polynomial expression for bare S-factor:

$$S(E) = S_b(E) \cdot f^{(AD)}_e; \quad S_b(E) = S_b(0) + S_1 E + S_2 E^2 + S_3 E^3,$$

using un implementation of the nonlinear least-squares Marquardt-Levenberg algorithm.

3.1 $^7\text{Li}(p, \alpha)^4\text{He}$

In Fig. 1 the S-factor of the reaction $^7\text{Li}(p, \alpha)^4\text{He}$ from several direct measurements are shown with error bars. Extracted S-factor data by THM are especially shown with the closed squares. [13] We performed fitting of the data by direct measurements in the incident energy region higher than 100keV using a cubic polynomial. In this energy region the screening enhancement is estimated to be 1% at utmost. The fitting parameters are shown in the first row of the table 2. The corresponding S-factor is shown with the thin solid curve in the figure 1. The curve supposed to give an extrapolation of low energy data, however it strays away from the trend of experimental data in the lower energy region. Instead of fitting higher-energy data, if we fit the whole data by the direct measurements in the form of Eq. 6 and $U_e^{(AD)} = 175\text{eV}$ [8], we obtain the fitting parameters in the second row of the table 2. The corresponding
Table 2
Fitting parameters of the reaction $^7\text{Li}(p,\alpha)^4\text{He}$ in the high energy region (the first row) and in the whole energy region (the second row).

<table>
<thead>
<tr>
<th></th>
<th>$S_b(0)$ [MeVb]</th>
<th>$S_1$ [b]</th>
<th>$S_2$ [MeV$^{-1}$b]</th>
<th>$S_3$ [MeV$^{-2}$b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_b(E)$</td>
<td>0.080 ± 0.008</td>
<td>0.04 ± 0.03</td>
<td>-0.06 ± 0.03</td>
<td>0.067 ± 0.006</td>
</tr>
<tr>
<td>$S_b(E) \cdot f_e^{(AD)}$</td>
<td>0.066 ± 0.002</td>
<td>0.08 ± 0.01</td>
<td>-0.10 ± 0.01</td>
<td>0.076 ± 0.004</td>
</tr>
</tbody>
</table>

S-factor is shown with the thick dashed curve in Fig. 1. Note that there is a big difference between these two curves. The extrapolation of the S-factor to the lower energy region should be done, therefore, with caution.

In Ref. [15] R-matrix fitting for higher energy region ($E > 40\text{keV}$) has been used to determine the S-factor. They obtained the zero-energy S-factor 0.067 ± 0.004 [MeVb] and the screening potential $U_e = 100\pm 25\text{eV}$, which is less than that within the adiabatic limit. The S-factor at zero energy from our result $S_b(0)=0.066 \pm 0.002 [\text{MeVb}]$ is in agreement with this result from R-matrix fitting but higher than $S_b(0)=0.055\pm 0.003[\text{MeVb}]$ by THM [13].

3.2 $^3\text{He} (^3\text{He},2p)^4\text{He}$

The S-factor of the reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$ from several measurements are shown with error bars in Fig. 2. At the minimum incident energy, which has been reached in an experiment by the LUNA collaboration [26], the screening enhancement is estimated to be more than 20%. In the case of the reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$, $^3\text{He}$ projectiles are likely to be $^3\text{He}^+$ or charge neutral state in the target medium. For the $^3\text{He}$ neutral projectile the adiabatic screening potential is $U_e^{(AD)} = 246.8 \text{eV}$ [26].

$$f_e^{(AD)}(^3\text{He}) = e^{\pi \eta(E) U_e^{(AD)} E}.$$  \hspace{1cm} (7)

For the $^3\text{He}^+$ projectile the adiabatic screening potential is calculated considering the charge symmetry of the system [30]. $U_e^{(AD)1} = 255.5 \text{eV}$ and $U_e^{(AD)2} = 122.2 \text{eV}$ in the cases where the system ends up with $^6\text{Be}^+(1s)^2(2s)$ state and $^6\text{Be}^+(1s)(2p)^2$ state respectively. The corresponding enhancement factor within the adiabatic limit is written as [4]

$$f_e^{(AD)}(^3\text{He}^+) = \frac{1}{2} \left( \exp \left[ \pi \eta(E) \frac{U_e^{(AD)1}}{E} \right] + \exp \left[ \pi \eta(E) \frac{U_e^{(AD)2}}{E} \right] \right).$$ \hspace{1cm} (8)

The results of fitting are shown in the table 3. The parameters in the second row are for $^3\text{He}$ neutral projectile and ones in the bottom row are for $^3\text{He}^+$.
Fig. 2. S-factor for the reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$ as a function of the incident center-of-mass energy. The experimental points are from [27](Backer67), from [28](Dwarakanath71), from [29](Krauss87) and from [26](Junker98).

Table 3
Fitting parameters of the reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$.

<table>
<thead>
<tr>
<th></th>
<th>$S_b(0)$ [MeVb]</th>
<th>$S_1$ [b]</th>
<th>$S_2$ [MeV$^{-1}$b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_b(E)$</td>
<td>$5.03 \pm 0.07$</td>
<td>$-1.9 \pm 0.2$</td>
<td>$0.75 \pm 0.08$</td>
</tr>
<tr>
<td>$S_b(E) \cdot f_e^{(AD)}(^3\text{He})$</td>
<td>$5.31 \pm 0.05$</td>
<td>$-2.6 \pm 0.2$</td>
<td>$1.0 \pm 0.1$</td>
</tr>
<tr>
<td>$S_b(E) \cdot f_e^{(AD)}(^3\text{He}^+)$</td>
<td>$5.41 \pm 0.06$</td>
<td>$-2.8 \pm 0.3$</td>
<td>$1.1 \pm 0.1$</td>
</tr>
</tbody>
</table>

projectile. The corresponding curve for $^3\text{He}$ neutral projectile case are shown in Fig. 2 together with the experimental points. We, again, see the difference between the extrapolation from high energy data (the top row in Tab. 3, the thin solid curve in Fig. 3) and the bare S-factor obtained by fitting the whole data, including low energy region (the bottom row in Tab. 3, the thick dashed curve in Fig. 3). It is clear that if we derive the screening potential by comparing the dashed and the thin curves, as it is often done in the previous studies, its screening potential will be larger than that in the adiabatic limit.

3.3 $D(p,\gamma)^3\text{He}$

The S-factor of the reaction $D(p,\gamma)^3\text{He}$ from several measurements are shown with error bars in Fig. 3. At the minimum incident energy, which has been reached in an experiment by the LUNA collaboration [31], the screening enhancement is estimated to be 7% at utmost. This enhancement within the
Fig. 3. S-factor for the reaction D(p,\γ)\(^3\)He as a function of the incident center-of-mass energy. The experimental points are from [32](Griffiths62), from [33](Warren63), from [34](Berman64), from [35](Wolfli67), from [36](Schmid95) and from [31](Casella02).

Table 4
Fitting parameters of the reaction D(p, \γ)\(^3\)He.

<table>
<thead>
<tr>
<th>S(_b)(0) [eVb]</th>
<th>S(_1) [b]</th>
<th>S(_2) [eV(^{-1})b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.217 ± 0.029</td>
<td>6.01 ± 0.25</td>
<td>3.04 ± 0.15</td>
</tr>
</tbody>
</table>

The obtained fitting parameters for the bare S-factor are shown in the table 4 and its is shown with the thick dashed curve in Fig. 3. The curve departs from the extrapolation, which is shown by the thin solid curve, using high energy data (E ≥ 100keV). Since the enhancement is less than 7% even at the lowest measured incident energy, it changes insignificantly the zero-energy S-factor: S(0)=0.220 ± 0.030[eVb] obtained by neglecting the enhancement differs only slightly from the bare S-factor at zero-energy S\(_b\)(0)=0.217 ± 0.029[eVb] from our fitting procedure. S\(_b\)(0)=0.217 ± 0.029[eVb] is consistent with the result from R-matrix fit in Ref. [15], S\(_b\)(0)=0.223±0.010[eVb], which
Fig. 4. S-factor for the reaction $^3\text{He}(\alpha, \gamma)^7\text{Be}$ as a function of the incident center-of-mass energy. The experimental points are from [37] (Parker63), from [38] (Kraewinkel82), from [39] (Osborne82), from [40] (Hilgemeier88) and from [17] (Bemmerer06).

Table 5
Fitting parameters of the reaction $^3\text{He}(\alpha, \gamma)^7\text{Be}$.

<table>
<thead>
<tr>
<th>$S_b(0)$ [keVb]</th>
<th>$S_1$ [b]</th>
<th>$S_2$ [keV$^{-1}$b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_b(E) \cdot f_e^{(AD)}(^4\text{He})$</td>
<td>0.49 ± 0.01</td>
<td>-0.30 ± 0.02</td>
</tr>
</tbody>
</table>

is obtained as a sum of M1 and E1 contributions, within the error-bars, despite they have not taken into account the screening enhancement in their analysis.

3.4 $^3\text{He}(\alpha, \gamma)^7\text{Be}$

The screening potential for the reaction $^3\text{He}(\alpha, \gamma)^7\text{Be}$ is estimated in the same way with the reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$. The estimated enhancement at the minimum incident energy within the adiabatic limit is 2% at utmost. In Fig. 4 the experimental data are shown with crosses, squares, triangles with error-bars. The thick curve is again the result of fitting by the polynomial expression and its fitting parameters are shown in Tab. 5. The S-factor at zero-energy $S_b(0)=0.49 ± 0.01$[keVb] from our procedure is in agreement with the result from R-matrix fitting $0.51±0.04$[keVb] in Ref. [15]. On account of consideration of the screening enhancement, our $S_b(0)$ is only slightly lower than the result of R-matrix fitting.
Fig. 5. S-factor for the reaction \(^7\)Be\((p,\gamma)\)^8\(\)B as a function of the incident center-of–mass energy. The experimental points are from \([19]\) (Junghans03).

### 3.5 \(^7\)Be\((p,\gamma)\)^8\(\)B

The reaction \(^7\)Be\((p,\gamma)\)^8\(\)B is a key process to produce the high energy solar neutrino through the \(\beta\)-decay of \(^8\)\(\)B and its S-factor is studied intensively by many groups by means of the direct capture\(\text{(DC)}\) reaction \([18, 19]\), the indirect Coulomb dissociation\(\text{(CD)}\) method \([20]\) and the asymptotic normalization coefficients\(\text{(ANC)}\) \([41]\). The experimental data of the S-factor by CD experiments give steeper energy dependence than that by DC experiments in the low energy region and the lower zero-energy S-factor as an average. \([19]\) This difference could be reduced by reanalyzing CD data including corrections to the far-field approximation and the dynamic polarization. \([21]\)

In addition to this contradictory situation, there are important contributions of resonances for this reaction in the low energy region. Therefore our approach with polynomial expressions is not adequate for this case. Nevertheless, if we were to venture determining S-factor making use of our approach by fitting the recent DC experiments data in Ref. \([19]\), we obtained \(S_b(0) = 18.6 \pm 0.3\) eVb. In this procedure we have avoided the 630keV M1 resonance contribution for the purpose of the fitting and made use of the enhancement factor \((5)\) with the adiabatic screening potential \(U_e^{(AD)} = 222.0\) eV. The corresponding bare S-factor is shown with the dashed curve in Fig. 5. Our result \(S_b(0) = 18.6 \pm 0.3\) eVb is considerably smaller, more than 10\%, than the zero-energy S-factor \(21.4 \pm 0.5\)(\text{expt}) \pm 0.6(\text{theor})\)eVb given as a mean of all modern direct measurements in Ref. \([19]\). The screening enhancement factor is of the order of 1\% at the minimum incident energy of the experiment in \([19]\).
4 Conclusions

We discussed the bound electron screening corrections to the S-factors of the reactions in PP-chains. We have assumed the screening enhancement within the adiabatic limit and have determined the bare S-factors for the reactions through the polynomial fitting of the experimental data. The obtained bare S-factors are significantly different from the simple extrapolation from high energy region, but $S_b$ at the zero incident energy is in agreement with the results from R-matrix fitting in Ref. [15], except the case of the reaction $^7$Be($p, \gamma$)$^8$B. We conclude that the polynomial expression of the bare S-factor for reactions without resonant component in the low energy region is valid, only if the experimental S-factor data in this region are available and one considers the screening effect properly.

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References


