Star Clusters with Primordial Binaries: III. Dynamical Interaction between Binaries and an Intermediate Mass Black Hole

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ABSTRACT

We present the first study of the dynamical evolution of an isolated star cluster that combines a significant population of primordial binaries with the presence of a central black hole. We use equal-mass direct N-body simulations, with $N$ ranging from $4096$ to $16384$ and a primordial binary ratio of $0 - 10\%$; the black hole mass is about one percent of the total mass of the cluster. The evolution of the binary population is strongly influenced by the presence of the black hole, which gives the cluster a large core with a central density cusp. Starting from a variety of initial conditions (Plummer and King models), we first encounter a phase, that last approximately 10 half-mass relaxation times, in which binaries are disrupted faster compared to analogous simulations without a black hole. Subsequently, however, binary disruption slows down significantly, due to the large core size. The dynamical interplay between the primordial binaries and the black hole thus introduces new features with respect to the scenarios investigated so far, where the influence of the black hole and of the binaries have been considered separately. A large core to half mass radius ratio appears to be a promising indirect evidence for the presence of an intermediate-mass black hole in old globular clusters.

Key words: stellar dynamics — globular clusters: general — methods: n-body simulations — binaries: general

1 INTRODUCTION

Over the last few years some tantalizing, but yet far from conclusive, evidence has been accumulating in support of the idea that some star clusters could harbor a central black hole (BH) with a mass of the order of $10^3 M_\odot$ or more. Detection of such an intermediate mass black hole (IMBH) has been claimed for $M15$ and $G1$ (Gerssen et al. 2003; Gebhardt et al. 2002, 2005). However, alternative dynamical models without a central BH have been proposed for these clusters (Baumgardt et al. 2003, 2004). Interestingly, the visual appearance of globulars containing an IMBH is not that of a so-called core-collapsed cluster, but rather that of a cluster with a still sizable core (Baumgardt et al. 2005).

IMBHs present a high theoretical and observational interest as these could be potential ultra-luminous X-ray sources and even emit gravitational waves, detectable by the next generation of gravitational wave detectors, as a result of close interactions with stars. However, despite this interest and the fact that theoretical studies of BHs in stellar systems started more than 30 years ago (e.g., see Peebles 1972; Bahcall & Wolf 1976), detailed direct N-body simulations to study the dynamics of an idealized model with single stars and a central BH have been performed only recently (Baumgardt et al. 2004) and see also the studies on the formation of IMBHs by runaway mergers of massive stars by Portegies Zwart et al. 2004).

One ingredient that complicates N-body simulations of globular clusters is the presence of primordial binaries. Often, these are neglected in large simulations despite the increasing observational evidence that many stars in a globular cluster have a companion (Hut et al. 1992; Albrow et al. 2001; Bellazzini et al. 2002; Zhao & Bailyn 2005). This frequent neglect is due to the dramatic increase in computational resources required in a simulation where the local dynamical timescale may be many orders of magnitude smaller than the global relaxation timescale (hard binaries have an orbital period of a few hours, while the half-mass relaxation time can be up to a few billion years). The study of the dynamics in the presence of primordial binaries has been mainly limited to Fokker-Planck or Monte Carlo approaches (Gao et al. 1991; Giersz & Spurzem 2000; Fregeau et al. 2003) and to direct simulations with rather modest particle numbers, from $N \approx 10^3$ (McMillan et al. 1990; McMillan & Hut 1994; Heggie & Aarseth 2000).
to recent higher resolution simulations, with \( N \) up to \( 16384 \) (Heegre, Trenti & Hut 2006; Trenti, Heegre & Hut 2006, hereafter PaperI and PaperII respectively). The results of these direct simulations have to be extrapolated to a realistic number of particles (e.g., by means of the Vesperini & Chernoff (1994) model) before being applied to the interpretation of the evolution of typical globular clusters, that have \( 10^5 - 10^6 \) stars. Some realistic simulations, including primordial binaries are available (Portegies Zwart & McMillan 2004), but these are limited only to the first stage of the life of young dense clusters. In the case of open clusters, M67 has been modeled in a 36,000-body simulation running for several Gigayears (Hurley et al. 2005).

The presence of either an IMBH or a significant population of primordial binaries leads to an early release of abundant energy, inhibiting the development of a deep core collapse and hence of the onset of gravothermal oscillations. An IMBH can generate energy by swallowing or tightly binding stars deep in its potential well [note also that energy can be generated through encounters of stars in the density cusp that is formed around the BH], while primordial binaries can generate energy by rapidly increasing binary binding energy through three and four body encounters. Energy thus generated in the core of the system fuels the expansion of the half-mass radius, leading to a self-similar expansion of the entire system (Hénon 1965).

In this work we present the first direct simulations (with a number of equal-mass particles up to \( N = 16384 \)) of the evolution of star clusters with both a significant fraction of primordial binaries (10%) as well as an IMBH with a mass of \( 0.8 - 3 \% \) of the total mass of the system. We address the following questions. Under the combined effect of the BH and of the binaries, what is the equilibrium size of the cluster core? How is the binary population affected by the presence of the central IMBH? Which physical processes dominate in the core? In principle two competing effects are possible: the BH may either enhance the disruption rate of binaries both indirectly, due to the creation of a density cusp, and directly, by tidal stripping (Pringle 2005), or it may reduce the probability of interactions between binaries and singles, by producing a low stellar density in a relatively large core (Baumgardt et al. 2004a). Which process is dominant is of fundamental importance. These questions are addressed in the next sections.

2 NUMERICAL SIMULATIONS: SETUP

Like in PaperI and PaperII, the simulations (see Table 1) have been performed using the NBODY-6 code (Aarseth 2003). For this project NBODY6 has been modified with the kind help of Dr. Aarseth to ensure a more efficient and accurate treatment of the dynamics around the BH by fine tuning the parameters controlling chain regularization.

2.1 Units

All our results are presented using standard units (Heegre & Mathie 1986) in which

\[
G = M = -4E_T = 1
\]

where \( G \) is the gravitational constant, \( M \) is the total mass, and \( E_T \) is the total energy of the system of bound objects. In other words, \( E_T \) does not include the internal binding energy of the binaries, only the kinetic energy of their center-of-mass motion and the potential energy contribution where each binary is considered to be a point mass. We denote the corresponding unit of time by

\[
t_d = GM^{3/2}/(-4E_T)^{3/2} \equiv 1.
\]

For the relaxation time, we use the following expression (Spitzer 1987, Giersz & Hegre 1994):

\[
t_{rh} = \frac{0.138N r_h^{3/2}}{\sqrt{GM \ln (0.11N)}},
\]

where \( r_h \) is the half-mass radius and \( N \) denotes the number of original objects (binaries + singles).

The core radius of the system is defined as the density averaged radius:

\[
r_c = \sqrt{\frac{\sum_{n=1}^N \rho_n^2}{\sum_{n=1}^N \rho_n^2}}.
\]

2.2 Initial conditions

The models considered in this paper are isolated with stars of equal mass \( m \), plus a BH, introduced as a massive star, with mass \( (m_{BH}) \) in the range \( 0.8 - 3 \% \) of the total mass of the system. The initial distribution is either a Plummer model (entries “PL” in Tab. 1) or a King model (entries “K” in Tab. 1) with concentration index \( W_0 = 3, 5, 7, 11 \). Our standard models have a primordial binary ratio of \( 0 - 10 \% \).

We define the primordial binary fraction as:

\[
f = n_b/(n_s + n_b)
\]

with \( n_s \) and \( n_b \) being the number of singles and binaries respectively.

We have considered runs with \( N \) in the range \( 4096 - 16384 \). Note that \( N \) denotes the number of original objects, i.e. \( N = n_s + n_b \), while the total number of stars is \( N_{tot} = n_s + 2n_b \); when we discuss a run with \( N = 8192 \) and 10% primordial binaries we are dealing with 9011 stars.

The initial binding energy distribution for the binaries is in the range 5 to \( 680 \) \( kT \), flat in logarithmic scale, similar to the initial conditions considered in the studies focused on the evolution of star clusters with primordial binaries, but without an IMBH (Gao et al. 1991; Fregeau et al. 2003; PaperI; PaperII). This form

\footnote{Note that with our definition of \( kT \) we are considering the same range of energy per particle.}
for the binding energy distribution is suggested by the observed properties of binaries in star clusters (Hut et al. 1992) and the upper and lower limits are physically motivated: very soft binaries, i.e. binaries with binding energies below a few kT, would be quickly destroyed (i.e. on a timescale below one relaxation time) due to three and four body encounters in the core of the the system; binaries harder than a few hundreds kT are, on the other hand, dynamically inert.

To initialize the simulation in a situation of approximate dynamical equilibrium in the presence of the BH we first generate our chosen initial configuration (Plummer or King model) made of $N_{\text{tot}}$ single stars only and we add the IMBH at rest at the center of the system. We then scale the velocities of the particles to dynamical equilibrium by experiencing mild virial oscillations (with amplitude below 6% of the equilibrium virial ratio) that are damped in crossing times $t_d$. At this stage we select randomly $n_b$ single stars that are eliminated from the simulation. Other $n_0$ single stars are randomly chosen to become binaries: for these a companion star is added with a semi-major axis ($a$) extracted from the assumed flat distribution in binary binding energy (that is translated in a distribution in $a$ proportional to $1/a$). The initial eccentricity for the binaries is chosen from a thermal distribution. At this stage the initialization is complete with the system in approximate dynamical equilibrium.

Our initial configurations have a limited choice of initial density profiles (Plummer model and King profiles with $W_0 = 3, 5, 7, 11$). This does however not influence or bias significantly the long-term, collisional evolution of the system. In fact, as we have shown in PaperII (see also Sec. 3.3, the memory of initial conditions is erased within a few half mass relaxation times. Runs starting from concentrated King models initially presents a core expansion, while runs starting with shallower cores have an initial contraction, with both processes leading to a common dynamical configuration that can be interpreted in terms of balance between energy production in the core and energy dissipation due to half mass radius expansion (see also Vesperini & Chernoff 1994). This argument also applies to justify our choice in the initialization procedure to distribute, like in PaperII, primordial binaries so as to exactly trace the distribution of single particles, without exploring the possibility of an initial mass segregation.

The evolution of the system is then followed up to $t \approx 25 t_{\text{rel}}(0)$ that corresponds to about 20 Gya in physical units for a globular cluster with $N = 3 \cdot 10^5$, $M = 3 \cdot 10^6 M_\odot$ and $r_h = 4$ pc at $t = 0$. For $t_{\text{rel}}(0)$ we mean $t_{\text{rel}}$ computed at $t = 0$. For our initial conditions with $N = 8192$, $t_{\text{rel}}(0) \approx 112 t_d$.

At $N = 8192$ the CPU time required for this typical run is of the order of 10 days on a state of the art pc Linux workstation. As here we are interested in the collisional evolution of the system our results are expressed in terms of the relaxation time. This means that the computational complexity for direct N-body simulations, if lasting for a constant number of relaxation times, scales as $N^3/ln(0.11 N)$: the factor $N^3$ comes from the direct gravitational interaction, while the additional $N/ln(0.11 N)$ is due to the increase of the two body relaxation time with respect to the dynamical time (Heggie & Hut 2003, Chapter 3).

### 2.3 BH properties and $N$ scaling

The mass of the black hole in our runs is in the range [0.8%, 3%] of the total mass of the system, a value that is slightly higher than the expected mass for an IMBH following the $m_{\text{BH}} \sim \sigma^2$ relation (Ferrarese & Merritt 2000, Gebhardt et al. 2000, Tremaine et al. 2002) applied to a typical globular cluster. E.g. using Tremaine et al. 2002, with a velocity dispersion of 10 km/s we obtain a $m_{\text{BH}} \approx 10^5 M_\odot$, which is about 0.3% to 1% of the total mass of a typical cluster. Our choice for the ratio $m_{\text{BH}}/M$ is motivated by the limits that the current combination of hardware and software set on the number of particles that can be employed in direct simulations of gravitational systems with the combined presence of a black hole and primordial binaries.

The number of particles that we can include in one simulation is between one and two orders of magnitude less than the typical number of stars in a globular cluster, so we are forced to introduce a scaling for the mass of the black hole. In principle two possibilities are available: we can keep constant either the ratio of the black hole mass to the stellar mass or the black hole mass to the total mass of the cluster. The first choice has the advantage of keeping unchanged the local interaction of stars with the black hole, while the second allows for a better comparison with observed star clusters. With respect to the properties of the interaction between the black hole and the primordial binary population it is not clear whether there is a preferred scaling. In fact, a black hole mass which is too big with respect to the total mass of the cluster may bias the results of the simulation, since the black hole may well be more massive than the core of the system, which usually contains a few percent of the total mass (see Baumgardt et al. 2004). But similarly it can be argued that also a low ratio of the black hole to stellar mass influences the results of the simulation, as it decreases the efficiency of tidal disruption of binaries around the black hole cusp, which is proportional to $m_{\text{BH}}/M$. In addition a black hole with a ratio $m_{\text{BH}}/m \approx 50$, i.e. a 1% mass black hole in a system with $5 \cdot 10^3$ particles, has a small, but non negligible, probability of be-

<table>
<thead>
<tr>
<th>ID</th>
<th>IC type</th>
<th>$N$</th>
<th>$m_{\text{BH}}/M$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>Plummer</td>
<td>8192</td>
<td>0.014</td>
<td>0.1</td>
</tr>
<tr>
<td>Pb</td>
<td>Plummer</td>
<td>8192</td>
<td>0.014</td>
<td>0</td>
</tr>
<tr>
<td>Pc</td>
<td>Plummer</td>
<td>8192</td>
<td>0.025</td>
<td>0.1</td>
</tr>
<tr>
<td>Pd</td>
<td>Plummer</td>
<td>8192</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>Pe</td>
<td>Plummer</td>
<td>8192</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pf</td>
<td>Plummer</td>
<td>16384</td>
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<td>0.1</td>
</tr>
<tr>
<td>Pg</td>
<td>Plummer</td>
<td>16384</td>
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<td>0.1</td>
</tr>
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<td>K5a1</td>
<td>King $W_0 = 5$</td>
<td>8192</td>
<td>0.014</td>
<td>0.1</td>
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<tr>
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<td>King $W_0 = 5$</td>
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<tr>
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<td>King $W_0 = 5$</td>
<td>8192</td>
<td>0.014</td>
<td>0</td>
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<td>King $W_0 = 5$</td>
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<td>0.014</td>
<td>0</td>
</tr>
<tr>
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<td>King $W_0 = 7$</td>
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<td>0.014</td>
<td>0.1</td>
</tr>
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<td>0.014</td>
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<tr>
<td>K7b1</td>
<td>King $W_0 = 7$</td>
<td>8192</td>
<td>0.014</td>
<td>0</td>
</tr>
<tr>
<td>K7b2</td>
<td>King $W_0 = 7$</td>
<td>8192</td>
<td>0.014</td>
<td>0</td>
</tr>
<tr>
<td>K3a</td>
<td>King $W_0 = 3$</td>
<td>4096</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>K7a</td>
<td>King $W_0 = 7$</td>
<td>4096</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>K11a</td>
<td>King $W_0 = 11$</td>
<td>4096</td>
<td>0.03</td>
<td>0</td>
</tr>
</tbody>
</table>

Initial conditions for our set of runs. Columns (left to right): identification mark for the simulation, type of initial density profile, number of particles, black-hole mass, fraction of primordial binaries.
ing ejected from the core as a result of a strong interaction with a binary. E.g. this happened during some test runs with $m_{BH} = 0.01$ and $N = 4096$ on a timescale of about $t = 20 t_{bh}(0)$, so that we were forced to stop these simulations at the moment of the ejection.

Finally here we consider the black hole as a massive particle, and no mass accretion due to tidal disruption of stars is taken into account. However, this is not expected to bias the results significantly: in fact (1) the mass accretion, whose rate is proportional to the ratio $(m_{BH}/m)^{1/27}$, is not very important in runs with $N \lesssim 10^4$ (see Baumgardt et al. 2004 Table 1); in addition (2) stars passing close to the black hole may be captured, so that they become inert with respect to the other stars (effectively behaving like as if they had been accreted onto the black hole) or ejected from the system at extremely high speed (above 100 times the mean velocity dispersion), so that also in this case the star is removed from the system (with the difference that its mass is not added to that of the BH).

3 GLOBAL EVOLUTION

After the initialization described in Sec. 2.2, our simulations start from a condition of dynamical equilibrium: the evolution of the system is due to collisional effects and happens on a $t_{bh}$ timescale. This means in particular that the system is in virial equilibrium ($2K/U = 1$, where $K$ is the kinetic energy and $U$ is the potential energy of the system). Only occasionally there are fluctuations of $2K/U$ at the order of one percent, when a strong interaction between a hierarchical system (e.g. a binary interacting with the IMBH) takes place leading to the consequent ejection of a high velocity particle from the system. The virial ratio returns to the equilibrium value on a dynamical time-scale.

The large scale structure evolution of the cluster is dominated by the heating related to the presence of the BH. Starting from a Plummer model only the inner regions of the system experience a mild collapse on a time-scale that, depending on the mass of the BH, is of the order of a few $t_{bh}$ (depicted in Fig. 1). Inside the core radius $r_c$ a cusp in the density profile is formed within the sphere of influence $r_i$ (with $r_i \approx 15 r_c m_{BH}/M$, see Baumgardt et al. 2004a) of the BH, with a profile proportional to $\approx 1/r^{1.7}$ (see Fig 2) and thus similar to the $1/r^{1.75}$ measured by Baumgardt et al. (2004a). For $m_{BH} = 0.014M$ the influence radius is approximately $0.2r_c$. By definition, the stellar mass within this radius is comparable to that of the BH, and thus around one percent of the total mass of the cluster.

The limited number of particles that we employ does not allow us to significantly characterize the anisotropy profile of the system, that appears to be consistent with quasi-isotropy, except for the outer parts of the system, where there is a mild excess of radial orbits. As the presence of a population of binaries does not influence, to a first approximation, the evolution of the anisotropy profile, we refer to the discussion in Baumgardt et al. (2004b) (see in particular their Fig. 8), where the authors take advantage of the absence of primordial binaries and run simulations with $O(10^2)$ particles.

3.1 Plummer Models

For a Plummer model with $N = 8192$ and $f = 10\%$, in case of $m_{BH} = 0.014M$, the core radius $r_c$ is reduced in $\approx 4 t_{bh}(0)$ from the initial value of 0.4 to 0.3 (in natural units, see Sec. 2.1). This is to be compared with a value of $\approx 0.1$ reached without the BH (Paper I; see also Tab. 2); when a BH (with the same mass) but...
no primordial binaries are present \( r_c \) goes down to \( \approx 0.28 \). From these values it is apparent that the IMBH dominates the global evolution of the system, as there is very little difference in simulations with and without primordial binaries if an IMBH is present.

After the first mild contraction, all Lagrangian radii (shown in Fig. 1) start to expand steadily and a self-similar regime sets in, with the half-mass radius growing in proportion to \( \approx t^{2/3} \), in agreement with the theoretical argument given by Hénon (1965) and as found for runs with single stars only by Baumgardt et al. (2004). The rate of expansion of the half-mass radius is set by the amount of energy generated in the core (Vesperini & Chernoff 1994). In our runs with binaries plus IMBH this is marginally bigger (by \( \approx 10 \% \) at \( 24 t_{\nu h}(0) \)) than in runs with binaries only; without a BH the half-mass expansion starts only after core collapse, which takes \( \approx 10 t_{\nu h}(0) \) in that case (PaperI). Conversely, if a BH but no binaries are present, the expansion rate of \( r_h \) is reduced by \( \approx 20 \% \). A summary of the properties of the Plummer runs is reported in Table 2.

This picture does not appear to depend much on the number of particles used, except quantitatively for the known logarithmic dependence on \( N \) of the ratio \( r_c/r_h \) (Vesperini & Chernoff 1994). To validate the results of our series of \( N = 8192 \) runs we have performed a run with \( N = 16384 \) and an IMBH mass \( m_{BH} = 0.008 \) (thus keeping quasi-constant the ratio \( m/m_{BH} \) and going toward a BH mass closer to the one expected by the \( m_{BH} = \sigma \) relation). The evolution of the system, reported in Fig. 3 is indeed very similar, both in terms of binary destruction rate and in terms of the Lagrangian radii evolution.

### 3.2 King Models

At fixed number of particles, IMBH mass and number of primordial binaries, there is no expectation that the long term evolution of the system depends much on the details of the initial conditions, as these are erased on a relaxation time-scale (see also PaperII). To verify this we have also started some of our runs (see Table 1) from an initial King profile, with different values of the concentration parameter \( W_b \). The evolution of the core radius with respect to the half mass radius is reported in Fig. 3. Both the \( W_b = 5 \) and the \( W_b = 7 \) models evolve toward \( r_c/r_h \approx 0.3 \) on a time-scale of \( \approx 5 t_{\nu h}(0) \) (this is the same value reached in the simulations starting from a Plummer profile). Interestingly the \( W_b = 7 \) model presents an initial expansion of the core, as the initial central concentration is so high that the energy released by three and four body encounters cannot be completely dissipated by expansion of the system, while the last entry (\( r_h \)) is the value of the half mass radius at \( t = 24 t_{\nu h}(0) \) in units of the initial half mass radius.

### Table 2. Global properties of Plummer runs with \( N = 8192 \)

<table>
<thead>
<tr>
<th>( m_{BH}/M )</th>
<th>( f )</th>
<th>( (r_c)<em>{cc}/r</em>{cc} )</th>
<th>( r_c/r_h )</th>
<th>( r_{hf}/r_{h0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014</td>
<td>0.1</td>
<td>0.75</td>
<td>0.29</td>
<td>2.46</td>
</tr>
<tr>
<td>0.025</td>
<td>0.1</td>
<td>0.80</td>
<td>0.31</td>
<td>2.83</td>
</tr>
<tr>
<td>0.014</td>
<td>0.0</td>
<td>0.70</td>
<td>0.27</td>
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</tr>
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<td>0</td>
<td>0.1</td>
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<tr>
<td>0</td>
<td>0.0</td>
<td>0.02</td>
<td>0.015</td>
<td>1.98</td>
</tr>
</tbody>
</table>

In the first column we report the BH mass, in the second the fraction \( f \) of primordial binaries, in the third the core radius at the end of the initial core contraction phase \( (r_c)_{cc} \) in units of the initial core radius \( r_{cc} \), the fourth entry is the core to half mass radius ratio during the self-similar expansion of the system, while the last entry \( (r_h) \) is the value of the half mass radius at \( t = 24 t_{\nu h}(0) \) in units of the initial half mass radius.

![Figure 3. Comparison between two simulations starting from a Plummer profile with \( f = 10\% \); one has \( N = 16384 \) with \( m_{BH} = 0.008 \) (solid line), the second has \( N = 8192 \) with \( m_{BH} = 0.014 \) (dashed line). In the upper panel we depict the relative number of binaries and singles, in the lower panel selected lagrangian radii (enclosing 5, 10, 20, 50 and 80 % of the total mass in stars). The Lagrangian radii have been smoothed with a triangular window filter of width 1.5 \( t_{\nu h}(0) \). Doubling the number of particles does not introduce significant changes in our simulations. There is only a small contraction of the Lagrangian radii, when the BH mass is reduced to 0.008.](image)

![Figure 4. Core to half mass radius ratio for a series of simulations with \( N = 8192, m_{BH} = 0.014 \) and 10% primordial binaries. Different initial conditions (King profiles with \( W_b = 5 \) or 7) converge toward a common value for \( r_c/r_h \). The results are presented by applying a triangular smoothing window of width 1.5 \( t_{\nu h}(0) \).](image)
half mass radius. This behavior is similar to that observed in runs
with primordial binaries only but with yet higher central concentra-
tions (Fregeau et al. 2003; PaperII). This picture is confirm ed also
with primordial binaries only but with yet higher central co ncentra-
half mass radius. This behavior is similar to that observed in runs

4 PROPERTIES OF THE Binary POPULATION

The evolution of the number of binaries in our simulations is driven
by two competing processes: the formation of new binaries and
the destruction of existing ones. As we discussed in detail in Pa-
perI (see in particular Sec. 2 in that paper), the probability of form-
ing new binaries is greatly suppressed with respect to the proba-
bility of destructing an existing binary. In fact, the leading forma-
tion channel for a new binary from single stars is a three body en-
counter, which is proportional to \( \rho_s^2 \), \( \rho_s \) is the number density of
single stars and this density is expressed in physical units, appro-
propriate for encounters, such that \( \rho = 1 \) corresponds to inter-particle
distances equal to the ninety-degree turnaround impact parameter
(Hut 1989). Therefore in this units we have \( \rho_s \ll 1 \). The leading
destruction channels are instead given by binary-single and binary-
binary encounters and therefore proportional to \( \rho_b \rho_s \) and \( \rho_b^2 \), where
\( \rho_b \) is the number density of binaries, in the same physical units con-
sidered for \( \rho_s \). As during our simulations the core number density
of binaries is similar to that of singles, these two last channel domi-
nates over the three single stars encounters. This expectation is con-
firmed empirically in our runs, where the number of dynamically
formed binaries that can be found after a few relaxation times in
the simulation (typically about \( 10^{-3} \) of the number of single stars)
is negligible not only with respect to the number of primordial bi-
naries but also with respect to the number of disrupted binaries.
Essentially, the dominant dynamical mechanism that changes the
net number of binaries is that of binary disruption. Of course, the
exchange of binary members during a binary-single encounter is
also frequent, but this does not alter the net number of binaries in
the cluster.

As the presence of the BH dominates the global dynamics of
the cluster, the evolution of the binaries presents some remarkable
differences from the scenario where no central BH is present. We
can distinguish two phases.

The presence of a BH significantly accelerates the rate at
which binaries are disrupted in the first few half-mass relaxation
times (see Fig. 5). The reason is that binaries that pass near the
BH can be quickly destroyed. The disruption rate is approximately
contant during the first \( 5 t_{rh}(0) \), while without a BH disruption
takes a while to get underway. The initial disruption rate depends
on the mass of the BH: a more massive BH starts burning binaries
at a higher rate, as can be seen from Fig. 5.

For our Plummer run with \( N = 8192, m_{BH} = 0.014 M \)
and \( f = 10\% \) we observe a binary depletion rate of \( \approx 0.15 \) binaries per
initial half-mass crossing time (\( t_d \)), during the first \( 5 t_{rh}(0) \) (here
\( t_{rh}(0) = 116.1 t_d \)).

For this time interval we present in Fig. 6 the distribution of the
radial positions at which binaries are destroyed. This is ob-
tained by looking in NBODY6 at the position at which binaries are
flagged by the code as too wide to continue meriting special nu-
merical treatment in terms of KS (Kustaanheimo & Stiefel 1965)
solutions (see Aarseth 2003 for a detailed description of the algo-

Figure 5. Time-dependence of the number of single (upper set of lines) and
binary stars (lower set of lines) expressed as a fraction of the initial values.
The data refer to simulations with \( N = 8192 \) and 10% primordial binaries.
There are three choices for BH mass: no BH (dotted line), a central BH with
\( m_{BH} = 0.014 M \) (solid line), and \( m_{BH} = 0.025 M \) (dashed line). The
unit of time is the initial half-mass relaxation time.

\[ r_t = \left( \frac{m_{BH}}{m} \right)^{1/3} a, \]

where \( a \) is the semi-major axis of the binary. With this respect, the
theoretical model recently proposed by Pfahl (2005) applied to our
\( N = 8192, m_{BH} = 0.014, f = 10\% \) simulation would give
(from his Eqs. 11-12) a tidal disruption rate of \( \approx 2 \cdot 10^{-2} \) binaries
per initial half-mass crossing time, so that we observe less tidally
striped binaries than expected (\( \approx 10 \)).

The disruption of binaries by the BH is thus mainly due to an
indirect effect: binaries venturing close to the BH, where the
density is higher, are more likely to interact with a single star or
with another binary before having the chance to being tidally de-
stroyed by the IMBH. In fact in our simulations we often observe
the presence of hierarchical systems within \( r_t \), with the BH playing
the role of a perturber. Fig. 6 nicely summarizes this picture: bina-
rices are disrupted at a significantly enhanced rate within \( r_t \) (right
panel), but the minimum distance from the BH is the order of \( 10 r_t \)
(left panel). The bi-modal distribution in the left panel of Fig. 6
is the number density of binaries, in the same physical units con-

there is a different series of runs - starting from \( W_0 = 3, 5, 7, 11 \) with
\( m_{BH} = 0.03 \) and no binaries - that exhibit a very similar behavior.
also reflects the enhanced probability of close encounters and binary disruption within the influence sphere of the BH, as can be seen from Fig. 6.

After the initial transient phase a self-similar expansion sets in, where the average core density is much less (approximately by a factor 10) than it would be without the presence of a central compact object. In this second phase we observe (see Fig. 5) a reduced rate of binary disruption (as this rate is proportional to the square of the density, e.g. see Vesperini & Chernoff 1994), so that the difference between a simulation with and without a BH becomes remarkable. The turning point is around 10 \( t_{\text{ch}}(0) \) when the number of surviving binaries for a simulation with a central object becomes greater than in the absence of a BH. Our Fig. 5 has been given in units of the initial half-mass relaxation time, but the picture remains qualitatively the same even if we consider a co-moving time coordinate to take into account the differences between the half-mass radii of a simulation with and without a BH.

Interestingly the spatial distribution of binaries is also different from the case without a BH (see Figs. 5). After \( \approx 20 t_{\text{ch}}(0) \) the number of binaries within the 0.05 Lagrangian radius is much less for the simulation with a BH. This is probably due to the disruption of binaries that approach close to the BH. In the presence of a BH, binaries tend to be more concentrated between the 0.05 Lagrangian radius and the half-mass radius, while in the absence of a BH, binaries can sink deeper into the central region of the cluster. As noted by Baumgardt et al. (2004) for simulations without primordial binaries, the mass segregation efficiency is different in presence of a central BH.

If we compare the evolution of the binding energy distribution of the binaries in a run with and without a central BH, we can see that the net effect of the BH at later times is to somewhat enhance the survival probability of binaries with binding energies of a few \( kT \)s (\( E_b \lesssim 16 kT \)), especially for binaries within the half-mass radius (see Fig. 5). This feature is especially apparent in a two-dimensional plot of the distribution of binaries as a function of binding energy and radial distance from the cluster center, depicted in the bottom panels of Fig. 5. The rather strong correlation between radius and binding energy that is observed (e.g., see Giersz & Spurzem 2000, PaperI) in systems without a central compact object is thus almost absent. The ratio of moderately hard (\( E_b/kT \leq 16 \)) to hard (64 \( \leq E_b/kT \leq 128 \)) binaries is greater by about a factor 2 at \( t \approx 20 t_{\text{ch}}(0) \) in our runs with the presence of an IMBH (see top panel of Fig. 5). The possible use of this ratio as a diagnostic to evidence the presence of an IMBH can be impaired if there is an initial segregation of primordial binaries depending on their semi-axis. Specifically if softer binaries are preferentially born in the core of the cluster then some of them could still survive after a few relaxation times, mimicking the signal due to an IMBH. However such a cluster would be characterized by a significantly smaller core radius with respect to a cluster truly hosting an IMBH.

This picture, presented for a \( N = 8192 \) Plummer model, is representative of all our set of simulations. We have shown in Sec. 4 that the global evolution of the system is not sensitive on the details of the initial conditions. Similarly the evolution of the binary abundance at fixed relaxation times does not significantly depend on the number of particles \( N \) (see Fig. 3) or on the details of the initial conditions: in Fig. 4 we depict the evolution of \( N_{\text{bin}}/N_0 \) for a number of different simulations and the pattern is consistently similar. The same trend shown in Fig. 3 is also obtained starting from different initial conditions, as we have verified in the runs starting from \( W_0 = 5 \) and \( W_0 = 7 \) King models. All these results show that the evolutionary differences in runs with and without a central black hole are physical and not just due to random fluctuations.

5 ESCAPERS

The N-body systems in our simulations, despite being isolated, lose mass, i.e. some particles acquire a positive energy and are consequently removed from the system (if they are unbound and reach a distance of \( 20 r_h \) from the center of system). There are two main contributions to this process. One is diffusion of particles in the energy space, that leads to the evaporation of particles with binding energy very close to 0 (that translates into a small residual velocity at infinity, which we call “ejection velocity”). The other is due to strong interactions of binaries with other stars and/or with the IMBH with the consequent ejection of a high velocity particle (the slingshot effect, see Aarseth 2005).

In our Plummer run with \( N = 8192, f = 10\% \) with \( m_{\text{BH}} = 0.014M \) we find that a fraction of \( 3 \cdot 10^{-3} \) of the stars of the cluster leave the system in a relaxation time and that \( \approx 1/3 \) of them have ejection velocities that exceed by at least five times the core velocity dispersion. Among these 54 were binaries; in addition one triple was ejected from the system. A distribution of the ejection veloc-
Figure 8. Distribution of binding energies for binaries (upper panel) at time $t = t_{*}(0)$ for a simulation starting from a Plummer model with $N = 8192$, 10% primordial binaries and $m_{BH} = 0.014M$ (solid line), compared to a similar simulation without the BH (dotted line). In the lower two panels we report the energy-radius distribution of binaries for the same simulations. The radii have been normalized to the Lagrangian radius containing 5% of the mass in the simulation. The dashed line gives the position of the half-mass radius, at $t = 24 t_{*}(0)$. The relatively large survival at $t = 24 t_{*}(0)$ of binaries within the half-mass radius with binding energy $E_b < 20 kT$ is clearly visible in the bottom panel, and is also reflected in the higher values for the solid line at the left-hand side of the upper panel.

Figure 9. Like Fig. 5 but for a series of simulations ($N = 8192$, $f = 10\%$, $m_{BH} = 0.014$) starting from King models with $W_0 = 5$ (solid line; two different runs) and $W_0 = 7$ (dotted line; two different runs). After an initial transient given by the different initial conditions, the binary destruction rate is very similar among all the runs and is quantitatively consistent with the one observed in Fig. 5 for an initial Plummer density profile.

These results, especially for the high velocity tail of the distribution, need to be confirmed by more realistic simulations that include a mass spectrum and account for possible stellar collisions during close encounters, but they suggest that the interaction of IMBH with a significant population of binaries may efficiently lead to the production of a number of ultra fast stars, i.e. stars with an ejection velocity of several hundreds $km/s$.

Like Fig. 5 but for a series of simulations ($N = 8192$, $f = 10\%$, $m_{BH} = 0.014$) starting from King models with $W_0 = 5$ (solid line; two different runs) and $W_0 = 7$ (dotted line; two different runs). After an initial transient given by the different initial conditions, the binary destruction rate is very similar among all the runs and is quantitatively consistent with the one observed in Fig. 5 for an initial Plummer density profile.

6 DISCUSSION AND CONCLUSIONS

In this paper we have presented for the first time the results of direct N-body simulations of the evolution of a star cluster with a population of primordial binaries that harbors a central IMBH, with a mass of the order of 1% of the total mass of the system. In order to begin to analyze the basic evolutionary processes we have restricted our simulations to isolated systems of stars of equal mass, to begin to analyze the basic evolutionary processes we have restricted our simulations to isolated systems of stars of equal mass, neglecting at this stage stellar evolution and physical collisions.

The environment around the BH turns out to be of great interest from a dynamical point of view. It represents a laboratory where complex interactions between hierarchical systems take place. Around the BH we observe usually one or more stars tightly bound to it. With a significant population of primordial binaries, it frequently happens that a binary approaches the center of the system. Interactions between these subsystems are important not only from a dynamical point of view, but also because they form a factory to produce exotic objects, such as tight (X-ray emitting) binaries and high velocities escapers, which we observe in high numbers in our simulations.

From a computational point of view the complex interactions between binaries and the BH have proved to be very challenging for current state of the art N-body algorithms. In our simulations we had to resort to several restarts of the computation to fine tune...
the integration parameters in order to achieve energy errors conservation below $5 \cdot 10^{-3}$ at the end of the simulation and below $5 \cdot 10^{-4}$ per dynamical time. Clearly, new regularization methods have to be developed to provide both more accuracy and higher stability. This will allow to perform automatic runs, a necessary condition to employ a more systematic study of systems with IMBH and binaries. We have shown that a BH with a mass of around $1\%$ of the cluster mass can produce most of the energy required to fuel the expansion of the star cluster, so that the evolution of the large scale structure of a star cluster with primordial binaries and a central BH is remarkably similar to that attained in the absence of binaries (see Baumgardt et al. 2004). Even so, the evolution of the population of primordial binaries is strongly influenced by the BH: binaries are disrupted primarily within the influence sphere of the BH, but we note a significant lack of binaries tidally disrupted by the BH with respect to predictions based on the loss cone theory. These theoretical estimates only take into account interactions between the binaries and the BH starting from the density of binaries in the core. Instead we observe that most of the binaries are destroyed by three and four body encounters with other stars once they enter the influence sphere of the BH and before they have the chance to venture close enough to the BH to be tidally shredded. These three and four body encounters within the influence sphere of the BH often lead to the ejection of stars with proper motions easily exceeding 10 times the core velocity dispersion of the cluster. We observe events with velocities up to 50 times the core velocity dispersion, that is of the order of $10 \text{ km/s}$ for a typical globular clusters. Binaries within the influence sphere of an IMBH can therefore originate a significant population of high velocity stars ejected from the system, whose presence could be detected observationally. Tidal shredding of binaries with main sequence components limits the ejection velocity to a few hundreds $\text{km/s}$, but higher velocities can be obtained if at least one member of the binary is a compact object (white dwarf, neutron star, or stellar mass BH). One related intriguing, although highly speculative possibility is indeed the dynamical origin of a $\approx 1000 \text{ km/s}$ pulsar candidate from the globular cluster remnant IRS13, which is considered to host an IMBH (Wang et al. 2006).

In principle, an indirect evidence for the presence of an IMBH in an old globular cluster could be obtained by observing the distribution of binaries: observational detection of an unexpectedly large fraction of only moderately hard binaries in the core could be used as circumstantial evidence for the presence of an IMBH in old star clusters, i.e. in those systems whose age would imply a depletion in the central region of binaries with binding energy below $\approx 15 kT$. With our runs we have shown that these binaries are able to survive in significant numbers when an IMBH is present. For a typical globular cluster this would imply to be able to measure in the core the ratio of binaries with semi-major axis of about $a \geq 0.5 \text{ AU}$ to those with $a \approx 0.05 \text{ AU}$, that appears to be particularly sensitive to the presence of an IMBH (see Fig. 8). This relative measure has the advantage of being relatively independent from the precise value of the primordial binary ratio (clearly unknown for observed cluster), but it is extremely challenging for the present observational techniques of binary detection. This would in fact mean to be able not only to detect a significant number of binaries in the dense core environment of a globular cluster, but also to accurately measure the semi-major axis of these binaries down to a fraction of AU. In addition, as we have mentioned in Sec. 4, if softer primordial binaries are preferentially born in the center of the cluster, this diagnostic fails.

A more promising observational evidence for the presence of an IMBH in old, relaxed, globular clusters can instead be given by a simpler measure, i.e. the ratio of the core to half mass radius. This quantity is set by the efficiency of energy generation in the core of the cluster and is robust with respect to changes in the details of the initial conditions. Systems made of single stars only have a very small $r_c/r_h$ ratio ($r_c/r_h \approx 0.01$), as no efficient energy production mechanism is available. When primordial binaries are present, $r_c/r_h$ is up to almost one order of magnitude larger and the ratio depends only on the initial fraction of binaries, saturating at $f \gtrsim 10\%$ (see PaperI and PaperII). Accounting for a small N-dependence of the ratio, a typical old globular cluster with primordial binaries is expected to have $r_c/r_h \lesssim 0.05$ (see Fregeau & Rasio 2006). Most of the observed globular clusters have instead much larger values for $r_c/r_h$, located in the range $[0.1; 1]$ (Fregeau & Rasio 2006). If we were to assume that old globular clusters host an IMBH, then we would naturally obtain $r_c/r_h \approx 0.3$ which is in good agreement with the observations.

Clearly, a quantitative comparison with observations would require us to proceed beyond the simple models presented here. To start with, we would need to include a realistic number of stars to avoid possible biases introduced by the scaling of the ratio of the BH mass to that of single stars. In addition, the introduction of a realistic initial mass function is likely to modify the concentration of binaries in the center of the system due to mass segregation, and stellar evolution will also influence the distribution of binary binding energy. However, the main effects presented in this paper are likely to be present, at least in qualitative ways, in more detailed realistic simulations.

Figure 10. Distribution of the escape velocity (in units of the core velocity dispersion at the escape time) as measured in a sample of 5 simulations with $N = 8192, 10\%$ primordial binaries, and $\eta_{BH} = 0.014$. If applied to a typical star cluster (where $\sigma_c \approx 10 \text{ km/s}$), this plot suggests that the combined action of an IMBH plus a population of primordial binaries can lead to the ejection of stars at several hundreds $\text{km/s}$.

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APPENDIX A: NUMERICAL ACCURACY

The Aarseth’s NBODY6 code represent one of the few state of the art integrators for the direct integration of collisional N-body systems. Its performance and accuracy have been extensively tested (see Aarseth 2003). The code employs a series of automatic internal accuracy tests at the level of individual timesteps and forces, treatment of hierarchical systems and, of course, conservation of the total integral of motions. If one of these tests fails, the code resumes the computation from a previously saved snapshot of the system, as the shortest dynamical timescale to be resolved may be of the order of days, compared a dynamical time of a few million of years for a typical globular cluster. This is reflected with relative errors in the conservation of the total energy at the level of $10^{-6}$ per dynamical time.
Figure A1. Relative total energy conservation per dynamical time $t_d$ for $N = 8192$ in a run with 10% primordial binaries (left panel) and in a run with 10% primordial binaries and an IMBH with mass 1.4% of the total mass of the system (right panel). The maximum allowed error per dynamical time is set at $5 \cdot 10^{-4}$; all our runs are stopped if the total energy error increases above $5 \cdot 10^{-3}$.

energy at the end of the simulation (the errors tend to have random signs so that the total error increases approximately with the square root of the time). The code has a similar performance also for runs that include single stars plus a IMBH (see Baumgardt et al. 2004a,b).

The introduction of an IMBH in simulations with primordial binaries introduces additional challenges for the code, as in this case several multiple interacting systems have to be treated in the sphere of influence of the IMBH. Despite the improvements introduced to increase the performance of the code (see Sec 2), significantly greater errors with respect to the standard runs for the code happened during our simulations (see Fig. A1), leading to a number of restart of the run performed manually in order to fine tune the integration options, depending on the kind of error message received.

To ensure reliable results we set a maximum allowed energy error per dynamical time of $5 \cdot 10^{-4}$ and we definitely stop a simulation that reaches a total energy error of $5 \cdot 10^{-3}$. This threshold value is typically reached after about 25 initial half mass relaxation times (thus at a time longer than the Hubble time for a typical globular cluster).

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