Rapidity Dependence of HBT radii based on a hydrodynamical model

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We calculate two-pion correlation functions at finite rapidities based on a hydrodynamical model which does not assume explicit boost invariance along the collision axis. Extracting the HBT radii through $\chi^2$ fits in both Cartesian and Yano-Koonin-Podgoretskii parametrizations, we compare them with the experimental results obtained by the PHOBOS. Based on the results, we discuss longitudinal expansion dynamics.

Keywords: hydrodynamical model, pion interferometry, boost invariance

I. INTRODUCTION

“Perfect fluidity” of the created matter at the Relativistic Heavy Ion Collider (RHIC) in BNL is one of the most exciting news in the field of high energy nuclear physics [1]. Experimental results and their comparison with theoretical calculation reveal that the matter created at Au+Au collisions should be something like a liquid of quarks and gluons, unlike a gas of almost free partons as naively expected [2]. One of the strong evidences of this finding is an observation of large elliptic flow ($v_2$) and its agreement with a perfect fluid-dynamical calculation [3]. In order to reproduce the experimental result, an equation of state assuming partonic degree of freedom at high temperature and a phase transition, and rapid thermalization time ($\tau_0 \leq 1$ fm/$c$) are required. Now the hydrodynamic model based on numerical solutions of the relativistic hydrodynamic equation for the perfect fluid becomes an indispensable tool for theoretical analyses of the relativistic heavy ion collisions. Furthermore, the model itself has been becoming more sophisticated to reproduce new experimental data with high statistics. Current most sophisticated ones are full three-dimensional (solving hydrodynamic equation without any symmetry) hydrodynamic expansion followed by a hadronic cascade [4,5]. These models can reproduce most of soft hadronic observables. Especially, simultaneous description of particle ratios, transverse momentum spectra and elliptic flow is possible only with such hybrid models.

However, there are still some insufficient ingredients in the hydrodynamic analyses. First, we don’t have a reasonable initial condition derived from the first principle. Recently, the Color Glass Condensate (CGC) has been proposed as a suitable initial condition for relativistic heavy ion collisions [6]. This picture has been examined as an initial condition for a hydrodynamic model in Ref. [7] and found to give a good description of some observables in the case of fully hydrodynamic description of the collision process. However, this initial condition fails if one takes hadronic dissipation into account [4]. This fact suggests there is still an open space for dissipative partonic phase, or improvement of the initial condition.

Second, the equation of state (EoS) of the QCD matter has not been fully understood yet. Since one of the most important merit of using hydrodynamic models is that it can be directly related to the EoS, detailed information of the EoS for appropriate region of temperature and baryonic chemical potential is indispensable. As for the RHIC energy, net baryon number observed at midrapidity is small enough to neglect it [8]. Nevertheless, EoS at finite baryonic chemical potential may play important role at forward rapidity region and heavy ion collisions at lower energies. Because of a well-known difficulty of lattice QCD at finite baryonic chemical potential [9], lattice QCD calculation has not provided the complete solution yet. For vanishing baryonic chemical potential, the lattice equation of state clearly shows a different behavior from the free parton gas [10] and a lattice-inspired EoS is implemented in a hydrodynamic calculation [11].

At last, in spite of the success in most of soft observables, results of two-pion momentum intensity correlation do not agree with experimental data yet. According to the symmetry of a wave function of two identical bosons, the two-particle correlation function can be related to sizes of the source from which particles are emitted. This fact is known as Hanbury Brown-Twiss (HBT) effect. Because it concerns with source sizes, which depend on momentum of particle pairs due to collective flow, the pion correlation function should be a diagnostic tool for the space-time evolution of the matter. However, none of hydrodynamic models can reproduce experimental data. Since the disagreement was firstly found with a (2+1)-dimensional model with boost invariance along the collision axis [12], many extensions such as an explicit longitudinal expansion [13,14], incorporating chemical freeze-out [14], chiral model EoS [15], opaque source [16], fluctuating initial conditions and continuous freeze-out [17] have been examined and the discrepancy has been getting improved, but the situation is still unsatisfactory, i.e., the HBT puzzle has not been solved yet.

So far discussion on the HBT radii at the RHIC has been limited to midrapidity because of acceptances of two experimental group, STAR and PHENIX. PHOBOS also has measured the two-pion correlation function. By virtue of the wider acceptance of the detector, measurements at non-zero rapidity windows can be done, and the data are now available in Ref. [18]. For analyses of such data in terms of the Cartesian parameterization [19,20], it should be noted that there exists an additional HBT radius called “out-long cross term” [21] which vanishes at midrapidity due to the symmetry in the case of head-on collisions. This radius contains information on the correlation between freeze-out positions on the transverse plane and those on the longitudinal direction. Hence, it
is expected that this quantity is sensitive to longitudinal expansion dynamics beyond boost-invariant approximation. Similar consideration also holds for the Yano-Koonin-Podgoretskii parametrization which has three radius parameters and one velocity parameter called YK velocity [22, 23]. The PHOBOS data also provide rapidity dependence of the YKP radii and YK velocity [18], which may impose a restriction on the longitudinal expansion dynamics. Indeed, the initial matter distribution as an input for hydrodynamic calculations has not been fixed yet. This is firstly indicated by Hirano in Ref. [24], in which two different initial energy density distributions can provide reasonable agreement with experimental data of pseudorapidity distribution of charged hadrons measured in 130A GeV Au+Au collisions at RHIC.

In this work, we employ two different initial energy density distributions for the hydrodynamic equations, as in Ref. [24]. We focus our discussion on central collisions. Both of them are so tuned that they reproduce the pseudorapidity distribution of charged hadrons measured in the most central events at 200A GeV Au+Au collisions. Then we compare the space-time evolution and shape of the freeze-out hypersurface of the fluids and see how the difference in the initial condition is reflected on them. We calculate the two-pion correlation function as the most promising experimental observable to see the difference. Extracting the HBT radii through Gaussian fits, we compare them with the experimental results and discuss the transverse momentum and rapidity dependence of the HBT radii. In the next section, we briefly review the hydrodynamical model used in this work. Initial conditions are given in Sec. III. In Sec. IV we show numerical solutions of hydrodynamical equations for the initial conditions given in Sec. III. Results for the HBT radii compared with the experimental data are given in Sec. V. Section VI is devoted to a summary.

II. HYDRODYNAMICAL MODEL

The basic equation of hydrodynamical models is the energy-momentum conservation law

$$\partial_{\mu}T^{\mu\nu} = 0,$$

where $T^{\mu\nu}$ is the energy-momentum tensor. For a perfect fluid,

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu},$$

where $g^{\mu\nu} = \text{diag}(+,-,-,-)$ and $\varepsilon$, $P$ and $u^\mu$ are energy density, pressure and the four velocities of the fluid, respectively. If one takes a conserved charge $i$ such as baryon number and strangeness into account, the conservation law

$$\partial_{\mu}(n_i u^\mu) = 0$$

is added to be solved. Providing an EoS $P = P(\varepsilon, n_i)$, one can solve these coupled equations numerically.

In this work, we consider the baryon number charge as a conserved charge and adopt an equation of state which exhibits a first order phase transition on the phase boundary in $T-\mu_B$ plane from the free massless partonic gas with three flavors to the free resonance gas including hadrons except for hyperons up to 2 GeV/c$^2$ of mass with excluded volume correction. See Ref. [25] for the detail. The critical temperature $T_c$ at vanishing baryonic chemical potential is set to 160 MeV. This model of current use is basically same with the one used in Refs. [13, 16].

Putting $z$-axis as the collision axis, we use a cylindrical coordinate system $(\tau, \eta, r, \phi)$ as follows:

$$t = \tau \cosh \eta_s,$$

$$z = \tau \sinh \eta_s,$$

$$r_s = r \cos \phi,$$

$$r_y = r \sin \phi.$$  

Here, $\tau = \sqrt{t^2 - z^2}$ is the proper time and $\eta_s = 1/2 \ln[(t + z)/(t - z)]$ is the space-time rapidity. Since we focus on central collisions, we assume azimuthally symmetric system. Then, by virtue of $u_\mu u^\mu = 1$, the four velocities are given in terms of a longitudinal flow rapidity $Y_L$ and a transverse flow rapidity $Y_T$ as

$$u^\tau = \cosh(Y_L - \eta_s) \cosh Y_T,$$

$$u^\eta = \sinh(Y_L - \eta_s) \cosh Y_T,$$

$$u^r = \sinh Y_L.$$  

To solve the equations numerically, we employed a method based on the Lagrangian hydrodynamics which traces flux of the current. The numerical procedure is described in Ref. [26]. For a treatment of the first order phase transition, we introduce a fraction of the volume of the QGP phase to express the energy density and net baryon number density at the mixed phase [25]. In this algorithm, we explicitly solve the entropy and baryon number conservation law. We checked that these quantities are conserved throughout the numerical calculation within 1% of accuracy for a time step $\delta \tau = 0.01$ fm/c, by choosing proper mesh sizes of $\eta_s$ and $r$ directions.

III. INITIAL CONDITIONS

Firstly, we choose an initial proper time as $\tau_0 = 1$ fm/c. Initial values for other variables are given on this hyperbola. Longitudinal flow rapidity is set to the Bjorken’s scaling ansatz $Y_L = \eta_s$ [27]. Transverse flow is simply neglected at the initial proper time. For the matter distributions, we assume that the energy and baryon number density are proportional to the number of binary collisions. Hence, for the Woods-Saxon profile of nucleon density in nuclei

$$\rho_n(r, z) = \frac{\rho_0}{e^{(\sqrt{R^2 + z^2} - R)/\xi} + 1},$$

where $R = 1.12A^{1/3} - 0.86A^{-1/3}$ fm is the radius of the nucleus with mass number $A$, $\xi = 0.54$ fm is the surface diffuseness and $\rho_0$ is the normal nuclear matter density, the density of binary collisions at vanishing impact parameter is given by

$$n_{BC}(r) = \sigma_0 \int_{-\infty}^{\infty} dz \rho_n(r, z)^2,$$
with $\sigma_0$ being the total inelastic nucleon-nucleon cross section which is absorbed into the proportionality constant between $n_{BC}$ and matter distributions.

Then, the energy density distribution is parameterized with a “flat+gaussian” form,

$$\varepsilon(\tau_0, \eta_k, r) = \varepsilon_0 \exp \left[ -\frac{(\eta_k - \eta_0)^2}{2\sigma_{\eta_0}^2} \theta(\vert \eta_k - \eta_0 \vert) \right] n_{BC}(r). \tag{13}$$

Here, $n_{BC}(r)$ is the normalized density of binary collisions \textsuperscript{(12)}. $\varepsilon_0$ the maximum energy density, and $\eta_0$ and $\sigma_\eta$ are parameters which determine the length of the flat region and width of the gaussian part, respectively. Similarly, the net baryon number density distribution is parameterized as

$$n_B(\tau_0, \eta_k, r) = n_{B0} \left\{ \exp \left[ -\frac{(\eta_k - \eta_{0D})^2}{2\sigma_{0D}^2} \right] \theta(\vert \eta_k - \eta_{0D} \vert) + \exp \left[ -\frac{(\eta_{0D} - \eta_0)^2}{2\sigma_{0D}^2} \right] \theta(\eta_0 - \vert \eta_k \vert) \right\} n_{BC}(r), \tag{14}$$

where $n_{B0}$ is the maximum net baryon number density and $\eta_{0D}$ and $\sigma_{0D}$ are the shape parameters as in Eq. (13).

To calculating final particle distribution, we use the Cooper-Frye prescription \textsuperscript{(28)}. The pseudorapidity distribution for a particle species $i$ is given by

$$\frac{dN_i}{d\eta} = \frac{d_\eta}{(2\pi)^2} \int_0^{\infty} dk' \int_{k'0}^k dk \frac{k|k|}{k'0} \sum \sigma f(k \cdot u, T, \mu_B), \tag{15}$$

where $k'$ is the momentum of thermally produced particles $i$ with $d_\eta$ being the degree of freedom, pseudorapidity $\eta$ defined by $\eta = 1/2 \ln [(|k' + k_2|)/(|k' - k_2|)]$ and $f(k \cdot u, T, \mu_B)$ the equilibrium distribution functions. We take into account not only directly produced particles but also resonance decay contribution. The freeze-out hypersurface $\Sigma$ is chosen to be a constant temperature one, $T = T_f = 140$ MeV. Here, we assume thermal and chemical freeze-out occur simultaneously. Since experimental data of particle yields can be well described by the statistical model with high chemical freeze-out temperature close to $T_f$, \textsuperscript{(29)} we cannot reproduce the correct particle yields with this lower freeze-out temperature. However, in hydrodynamic analyses, thermal freeze-out temperature is sensitive to $k_t$ spectra which mainly reflects transverse expansion. In this calculation, we set the freeze-out temperature so that pion $k_t$ spectrum is roughly reproduced. In order to reproduce both of the particle yields and the $k_t$ spectra in dynamical regimes, one should introduce separate freeze-out temperatures \textsuperscript{(14, 30)} or go to hybrid approach. \textsuperscript{(4, 5, 31, 32)} In this work, however, our main argument will not be so affected by the description of the freeze-out because we focus on longitudinal expansion.

In Table I, two sets of the initial parameters are listed. The corresponding initial energy density distributions and resultant pseudorapidity distributions are illustrated in Fig. 1 and Fig. 2 respectively. We have chosen two initial conditions both of which reproduce the experimental data. The difference of these two initial conditions is characterized by two parameters, $\eta_{0D}$ and $\sigma_{0D}$. One has small $\eta_{0D}$ and large $\sigma_{0D}$, which we denote initial condition A (IC.A). The other which we represent IC.B has an opposite feature; $\eta_{0D}$ is large and $\sigma_{0D}$ is small. We calculated the pseudorapidity distributions for not only these two initial conditions but also intermediate ones by varying $\eta_{0D}$ from 1.0 to 3.0 and found that they can also reproduce the experimental data by adjusting other parameters appropriately. Perhaps the best fit will exist in the middle of this parameter range \textsuperscript{(34)}. Here, we choose the extreme cases in order to see differences in the space-time evolution of the fluids originating from the difference in the initial conditions.

<table>
<thead>
<tr>
<th>TABLE I: Parameters in initial matter distributions</th>
<th>$\varepsilon_0$ [GeV/fm$^3$]</th>
<th>$\eta_{0D}$</th>
<th>$\sigma_{0D}$</th>
<th>$n_{B0}$ [fm$^{-3}$]</th>
<th>$\eta_0$</th>
<th>$\sigma_\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IC.A</td>
<td>23.0</td>
<td>1.0</td>
<td>1.48</td>
<td>0.47</td>
<td>2.2</td>
<td>0.9</td>
</tr>
<tr>
<td>IC.B</td>
<td>20.5</td>
<td>3.0</td>
<td>0.33</td>
<td>0.55</td>
<td>2.2</td>
<td>0.75</td>
</tr>
</tbody>
</table>

FIG. 1: Initial energy density distributions

FIG. 2: Pseudorapidity distribution. The solid line and dashed line stand for the initial condition A and B, respectively. Experimental data measured by PHOBOS are taken from Ref. [33]

IV. SPACE-TIME EVOLUTION OF THE FLUIDS

Figures 3 and 4 show the space-time evolution of the temperature distributions and deviation from the scaling solution $Y_t = \eta_k$, as a function of $\eta_k$ at $r = 0$ for various $\tau$, respectively. From these figures, we find that the space-time evolution at forward rapidity is quite different between IC.A and IC.B in spite of the fact that both solutions give similar pseudorapidity distributions of hadrons. In IC.B, sharp decrease of

FIG. 3: Temperature distribution.
FIG. 4: Pseudorapidity distribution.
temperature which is identical to steep pressure gradient at the
forward rapidity causes rapid acceleration of the longitudinal
flow at the edge of the fluid. On the other hand, in IC.A, the
pressure gradient is rather gradual. Hence, resultant deviation
from the scaling solution is smaller. Because the pressure gra-
dient exists at smaller $\eta_s$ in IC.A, however, such deviations
take place at $\eta_s \simeq 1$ while the flow keeps the scaling solutions
up to $\eta_s \simeq 2$ in IC.B. This fact explains slightly larger $\epsilon_0$ in
IC.A since faster longitudinal expansion than the scaling ex-
pansion pushes entropy per unit rapidity to forward rapidity
[13,35].

Although Figs. 3 and 4 show that there exist differences be-
tween IC.A and IC.B in the space-time evolution, it is not triv-
ial that such differences can survive at the freeze-out hyper-
surfaces. Since hadrons strongly interact and have information
only at the thermal freeze-out, differences on the freeze-out
hypersurfaces are necessary to find the signature in hadronic
experimental observables.

We show freeze-out proper time $\tau_f$ of the all fluid elements
in Fig. 5. This characterizes the shape of the freeze-out hyper-
surface, which is expected to affect the HBT radii. In Fig. 5
we can see that the system expands in the transverse direction
in both of the fluids. Due to the same transverse profile, there
is no apparent difference in the transverse direction. On the
other hand, the shape of the hypersurface in the $\eta_s$ direction
shows some variations. In IC.B, expansion appears and the
freeze-out proper time is mostly constant in the broad range
of $\eta_s$ while it moderately decreases with $\eta_s$ in IC.A. This is
a consequence of the different longitudinal flow profile (Fig. 4).
We also plot the deviation from the scaling solution at the
freeze-out in Fig. 6. Large deviation seen at forward rapidity
in IC.B (Fig. 4) survives at the freeze-out. We will see how
these differences affect the HBT radii in the next section.

V. HBT RADI

A. Two-pion correlation function

Assuming that the source is completely chaotic, we can cal-
culate the two-particle correlation momentum intensity correla-
tion function through this formula [36]

$$C_2(q,K) = 1 + \frac{|I(q,K)|^2}{I(0,k_1)I(0,k_2)}, \quad (16)$$

where $q = k_1 - k_2$ is the four-relative momentum and $K = 1/(k_1 + k_2)$ is the four-average momentum, with $k_i$ being
on-shell momentum of emitted pions. The interference term
$I(q,K)$ can be chosen as

$$I(q,K) = \int K \cdot d\sigma e^{i\sigma \cdot q} f(u\cdot K,T), \quad (17)$$

so that $I(0,k_i)$ reduces to the Cooper-Frye formula [37].

Experimentally, the two-pion correlation function is defined
as

$$C_2(q) = \frac{A(q)}{B(q)}, \quad (18)$$

where $A(q)$ is the measured two-pion pair distribution with
momentum difference $q$ and $B(q)$ is the background pair dis-
tribution generated from mixed events. Momentum accep-
tances are imposed separately in the numerator and the de-
ominator. Accounting for the large acceptance in the PHO-
BOS experiment, $0.4 < Y_{\pi\pi} < 1.3$ for three $K_T$ bins and $0.1 <
K_T < 1.4$ GeV/$c$ for three rapidity bins, we integrate the cor-
relation function as follows:

$$C(q;K_T) = 1 + \frac{\int_{0}^{1.4} dY_{\pi\pi}|I(q,K)|^2}{\int_{0}^{1.3} dY_{\pi\pi}I(0,k_1)I(0,k_2)} \quad (19)$$

$$C(q;Y_{\pi\pi}) = 1 + \frac{\int_{0}^{1.4} dK_T K_T |I(q,K)|^2}{\int_{0}^{1.4} dK_T K_T I(0,k_1)I(0,k_2)}. \quad (20)$$

For simplicity, we consider only directly emitted pions and
neglect resonance decay contributions.
B. $K_T$ dependence of the HBT radii in the Cartesian parametrization

Physical meaning of the HBT radii depends on the choice of three independent components of the relative momentum $q$. The most standard choice is the so-called Cartesian Bertch-Pratt parametrization \cite{19, 20} for $q = (q_{\text{out}}, q_{\text{side}}, q_{\text{long}})$ in which “long” means parallel to the collision axis, “side” perpendicular to the transverse component of the average momentum $K_T$ and “out” parallel to $K_T$. In the case of azimuthally symmetric system as considered here, one can put $K_T = (K_T, 0)$ so that $q_{\text{out}} = q_x$ and $q_{\text{side}} = q_y$. Note that $q_{\text{long}} = q_z$. Then, the gaussian form of the two-pion correlation function is given as \cite{21}

$$C_{2\text{fit}}(q) = 1 + \lambda \exp(-q_{\text{out}}^2 R_{\text{out}}^2 - q_{\text{side}}^2 R_{\text{side}}^2 - q_{\text{long}}^2 R_{\text{long}}^2 - 2q_{\text{out}}q_{\text{long}} R_{\text{out}} R_{\text{long}}).$$  \hspace{1cm} (21)

The HBT radii $R_i$ can be extracted by a $\chi^2$-fit to the above fitting function. For a chaotic source, the chaoticity parameter $\lambda$ should become unity. However, experimentally observed chaoticity is smaller than 1 because of such contributions as long-lived resonance decay \cite{38}. Here we fix $\lambda = 1$ in the Gaussian fit to the calculated correlation functions with Eqs. (19) and (20).

By expanding the correlation function (16) for $q \cdot x \ll 1$, the size parameters $R_i$ can be related to second order moments of the source function \cite{21}. In the Cartesian parametrization, taking the longitudinal comoving system (LCMS) makes the expression simple:

$$R_{\text{out}}^2 = \langle (\vec{r}_x - \beta_\perp \vec{r})^2 \rangle = \langle \vec{r}_x^2 \rangle - 2 \beta_\perp \langle \vec{r}_x \vec{r} \rangle + \beta_\perp^2 \langle \vec{r}^2 \rangle,$$ \hspace{1cm} (22)

$$R_{\text{side}}^2 = \langle \vec{r}_y^2 \rangle,$$ \hspace{1cm} (23)

$$R_{\text{long}}^2 = \langle \vec{r}_z^2 \rangle,$$ \hspace{1cm} (24)

$$R_{\text{col}}^2 = \langle (\vec{r}_x - \beta_\perp \vec{r}) \vec{z} \rangle,$$ \hspace{1cm} (25)

where

$$\langle A(x) \rangle = \frac{\int_E k \cdot d\sigma f(u \cdot k, T) A(x)}{\int_E k \cdot d\sigma f(u \cdot k, T)},$$ \hspace{1cm} (26)

$\vec{x} \equiv x - \langle x \rangle$, and $\beta_\perp = k_T/E_k$. Hence, $R_{\text{out}}$, $R_{\text{side}}$ and $R_{\text{long}}$ can be interpreted as a mixture of thickness of the source and emission duration, transverse source size and longitudinal source, seen from the LCMS, respectively. Validity of these expressions for a hydrodynamical model is discussed in Ref. \cite{39}. Although they have been shown to be good approximations, it is also pointed out that there are still some discrepancies and one should use fitted HBT radii for comparison with the experimental data which are obtained from the fit \cite{40}.

Figure 7 shows results for the four HBT radii compared with the experimental data measured by PHOBOS \cite{18}. For comparison of the initial conditions, any qualitative and quantitative difference cannot be seen in $R_{\text{out}}$ and $R_{\text{side}}$, as expected from Fig. 5 $R_{\text{long}}$ of IC.A is about 1 fm smaller than that of IC.B, in the lowest $k_T$ bin. This can be considered as a consequence of the fact that the deviation from the scaling solution at small $\eta_s$ is larger in IC.A, because faster flow causes more thermal suppression of the emission region \cite{13}. For these three radii, our calculation cannot reproduce the experimental results and show similar behavior with other perfect

FIG. 7: $K_T$ dependence of Cartesian HBT radii. Closed squares and open squares denote our results for IC.A and IC.B, respectively. Experimental data are taken from Ref. \cite{18}. Errorbars for the experimental data are statistical only.
fluid dynamical calculations. In the bottom of Fig. 7 result of the out-long cross term is presented. Reflected the uniform shape of the freeze-out hypersurface in Fig. 5 the value of $R_{Q1}$ of IC.B is smaller than that of IC.A. At the lowest $K_\perp$ bin, the difference is about 4 fm. Unfortunately, experimental uncertainty is still too large to distinguish which initial condition is favored. However, it should be noted that both of the two results agree with the experimental data, in spite of the disagreement of other radii.

C. Rapidity dependence of the HBT radii in the YKP parametrization

In the YKP parametrization, three independent components of the relative momentum $q$ are $q_\perp = \sqrt{q_\perp^2 + q_\parallel^2}$, $q_\parallel = q_\parallel = q_\parallel = q_{long}$ and $q_\tau = E_1 - E_2$. Then, the Gaussian fitting correlation function is given as

\[
C_{2\text{YKP}}(q) = 1 + \lambda \exp \left[ -R^2_{\perp} q^2_{\perp} - R^2_{\parallel} (q^2_\parallel - q^2_{\tau}) \right. \\
- (R^2_{\perp} + R^2_{\parallel}) (q \cdot U)^2 \right],
\]

where $U^\mu = \gamma(1,0,0,v_{YK})$, $\gamma = 1/\sqrt{1 - v^2_{YK}}$ and $v_{YK}$ is the fourth fitting parameter called YK velocity. The three HBT radii, $R_{\perp}$, $R_{\parallel}$ and $R_\tau$ are invariant under a longitudinal boost. Physical meaning of the parameters can be given in a similar manner \[23\] and becomes the simplest as follows, if one adopt the YK frame where $v_{YK} = 0$.

\[
\begin{align*}
R^2_{\perp} &= \langle r^2_{\perp} \rangle = R^2_{\text{side}}, \\
R^2_{\parallel} &\approx \langle z^2 \rangle = R^2_{\text{long}}, \\
R^2_\tau &\approx \langle t^2 \rangle,
\end{align*}
\]

The main advantage of using YKP parametrization is that the three HBT radii directly give the transverse, longitudinal and temporal source size, which are seen from the YK frame. However, one should note that the latter two, \[29\] and \[30\], are approximate expressions which hold only if the source is not opaque \[39\]. Hence, $R_{\parallel}$ and $R_\tau$ cannot always be regarded as the source sizes in the presence of strong transverse flow which makes the source highly opaque \[16\]. The general expression of $v_{YK}$ is complicated one \[23\] but it can be regarded as a longitudinal flow velocity of the source measured in an observer’s frame.

We plot results of HBT radii for the YKP parametrization in Fig. 8. Though PHOBOS measures only at small rapidity bins, we calculate the HBT radii for $Y_{\text{YK}} = 0.602, 0.877, 1.122, 1.5, 2.0, 2.5, 3.0, 3.5$ and 4.0 and show the results as a prediction. For comparison between IC.A and IC.B, $R_\perp$ seems to barely reflect the uniform structure along $\eta_6$ direction in IC.B. While $R_\perp$ shows the difference of order 1 fm at small rapidity coming from the deviation of the scaling solution as well as in the third of Fig. 7 $R_\tau$ shows little difference but agrees with the experiment. Significant differences at forward rapidity expected from Figs. 5 and 6 cannot be seen.

This will come from the fact that the number of produced particles at a proper time is roughly proportional to $t_\perp$, because of $dt_\perp = \tau d\tau d\eta_6 d^2 r$, and such differences occur rather early freeze-out proper time.

Finally the Yano-Koonin rapidity $Y_{YK} = 1/2 \ln[(1 + v_{YK})/(1 - v_{YK})]$ is shown as a function of $Y_{\pi\pi}$ in Fig. 9. Both of results from IC.A and IC.B surprisingly agree with the experimental data and show no difference between the two. At forward rapidity region, our results show deviation from the infinite boost invariant case, which is indicated by the straight line. Although our solutions of longitudinal flow show deviation from the scaling solution (Figs. 4 and 6), the result would have to larger $Y_{YK}$ than a given $Y_{\pi\pi}$ if $Y_{YK}$ correctly represents the longitudinal source velocity. Hence, this deviation will be caused by the finite size effect \[39\] which becomes more significant at forward rapidity rather than the difference in the flow velocity.
FIG. 9: The Yano-Koonin rapidity $Y_{\text{YK}}$. The identification of the symbols is the same as in Figs. 7 and 8. The solid line indicates the case of the infinite boost-invariant source.

VI. SUMMARY

In summary, we calculate the two-pion correlation function for two sources which are given by a hydrodynamical model without explicit boost invariance along the collision axis. The two initial conditions are so chosen that both of them give consistent pseudorapidity distribution with the experimental data and have different shape in the longitudinal direction. We find that there exist some differences in the space-time evolution of the fluids in spite of the fact that both fluids give similar particle distribution. The HBT radii are extracted from the two-pion correlation functions and compared with the experiment. In the Cartesian parametrization, the out-long cross term which arises at nonzero rapidities shows a difference between two initial conditions and the good agreements with the experimental data. The correlation function is also analyzed with the YKP parametrization. We find a small difference between the two initial conditions in $R_\parallel$ which reflects deviation from the scaling solution in the longitudinal expansion. In spite of the disagreement of the HBT radii, the YK rapidity shows a good agreement with the experimental data.

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