Fundamental spatiotemporal decoherence: 
a key to solving the conceptual problems of black holes, 
cosmology and quantum mechanics

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Abstract

Unitarity is a pillar of quantum theory. Nevertheless, it is also a source of several of its conceptual problems. We note that in a world where measurements are relational, as is the case in gravitation, quantum mechanics exhibits a fundamental level of loss of coherence. This can be the key to solving, among others, the puzzles posed by the black hole information paradox, the formation of inhomogeneities in cosmology and the measurement problem in quantum mechanics.
In ordinary quantum field theory space-time is taken to be a classical variable that can be measured with arbitrarily high precision. We know however, that this is only an approximation. In reality there exist fundamental limitations to how accurately we can measure space and time. In recent papers we have argued that the lack of accuracy in measuring time leads to a loss of unitarity in ordinary quantum mechanics \[1\]. This loss of unitarity was sufficiently significant to render the black hole information puzzle unobservable \[2\]. In this essay we would like to explore the implications of the fact that space cannot be measured with arbitrary precision either.

Pioneering work in this area was done by Salecker and Wigner \[3\], further elaborated by Ng and Van Dam \[4\] who noted that there are fundamental limitations to the accuracy with which space-time can be measured. It should be noted that these limitations are due to the measurement apparatus and they tend to be considerably larger than the intrinsic uncertainties due to the quantum fluctuations of the metric. That is, even if we ignore the latter and assume space-time to be classical, there will still be limitations in the accuracy with which the former can be measured. Covariantly, for a space-time interval \( s \) the bound they find for the error of measurement is

\[
\delta |s| \sim L_{\text{Planck}}^{3/4} |s| / L_{\text{Planck}}.
\]

We will assume that space-time is flat for the rest of this paper, though it is possible to generalize the results with the usual caveats of quantum field theory on curved space-time. In spite of this, we are entering into the discussion of quantum field theory crucial elements that have to be used in formulating a theory of quantum gravity. Namely, in quantum gravity (at least formulated canonically) one starts with a spatial manifold with coordinates \( r \), that is due to a slicing of a space-time in which a time parameter \( t \) has been identified. Both the spatial and temporal variables \( t, r \) are fiducial variables in the construction and do not have physical meaning. To give physical meaning to space-time points one has to recourse to values of physical variables. For instance dust and scalar fields have been used to give physical meaning to the coordinates \[5\], see also \[6\].

We will consider a set of fields \( \phi(\vec{r}, t) \) and we will assume that one can, in the spirit of \[5\], construct observables of the theory whose level sets can be used to label physical points \( \vec{R}(\phi(\vec{r}, t)) \) and \( T(\phi(\vec{r}, t)) \) (from now on we will drop the vector symbol on the \( r \) to simplify notation). Quantum mechanically, one would like to ask questions like “what is the probability that an observable of the field \( \psi \) take a certain value \( \psi_0 \) at a given coordinate point \( \vec{R}, T \) in space-time”. Such question will be formulated as a conditional probability,
following the same lines as in the quantum mechanical case [1].

To make contact with ordinary quantum field theory we will assume a semi-classical behavior for the clock and measuring rods in order to compare with the ideal case. Given the similarities with previously obtained results in the context of quantum mechanics [1] we will directly jump into the final stages of the calculation. We assume the quantum state and evolution operators factorize into a “clock and rods” and a system under study. We also assume the density matrix for the system under study is given by $\rho_{\text{sys}}$. One ends up with an effective density matrix as a function of the clock and rod variables,

$$\rho(T, R) \equiv \int_{-\infty}^{\infty} dt dR \rho_{\text{sys}} U_{\text{sys}}(t, r) \rho_{\text{sys}} U_{\text{sys}}^\dagger P_{t, r}(T, R),$$

(1)

with $P_{t, r}(T, R)$ the probability that the variables $T, R$ occur as a function of $t, r$, and from now on we drop the suffix “sys” in the density matrix. Again we refer the reader to the quantum mechanical case for details [1].

We have therefore ended with the standard probability expression with an “effective” density matrix in the Schrödinger picture given by $\rho(T, R)$. In this unorthodox representation the space-time dependence of one field operator is shifted to the density matrix. We do this to keep things as parallel as possible to the quantum mechanical case, but one later pays a price at the time of computing correlation functions, which are the usual observables of quantum field theory. Now that we have identified what will play the role of a density matrix in terms of “real clocks and rods”, we would like to see what happens if we assume they are behaving semi-classically. We start by assuming that the probability for a measurement of $R, T$ for a given value of $r, t$ is given by a function $P_{t, r}(T, R)$ centered at $t, r$ and with a width $b(s)$ where $(T, R)^\mu = sn^\mu$ and $n^\mu$ is a unitary four dimensional vector (we exclude in a first analysis the use of null intervals for simplicity). Under these assumptions the Lorentz invariant equation for the density matrix turns out to be,

$$\frac{\partial \rho(s)}{\partial s} = i[\rho, P] + \sigma(s)[P, [P, \rho]] + ...$$

(2)

with $P \equiv P^\mu n_\mu$ and $P^\mu$ is the spacetime canonical momentum operator. The extra term is determined by the rate of variation of the width of the distribution, $\sigma(s) = \partial b(s)/\partial s$. The equation is function only of the invariant space-time interval from the point $R, T$ at which the field operator was originally defined to the origin selected for the measurement of these “quasi-Lorentzian” coordinates. A similar equation was recently considered by Milburn [8].
though he did not derive it from the relational measurement of space time points. In his approach he assumed $\sigma$ to be a constant, whereas for us it follows from the fundamental uncertainty relation $\delta s = L_{\text{Planck}}^{2/3} s^{1/3}$, which yields $b(s) = L_{\text{Planck}}^{4/3} (s_{\text{max}}^{2/3} - (s_{\text{max}} - s)^{2/3})$ where $s_{\text{max}}$ is the spatiotemporal “length” of the experiment. This leads to a decoherence effect in the four-momentum basis,

$$\rho(s)_{k_n,k_m} = \rho_{k_n,k_m}(0) \exp \left( -is(k^\mu_n - k^\mu_m)n_\mu \right) \exp \left( -\left[ (k^\mu_n - k^\mu_m)n_\mu \right]^2 L_{\text{Planck}}^{4/3} s^{2/3} \right).$$

(3)

where we have chosen the “optimal” measurement device, i.e. $s_{\text{max}} = s$. This is the spacetime extension of our previous result that includes temporal decoherence as a particular case $n^i = 0, n^0 = 1, s = T$. We have presented results in terms of the density matrix. We have not completed an analysis in terms of the more familiar observables of quantum field theory, correlation functions, but preliminary calculations show a similar dependence with distance and the difference of momentum components.

Is there a chance of observing experimentally these effects? Simon and Jaksch [9] have argued that there is a better chance of detecting spatial decoherence than the purely temporal one we had proposed. The purely temporal decoherence needed the construction of quantum state superpositions with energy differences between levels of $10^{10} eV$. Though not out of the question in the future, such states are not available in the lab today. For the spatial decoherence introduced here, better experiments could be conceived.

Now that we have established the basis for spatio-temporal decoherence, let us remark briefly on the possible implications of this mechanism for some of the most basic conceptual problems of modern physics:

- The construction we have carried out is the most natural to deal with space and time in a diffeomorphism invariant theory, leading to a relational description of nature that solves “the problem of time” in a well defined implementation of canonical quantum gravity where the conditional probabilities can be computed without conceptual obstructions and which originated this point of view [10, 11]. As such, it appears as natural and conceptually strong, yet it leads to a fresh perspective on several conceptual problems in physics.

- As we argued in [2] having a fundamental mechanism of loss of coherence is able to render the black hole information puzzle unobservable. The fact that pure states evolve naturally into mixed states implies that there is no puzzle in such an evolution when it occurs when a black hole evaporates. In addition to this a detailed calculation [2] shows
that the order of magnitude of the speed of the effect is appropriate in the context of black hole evaporation.

- The mechanism for fundamental decoherence can lead to a better understanding of the measurement problem in quantum mechanics. In the usual system-environment interaction the off-diagonal terms of the density matrix oscillate as a function of time. Since the environment is usually considered to contain a very large number of degrees of freedom, the common period of oscillation for the off-diagonal terms to recover non-vanishing values is very large, in many cases larger than the life of the universe. This allows to consider the problem solved in practical terms, yet one is left with a conceptual puzzle: could the off diagonal terms at least in principle reappear? When one adds the effect we discussed, since it suppresses exponentially the off-diagonal terms, one never has the possibility that the latter will see their initial values restored, no matter how long one waits.

- In the standard inflationary paradigm for cosmology it is assumed that quantum fluctuations in the inflaton give rise to the cosmological perturbations that seed the formation of structure in the universe. A puzzle arises since the fluctuations in the inflaton are quantum in nature whereas the seeds for structure formation are classical. Several mechanisms related to environmental decoherence have been proposed to solve the puzzle. A recent critique [12] however argues strongly that new physics is needed to fully explain the quantum to classical transition. Our mechanism can provide the needed “new physics” required for such a transition. Given its momentum dependence, our effect would naturally destroy correlations of short wavelength. Recalling that the environmental decoherence makes the reduced density matrix for the large modes obtained by tracing out the unobservable short wavelength modes, take a quasi diagonal form, our effect would allow to explain the passage from this reduced matrix to a true statistical mixture of long wavelength inhomogeneities that will seed the formation of structure.

Summarizing, the use of the natural relational ideas needed to discuss the physics of gravity yields modifications in quantum theory. Though too small to be observed in the lab today, the modifications are profound enough to alter our understanding of several of the most challenging conceptual problems of modern physics.

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