Hints of Isocurvature Perturbations in the Cosmic Microwave Background

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The improved data on the cosmic microwave background (CMB) anisotropy allows a better determination of the adiabaticity of the primordial perturbation. Interestingly, we find that the CMB favors a significant contribution of a primordial isocurvature mode where the entropy perturbation is positively correlated with the primordial curvature perturbation and has a large spectral index ($n_{\text{iso}} \sim 3$). With 4 additional parameters we obtain a better fit to the CMB data by $\Delta \chi^2 = 9.6$ compared to an adiabatic model. At more than 95% C.L., the nonadiabatic contribution to the CMB temperature variance is nonzero; indeed positive. For the best-fit model it is 4%.

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Introduction.—A fundamental question in cosmology is the origin of the density perturbation, from which the structure of the universe (galaxies, and galaxy clusters) has grown. Today a popular scenario is inflation, where the perturbation originates as a quantum fluctuation of the inflaton field.

Clues to the origin of structure can be sought in the nature of this perturbation. Simple inflation models produce adiabatic perturbations, but more complicated (multi-field) models may also produce nonadiabatic perturbations.

Current observations are consistent with adiabatic primordial perturbations, but sizable deviations from adiabaticity remain allowed. Here “primordial” refers to the early radiation-dominated epoch (e.g., at some time soon after big bang nucleosynthesis), when all cosmological scales ($\gtrsim 1$ Mpc) are well outside the horizon. Adiabatic perturbations remain adiabatic while outside the horizon, but give rise to entropy perturbations as they enter the horizon. Adiabatic perturbations are completely characterized by the associated (comoving gauge) curvature perturbation $\mathcal{R}$, whereas nonadiabatic perturbations have entropy perturbations $S_{ij} \equiv \delta_i/(1+w_i)-\delta_j/(1+w_j)$ between different constituents $i, j$ to the energy density. Here $\delta$ is the dimensionless relative density perturbation and $w \equiv p/\rho$ is the ratio of pressure to energy density.

A general perturbation can be divided into an adiabatic mode and a number of isocurvature modes, which evolve independently. The adiabatic mode has $S_{ij} = 0$ initially (i.e., outside the horizon in the radiation-dominated epoch), whereas isocurvature modes have $\mathcal{R} = 0$ and $S_{ij} \neq 0$ initially. There are four different types of isocurvature perturbations $[2]$ — the cold dark matter (CDM), baryon, neutrino density, and neutrino velocity isocurvature modes. For simplicity, we consider here only the CDM mode, with an initial entropy perturbation

$$S \equiv \delta_c - \frac{3}{4} \delta_\gamma .$$

The baryon mode is observationally very similar.

Even though purely isocurvature perturbations have been ruled out $[2]$, the data still do allow a subdominant ($\sim 10\%$ level $[3]$) isocurvature contribution. Depending on how the primordial $\mathcal{R}$ and $S$ perturbations were generated, they may be correlated with each other $[2]$.

Observationally the CDM isocurvature mode differs from the adiabatic mode in the locations of the acoustic peaks in the CMB angular power spectrum $C_\ell$. The separation between neighboring peaks in $C_\ell$ is determined by the sound horizon angle $\theta = r_s/d_A$, where $r_s$ is the sound horizon at last scattering and $d_A$ the angular diameter distance to last scattering. Since $\theta$ does not depend on the nature of the perturbations, the peak separation is the same for adiabatic and isocurvature modes. But for the CDM isocurvature mode the peak positions are in the opposite phase compared to the adiabatic mode. If the perturbations are correlated, this produces a contribution to $C_\ell$ which is intermediate in amplitude and peak position between the uncorrelated contributions. Thus the presence of an isocurvature contribution, especially a correlated one, appears in $C_\ell$ as a change in the ratio of peak separation to the first peak position. Since the first peak is well fixed by the present data, an isocurvature contribution would appear as a shift in the position of the other peaks, i.e., as a reduction in peak separation. CMB measurements of increasing accuracy allow thus a tighter constraint on the isocurvature contribution. For studies utilizing the Wilkinson Microwave Anisotropy Probe (WMAP) 1-year data $[6]$, see $[4, 7, 8, 9, 10, 11, 12, 13, 14]$.

The WMAP 3-year data $[15]$ is an improvement in this respect $[16, 17, 18]$. While the first peak was already measured very accurately in the first year, the determination of the second peak shape and location is improved with the 3-year data. Likewise the new Boomerang data $[19]$ begins to define the third peak. This motivates a new study to update our earlier results $[6]$.

The focus of this letter is on what the CMB data say about the nature of primordial perturbations. Thus we use CMB and large-scale structure (LSS) data only, but
address other cosmological data in the end. Inclusion of LSS data was needed to break certain parameter degeneracies [20] and to constrain extreme values for spectral indices. Current CMB data do not cover with good accuracy a sufficient range of scales to constrain well the several independent spectral indices of our model. The Planck satellite will eventually fix this situation.

Model.—We consider a flat ($\Omega_0 = 1$) $\Lambda$CDM model with primordial curvature and entropy perturbations, which may be correlated. For details of our model and parameterization, see [3]. We repeat below just the main points.

We divide the primordial curvature perturbation into an uncorrelated and a fully correlated part. We assume the power spectra of these perturbations and correlations can be characterized by simple power laws, but allow different spectral indices for the entropy and curvature perturbation. Thus the spectra can be written as

\[
P_R(k) = C_R^R(k) = A^2_\ell \hat{k}^{\alpha_{ad1}} + A^2_\ell \hat{k}^{\alpha_{ad2}} - 1,
\]

\[
P_S(k) = C_S^S(k) = B^2 \hat{k}^{\alpha_{iso}} - 1,
\]

\[
C_{RS}(k) = C_{SR}(k) = A_\ell B \hat{k}^{\alpha_{cor}} - 1,
\]

where $\hat{k} = k/k_0$ and $k_0 = 0.01$ Mpc$^{-1}$ (corresponding roughly to CMB multipole $\ell \sim 140$) is the pivot scale at which the amplitudes are defined. The spectral index $\alpha_{cor}$ is not independent but defined by the correlated quantities, $\alpha_{cor} = (\alpha_{ad2} + \alpha_{iso})/2$. For motivation and discussion of this parametrization in the context of inflation, see [1, 21, 22, 23].

The total CMB angular power spectrum $C_\ell$ can now be divided into four components: the uncorrelated adiabatic part, the correlated adiabatic part, the isocurvature part, and the correlation between the last two. We write the total $C_\ell$ as

\[
C_\ell = A^2_\ell \left[(1 - \alpha)(1 - |\gamma|)\hat{\gamma}_{ad1} + (1 - \alpha)|\gamma|\hat{\gamma}_{ad2}ight]
+ \alpha \hat{\gamma}_{iso} + \text{sign}(\gamma) \sqrt{\alpha(1 - \alpha)|\gamma|} \hat{\gamma}_{cor}
\equiv C_{ad1} + C_{ad2} + C_{iso} + C_{cor},
\]

where we have defined the total amplitude, the isocurvature fraction, and the correlation at the pivot scale

\[
A^2 \equiv A^2_\ell + A^2_\ell + B^2, \quad \alpha \equiv \frac{B^2}{A^2}, \quad \alpha \in [0, 1],
\]

\[
\gamma \equiv \text{sign}(A_\ell B) \frac{A^2_\ell}{A^2_\ell + A^2_\ell}, \quad \gamma \in [-1, 1].
\]

The $\hat{\gamma}_{\ell}$ denote spectra obtained with unit amplitude ($A_\ell = 1$, $A_\ell = 1$, or $B = 1$).

Analysis.—We created 8 Monte Carlo Markov chains (MCMC) using the CosmoMC [24] engine, which we have modified to handle correlated adiabatic and isocurvature modes. We accumulated a total of 295,809 steps with average multiplicity 17.1, summing up to a total of 5,065,805 samples.

Likelihood of a model was assessed using the WMAP 3-year data and likelihood code [13, 25, 26], Boomerang [19] and ACBAR [27] data, and the LSS data from the SDSS data release 4 luminous red galaxy sample [20, 28]. To ignore nonlinear corrections to large scale structure formation, only the first 14 $k$-bands were used. We increased the precision of the beam and point source correction in the WMAP likelihood code following [29].

Our model has 10 primary parameters for which we assign flat prior probabilities. The six that are present also in the adiabatic model are the physical baryon density ($\omega_b = h^2 \Omega_b$), the physical CDM density ($\omega_c = h^2 \Omega_c$), the sound horizon angle ($\theta$), the optical depth to reionization ($\tau$), logarithm of the overall primordial perturbation amplitude ($\ln A^2$), and the adiabatic spectral index ($n_{ad}$, or $n_{ad1}$). We marginalize analytically over the galaxy bias parameter $b$. Correlated isocurvature brings in four additional parameters: the spectral indices $n_{ad2}$, $n_{iso}$ introduced in [2], and the fractional amplitudes $\alpha$ and $\gamma$ introduced in [1] and [3].

Results.—We show 1-d marginalized likelihoods for selected primary and derived parameters in Fig. 1.

The current data lead to likelihood peaks at clearly nonzero values for $\alpha$ and favor a positive $\gamma$. (This significantly reduces the pivot-scale dependence of the likelihoods discussed in [3].) The isocurvature fraction $\alpha$ gives the ratio of the primordial entropy perturbation power to the total perturbation power at the pivot scale. If the subdominant entropy perturbation is correlated with the dominant curvature perturbation it has a much stronger effect on the observables. To account for this we also show $\alpha_{cor} \equiv \text{sign}(\gamma) \sqrt{\alpha(1 - \alpha)|\gamma|}$, which gives the relative “weight” of the correlation spectrum $C_{cor}^\ell$ in $C_\ell$, see Eq. 3. The values we find for these parameters are $\alpha = 0.072 \pm 0.051$ (mean ± stdev), with $\alpha < 0.169$ at 95% C.L. and $\alpha_{cor} = 0.109 \pm 0.069$ (or median and 68% C.L. $\alpha_{cor} = 0.106^{+0.071}_{-0.063}$), with $-0.021 < \alpha_{cor} < 0.258$ at 95% C.L. The correlation between the isocurvature and adiabatic modes is positive, $\gamma > 0.0021$ at 95% C.L.

Since the definitions of these parameters depend on the choice of pivot scale, we also define

\[
\alpha_T \equiv \frac{\sum(2\ell + 1)C_{iso}^\ell + C_{cor}^\ell}{\sum(2\ell + 1)C_\ell^\ell},
\]

which gives the total nonadiabatic contribution to the CMB temperature variance, see [2]. We find $\alpha_T = 0.037 \pm 0.016$. Most interestingly this is positive both at 68% C.L. (0.022 < $\alpha_T < 0.052$ ) and at 95% C.L. (0.008 < $\alpha_T < 0.070$). Thus the CMB data favors a ∼ 4% nonadiabatic contribution.

We give the parameter values for our best-fit model and also those of the best-fit adiabatic model in Table I. A comparison between these models is shown in Fig. 2. The WMAP 3-year data prefer a slightly narrower 2nd peak than the adiabatic model can produce. This holds
Figure 1: Marginalized likelihood functions for selected primary and derived parameters. The solid black curves are our new results using WMAP 3-year data and Boomerang data from the 2003 flight (+ ACBAR & SDSS). The red/gray curves are for an adiabatic model using the same data. The dashed blue curves are from our previous study [4] using data available in 2004.

Table I: The best-fit models. A: The full model with a correlated isocurvature mode. B: The adiabatic model.

<table>
<thead>
<tr>
<th>ω_b</th>
<th>ω_c</th>
<th>100θ</th>
<th>τ</th>
<th>n_ad1</th>
<th>n_ad2</th>
<th>n_iso</th>
<th>α</th>
<th>γ</th>
<th>Ω_L</th>
<th>H_0</th>
<th>σ_S</th>
<th>ζ_ad</th>
<th>ζ_T</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0219</td>
<td>0.1051</td>
<td>1.065</td>
<td>0.0941</td>
<td>0.978</td>
<td>0.938</td>
<td>3.51</td>
<td>0.0513</td>
<td>0.315</td>
<td>0.810</td>
<td>81.7</td>
<td>1.12</td>
<td>0.124</td>
</tr>
<tr>
<td>B</td>
<td>0.0227</td>
<td>0.1124</td>
<td>1.043</td>
<td>0.0846</td>
<td>0.960</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.736</td>
<td>71.5</td>
<td>0.80</td>
<td></td>
</tr>
</tbody>
</table>

This improvement comes entirely from the fit to the WMAP and Boomerang temperature $C_{ℓ}$; for the WMAP part entirely from the 2nd peak. For the first time the isocurvature parameters are now clearly nonzero.

The other major effect is that the correlated isocurvature model favors a smaller CDM density $ω_c$ (also the baryonic density $ω_b$ is down, but not by as much). This can be understood as an effect of the correlation component $C_{ℓ}^{cor}$, which raises the first and third peaks with respect to the second one. See Fig. 2. This has to be compensated by $ω_m$ and $ω_b$. Lowering both of them, the second peak is raised with respect to the first and third.

For fixed $ω_c$ and $ω_b$, increasing $θ$ leads to a larger $Ω_L$ and a larger Hubble constant $H_0$. For fixed $θ$ and $ω_b$, a lower $ω_c$ requires an even larger $Ω_L$ and $H_0$.

Thus these models have a large $H_0$ and a small matter density parameter $Ω_m = 1 - Ω_L$, causing some tension with other cosmological data, like the Hubble Space Telescope (HST) value $H_0 = 72 ± 8$ km/s/Mpc [30] and the estimates of $Ω_m$ from Supernova Ia data ($Ω_m = 0.29^{+0.03}_{−0.05}$ from [31], or $Ω_m = 0.263 ± 0.042 ± 0.032$ from [32]). To assess this, we postprocessed our likelihoods by weighting (importance sampling [24]) our MCMC chains with corresponding Gaussian distributions. The effect of the HST $H_0$ constraint was minor and did not eliminate the preference for nonzero $α$ and positive $α_T$. The same is true for the weaker $Ω_m = 0.263 ± 0.074$ Gaussian prior. On the other hand, a Gaussian $Ω_m = 0.30±0.04$ prior is tight enough to move the maximum 1-d likelihood of $α$ to zero (but the 95% C.L. upper limit does not change much), and to shift the $α_T$ distribution to the left so that zero is included in the 95% C.L. range, $−0.006 < α_T < 0.054$.

Figure 2: The CMB temperature angular power spectrum for our best-fit model (black) compared to the best-fit adiabatic model (red/gray). The dashed blue curve shows the nonadiabatic contribution. The inset shows the 2nd and 3rd peaks.

for the third peak also, but now the peak position and width are determined by the Boomerang data.

This feature in the data can be accounted for by a contribution from the correlated isocurvature component. It can narrow down the 2nd peak without affecting the 1st peak position, which is very accurately determined by the data. Increasing $θ$ shifts the whole peak structure to the left, making the peaks narrower. Adding a small positively correlated isocurvature component returns the 1st peak to its place.

Compared to the adiabatic model, we have added 4 parameters and achieved an improvement of $Δχ^2 = 9.6$. 

As discussed in [4], the spectral index $n_{\text{iso}} \simeq 3$ (when $n_{\text{ad}} \simeq 1$) means that the relative isocurvature contribution to $C_l$ is roughly the same on all scales (multipoles). On the other hand, in the LSS matter power spectrum the isocurvature and correlation components overtake the adiabatic ones at small scales. Thus there is more (matter) power at small scales (large $k$), reflected in a larger value of the rms mass fluctuation on 8 Mpc scale $\sigma_8$. While the adiabatic model favors $\sigma_8 \approx 0.8$, our model favors $\sigma_8 \approx 1$. Assuming that the power law is a good approximation for the primordial spectra also for smaller scales, Lyman-\(\alpha\) data can significantly constrain $n_{\text{iso}}$ [32]. Creating new MCMC chains with Lyman-\(\alpha\) data included like in [33] we checked that the peak in the likelihood of $n_{\text{iso}}$ was shifted to 2.1, and $\alpha$ to zero. In addition, the other likelihoods became more like in the adiabatic case. The peak in $\alpha_T$ remained clearly positive, but the 68% C.L. region, $\alpha_T = 0.022^{+0.017}_{-0.025}$, now marginally included $\alpha_T = 0$.

**Conclusion.**—The acoustic peak structure in the latest CMB data clearly favors a few per cent nonadiabatic contribution. Our best fit to the CMB + LSS data has a significant contribution from a positively correlated isocurvature mode, so that the isocurvature fraction, i.e., has a significant contribution from a positively correlated batic contribution. Our best fit to the CMB + LSS data clearly favors a few per cent nonadiabatic models that give the best fits to the CMB data. Including like in [33] we checked that the peak in the likelihood of $n_{\text{iso}}$ was shifted to 2.1, and $\alpha$ to zero. But the 68% C.L. region, $\alpha_T = 0.022^{+0.017}_{-0.025}$, now marginally included $\alpha_T = 0$.

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