Abstract

There is something missing in our understanding of the origin of the seeds of Cosmic Structure. The fact that the fluctuation spectrum can be extracted from the inflationary scenario through an analysis that involves quantum field theory in curved space-time, and that it coincides with the observational data has lead to a certain complacency in the community, which prevents the critical analysis of the obscure spots in the derivation. The point is that the inhomogeneity and anisotropy of our universe seems to emerge from an exactly homogeneous and isotropic initial state through processes that do not break those symmetries. This article gives a brief recount of the problems faced by the arguments based on established physics, which comprise the point of view held by a large majority of researchers in the field. The conclusion is that we need some new physics to be able to fully address the problem. The article then exposes one avenue that has been used to address the central issue and elaborates on the degree to which, the new approach makes different predictions from the standard analyses. The approach is inspired on Penrose’s proposals that Quantum Gravity might lead to a real, dynamical collapse of the wave function, a process that we argue has the properties needed to extract us from the theoretical impasse described above.

1 Introduction

The last decade has undoubtedly been one of great advances in physical cosmology. One of the most important achievements is the precision measurements of the anisotropies in the CMB\cite{1} together with what seems to be their natural explanation within the context of the inflationary scenarios\cite{2}. There is however a serious hole in this seemingly blemish-less suit of the Emperor: The description of our Universe – or the relevant part thereof- starts\cite{3} with an initial condition which is homogeneous and isotropic both in the background space-time and in the quantum state that is supposed to describe the “fluctuations”, and it is quite easy to see that the subsequent evolution through dynamics that do not break these symmetries can only lead to an equally homogeneous and anisotropic universe\cite{4}. In fact, if we were to think in terms of first principles, we would start by acknowledging that the correct description of the problem at hand would involve a full theory of quantum gravity coupled to a theory of all the matter quantum fields,

\footnote{1}{Here we refer to the era relevant to the starting point of the analysis that leads to the “fluctuation spectrum”. In the standard view of inflation, the relevant region of the universe starts with a Plank regime containing large fluctuations of essentially all relevant quantities, but then, a large number of inflationary e-folds leads to an homogeneous and isotropic universe which is in fact the starting point of the analysis that takes us to the primordial fluctuation spectrum. One might wish, instead, to regard such fluctuation spectrum as a remnant of the earlier anisotropic and inhomogeneous conditions but then one ends up giving up any pretense that one can explain its origin and account for its specific form.}

\footnote{2}{In fact many arguments have been put forward in order to deal with this issue, that is often phrased in terms of the Quantum to Classical transition – without focussing on the required concomitant breakdown of homogeneity and isotropy in the state– the most popular ones associated with the notion of decoherence. These the alternatives have been critically discussed in\cite{4}.}
and that there, the issue would be whether we start with a quantum state that is homogeneous and isotropic or not?. Even if these notions do not make sense within that level of description, a fair question is whether or not, the inhomogeneities and anisotropies we are interested on, can be traced to aspects of the description that have no contra-part in the approximation we are using. Recall that such description involves the separation of background vs. fluctuations and thus must be viewed only as an approximation, that allows us to separate the nonlinearities in the system—as well as those aspects that are inherent to quantum gravity—from the linear part of problem represented by the fluctuations, which are treated in terms of linear quantum field theory. In this sense, we might be tempted to ignore the problem and view it as something inherent to such approximation. This would be fine, but we could not argue that we understand the origin of the CMB spectrum, if we view the asymmetries it embodies as arising from some aspect of the theory we do not rely or touch upon. In fact, what we propose in the following treatment is to bring up one element or aspect, that we view as part of the quantum gravity realm, to the forefront in order to modify—in a minimalistic way—the semiclassical treatment, that, as we said, we find lacking. It is of course not at all clear that the problem we are discussing should be related to quantum gravity, but since that is the only sphere of fundamental physics for which we have so far failed to find a satisfactory conceptual understanding, we find quite natural to associate the two. In this sense we would be following the ideas of Penrose regarding the fundamental changes, that he argues, are needed in quantum mechanics and their connection to quantum gravity. He argues that quantum gravity might play a role in triggering a real dynamical collapse of the wave function of systems. His proposals would have a system collapsing whenever the gravitational interaction energy between two alternative realizations that appear as superposed in a wave function of a system reaches a certain threshold which is identified with $M_{\text{Planck}}$. The ideas can, in principle, lead to observable effects and, in fact, experiments to test them are currently being contemplated (although it seems that the available technology can not yet be pushed to the level where actual tests might be expected to become a reality soon). We have considered in a situation for which there exists already a wealth of empirical information and which we have argued can not be fully understood without involving some New Physics, which required features would seem to be quite close to Penrose’s proposals.

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There are of course many open issues in fundamental understanding of physics that are not in principle connected with the issue of quantum gravity, however it is only in this latter field that the problems seem to be connected with deep conceptual issues and where one can envision the possibility that their resolution might require a fundamental change of paradigm, as would be the case if we find we must modify the laws of quantum mechanics.
2 The quantum origin of the seeds of cosmic structure

One of the major claimed successes of Inflationary cosmology is its reported ability to predict the correct spectrum for the primordial density fluctuations that seed the growth of structure in our Universe. However when one thinks about it one immediately notes that there is something truly remarkable about it, namely that out of an initial situation which is taken to be perfectly isotropic and homogeneous and based on a dynamics that preserves those symmetries one ends with a non-homogeneous and non-isotropic situation. Most of our colleagues who have been working in this field for a long time would reassure us, that there is no problem at all by invoking a variety of arguments. It is noteworthy that these arguments would tend to differ in general from one inflationary cosmologist to another \cite{6}. Other cosmologists do acknowledge that there seems to be something unclear at this point \cite{7}. In a recent paper \cite{4}, a critical analysis of such proposals has been carried out indicating that all the existing justifications fail to be fully satisfactory. In fact, the situation at hand, is quite different from any other situation usually treated using quantum mechanics as can be seen by noting that, while in analyzing ordinary situations, quantum mechanics offers us, at least one self-consistent assignment at each time of a state of the Hilbert space to our physical system (we are of course thinking of the Schroedinger picture), the same is not true for the standard analysis of the current problem. It is well known, that in certain instances there might be several mutually incompatible assignments of that sort, as for instance when contemplating the two descriptions offered by two different inertial observers who consider a given a specific EPR experiment. However, as we said, in all other known cases, one has at least one description available. The reader should try the consideration of such assignment – of a state at each time – when presented with any of the proposed justifications offered to deal with the issue of the transition from a symmetric universe to a non-symmetric one. The reader will find that in each case he/she will be asked to accept one of the following: i) our universe was not really in that symmetric state (corresponding to the vacuum of the quantum field), ii) our universe is still described by a symmetric state, iii) at least at some points in the past the description of the state of our universe could not be done within quantum mechanics, iv) quantum mechanics does not correspond to the full description of a system at all times, or v) our own observations of the universe mark the transition from a symmetric to an asymmetric state. It should be clear that none of these represent a satisfactory alternative. In particular, if we want to claim, that we understand the evolution of our universe and its structure – including ourselves – as the result of the fluctuations of quantum origin in the very early stages of our cosmology. Needless to say that none of these options will be explicitly called upon in the arguments one is presented with, however one or more would be hidden, perhaps in a subtle way, underneath some of the aspects of the explanation. For a more thorough discussion we refer the reader to \cite{3}.

The interesting part of this situation is that one is forced to call upon some novel
physical process to fill in the missing or unacceptable part of the justification of the steps that are used to take us from that early and symmetric state, to the asymmetric state of our universe today, or the state of the universe we photograph when we look at the surface of last scattering in the pictures of the CMB. In [4] we have considered in this cosmological context a proposal calling for a self induced collapse of the wave function along the general lines conceived by Penrose, and have shown that the requirement that one should obtain results compatible with current observations is already sufficient to restrict in important ways some specific aspects of these novel physics. Thus, when we consider that the origin of such new physics could be traced to some aspects of quantum gravity, one would be already in a position of setting phenomenological constraints on at least this aspect of the quantum theory of gravitation.

In the following we give a short description of this analysis. The starting point is as usual the action of a scalar field coupled to gravity

$$ S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R[g] - \frac{1}{2} \nabla_a \phi \nabla_b \phi g^{ab} - V(\phi) \right], $$

(1)

where $\phi$ stands for the inflaton or scalar field responsible for inflation and $V$ for the inflaton’s potential. One then splits both, metric and scalar field into a spatially homogeneous (‘background’) part and an inhomogeneous part (‘fluctuation’), i.e. $g = g_0 + \delta g$, $\phi = \phi_0 + \delta \phi$.

The equations governing the background unperturbed Friedman-Robertson universe with line element $ds^2 = a(\eta)^2 [-d\eta^2 + \delta_{ij} dx^i dx^j]$, and the homogeneous scalar field $\phi_0(\eta)$ are, the scalar field equation

$$ \ddot{\phi}_0 + 2 \frac{\dot{a}}{a} \dot{\phi}_0 + a^2 \partial_\phi V[\phi] = 0, $$

(2)

and Friedman’s equation

$$ 3 \frac{\dot{a}^2}{a^2} = 4\pi G(\dot{\phi}_0^2 + 2a^2 V[\phi_0]). $$

(3)

The background solution corresponds to the standard inflationary cosmology which written using a conformal time, has, during the inflationary era a scale factor $a(\eta) = -\frac{1}{H_\eta}$, with $H_\eta^2 \approx (8\pi/3)GV$ and with the scalar $\phi_0$ field in the slow roll regime so $\dot{\phi}_0 = -(a^3/3\dot{a})V'$. This era is supposed to give rise to a reheating period whereby the universe is repopulated with ordinary matter fields, and then to a standard hot big bang cosmology leading up to the present cosmological time. The functional form of $a(\eta)$ during these latter periods is of course different but we will ignore such details on the account that most of the change in the value of $a$ occurs during the inflationary regime. The overall normalization of the scale factor will be set so $a = 1$ at the “present

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We will be using units where $c = 1$ but will keep $\hbar$ (with units of Mass $M$ times Length $L$ ), and $G$ (with units of $L^2/M$ ) explicitly throughout the manuscript. The coordinates in the metric $\eta, x^i$ will have units of length $L$ but the metric components, such as the scale factor $a$ will be dimensionless. The field $\phi$ has units of $(M/L)^{1/2}$, and the potential $V$ has units of $M/L^3$.
cosmological time”. The inflationary regime would end for a value of $\eta = \eta_0$, negative and very small in absolute terms.

The perturbed metric can be written

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi) d\eta^2 + (1 - 2\Psi) \delta_{ij} dx^i dx^j \right],$$

where $\Psi$ stands for the relevant perturbation and is called the Newtonian potential.

The perturbation of the scalar field leads to a perturbation of the energy momentum tensor, and thus Einstein’s equations at lowest order lead to

$$\nabla^2 \Psi = 4\pi G \dot{\phi} \delta \dot{\phi} \equiv s \Gamma$$

where we introduced the abbreviation $s = 4\pi G \dot{\phi} \delta \dot{\phi}$ and the quantity $\Gamma$ as the aspect of the field that acts as a source of the Newtonian Potential, which for slow roll approximation considered here is just $\Gamma = \delta \dot{\phi}$. Now, write the quantum theory of the field $\delta \phi$. It is convenient to consider instead the field $y = a \delta \phi$. We consider the field in a box of side $L$, and decompose the real field $y$ into plane waves

$$y(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} \left( \hat{a}_k y_k(\eta) e^{i\vec{k} \cdot \vec{x}} + \hat{a}^\dagger_k \bar{y}_k(\eta) e^{-i\vec{k} \cdot \vec{x}} \right),$$

where the sum is over the wave vectors $\vec{k}$ satisfying $k_i L = 2\pi n_i$ for $i = 1, 2, 3$ with $n_i$ integers.

It is convenient to rewrite the field and momentum operators as

$$\hat{y}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} \hat{y}_k(\eta), \quad \hat{\pi}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} \hat{\pi}_k(\eta),$$

where $\hat{y}_k(\eta) \equiv y_k(\eta) \hat{a}_k + \bar{y}_k(\eta) \hat{a}_k^\dagger$ and $\hat{\pi}_k(\eta) \equiv g_k(\eta) \hat{a}_k + \bar{g}_k(\eta) \hat{a}_k^\dagger$ with

$$y_k(\pm)(\eta) = \frac{1}{\sqrt{2k}} \left( 1 \pm \frac{i}{\eta k} \right) \exp(\pm ik\eta), \quad g_k(\eta) = \pm \frac{i}{\sqrt{2}} \exp(\pm ik\eta).$$

As we will be interested in considering a kind of self induced collapse which operates in close analogy with a “measurement”, we proceed to work with Hermitian operators, which in ordinary quantum mechanics are the ones susceptible of direct measurement. Thus we decompose both $\hat{y}_k(\eta)$ and $\hat{\pi}_k(\eta)$ into their real and imaginary parts $\hat{y}_k(\eta) = \hat{y}_k R(\eta) + i \hat{y}_k I(\eta)$ and $\hat{\pi}_k(\eta) = \hat{\pi}_k R(\eta) + i \hat{\pi}_k I(\eta)$ where

$$\hat{y}_k R,I(\eta) = \frac{1}{\sqrt{2}} \left( y_k(\eta) \hat{a}_k R,I + \bar{y}_k(\eta) \hat{a}_k^\dagger R,I \right), \quad \hat{\pi}_k R,I(\eta) = \frac{1}{\sqrt{2}} \left( g_k(\eta) \hat{a}_k R,I + \bar{g}_k(\eta) \hat{a}_k^\dagger R,I \right).$$

We note that the operators $\hat{y}_k R,I(\eta)$ and $\hat{\pi}_k R,I(\eta)$ are therefore hermitian operators. Note that the operators corresponding to $k$ and $-k$ are identical in the real case (and identical up to a sign in the imaginary case).
Next we specify our model of collapse, and follow the field evolution through collapse to the end of inflation. We will assume that the collapse is somehow analogous to an imprecise measurement of the operators $\hat{y}_k^{\mathcal{R},\mathcal{I}}(\eta)$ and $\hat{\pi}_k^{\mathcal{R},\mathcal{I}}(\eta)$ which, as we pointed out are hermitian operators and thus reasonable observables. These field operators contain complete information about the field (we ignore here for simplicity the relations between the modes $k$ and $-k$).

Let $|\Xi\rangle$ be any state in the Fock space of $\hat{y}$. Let us introduce the following quantity: $d_{R,I}^{k} = (\hat{a}_{R,I}^{k})_{\Xi}$. Thus the expectation values of the modes are expressible as

$$\langle \hat{y}_k^{R,I} \rangle_{\Xi} = \sqrt{2}\Re(y_k d_{R,I}^{k}), \quad \langle \hat{\pi}_k^{(y)R,I} \rangle_{\Xi} = \sqrt{2}\Re(g_k d_{R,I}^{k}).$$

(10)

For the vacuum state $|0\rangle$ we have of course: $\langle \hat{y}_k^{R,I} \rangle_0 = 0, \langle \hat{\pi}_k^{(y)R,I} \rangle_0 = 0$, while their corresponding uncertainties are

$$(\Delta \hat{y}_k^{R,I})_0^2 = (1/2)|y_k|^2(hL^3), \quad (\Delta \hat{\pi}_k^{(y)R,I})_0^2 = (1/2)|g_k|^2(hL^3).$$

(11)

The collapse

Now we will specify the rules according to which collapse happens. Again, at this point our criteria will be simplicity and naturalness. Other possibilities do exist, and may lead to different predictions.

What we have to describe is the state $|\Theta\rangle$ after the collapse. We need to specify $d_{R,I}^{k} = (\Theta|\hat{a}_{R,I}^{k}|\Theta\rangle$. In the vacuum state, $\hat{y}_k$ and $\hat{\pi}_k^{(y)}$ individually are distributed according to Gaussian distributions centered at 0 with spread $(\Delta \hat{y}_k)^2$ and $(\Delta \hat{\pi}_k^{(y)})^2$ respectively. However, since they are mutually non-commuting, their distributions are certainly not independent. In our collapse model, we do not want to distinguish one over the other, so we will ignore the non-commutativity and make the following assumption about the (distribution of) state(s) $|\Theta\rangle$ after collapse:

$$\langle \hat{y}_k^{R,I}(\eta_k^c) \rangle_{\Theta} = x_{k,1}^{R,I} \sqrt{(\Delta \hat{y}_k^{R,I})_0^2} = x_{k,1}^{R,I} |y_k(\eta_k^c)| \sqrt{hL^3/2},$$

$$\langle \hat{\pi}_k^{(y)R,I}(\eta_k^c) \rangle_{\Theta} = x_{k,2}^{R,I} \sqrt{(\Delta \hat{\pi}_k^{(y)R,I})_0^2} = x_{k,2}^{R,I} |g_k(\eta_k^c)| \sqrt{hL^3/2},$$

(12) (13)

where $x_{k,1}, x_{k,2}$ are selected randomly from within a Gaussian distribution centered at zero with spread one. From these equations we solve for $d_{R,I}^{k}$. Here we must emphasize that our universe, corresponds to a single realization of the random variables, and thus each of the quantities $x_{R,I}^{k,1,2}$ has a single specific value. Later, we will see how to make relatively specific predictions, despite these features.

Next we focus on the expectation value of the quantum operator which appears in our basic formula Eq.(5). In the slow roll approximation we have $\Gamma = a^{-1}\pi^\nu$. Our general approach indicates that, upon quantization, the above equation turns into

$$\nabla^2 \Psi = s\langle \Gamma \rangle.$$

(14)
Before the collapse occurs, the expectation value on the right hand side is zero. Let us now determine what happens after the collapse: To this end, take the Fourier transform of Eq.\,(14) and rewrite it as

$$\Psi_k(\eta) = \frac{s}{k^2} \langle \hat{\Gamma}_k \rangle \Theta. \tag{15}$$

Let us focus now on the slow roll approximation and compute the right hand side, we note that \( \delta \dot{\phi} = a^{-1} \hat{\pi}(y) \) and hence we find

$$\langle \Gamma_k \rangle \Theta = \sqrt{\hbar L^3 k} \frac{1}{2a} F(k),$$

where

$$F(k) = (1/2)[A_k(x_{k,1}^R + ix_{k,1}^I) + B_k(x_{k,2}^R + ix_{k,2}^I)], \tag{16}$$

with

$$A_k = \frac{\sqrt{1 + z_k^2}}{z_k} \sin(\Delta_k); \quad B_k = \cos(\Delta_k) + \left(1/z_k \right) \sin(\Delta_k) \tag{17}$$

and where \( \Delta_k = k\eta - z_k \) with \( z_k = \eta \theta^k k \).

Next we turn to the experimental results. We will, for the most part, disregard the changes to dynamics that happen after re-heating and due to the transition to standard (radiation dominated) evolution. The quantity that is measured is \( \Delta T(\theta, \phi) \) which is a function of the coordinates on the celestial two-sphere which is expressed as \( \sum_{lm} \alpha_{lm} Y_{l,m}(\theta, \phi) \). The angular variations of the temperature are then identified with the corresponding variations in the “Newtonian Potential” \( \Psi \), by the understanding that they are the result of gravitational red-shift in the CMB photon frequency \( \nu \) so \( \delta T = \frac{\delta \nu}{\nu} = \frac{k(\sqrt{g_{00}})}{\sqrt{g_{00}}} \approx \Psi \).

The quantity that is presented as the result of observations is \( OB_l = l(l+1)C_l \) where \( C_l = (2l+1)^{-1} \sum_{lm} |\alpha_{lm}|^2 \). The observations indicate that (ignoring the acoustic oscillations, which is anyway an aspect that is not being considered in this work) the quantity \( OB_l \) is essentially independent of \( l \) and this is interpreted as a reflection of the “scale invariance” of the primordial spectrum of fluctuations.

Then, as we noted the measured quantity is the “Newtonian potential” on the surface of last scattering: \( \Psi(\eta_D, \vec{x}_D) \), from where one extracts

$$a_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y_{l,m}^* d^2\Omega. \tag{18}$$

To evaluate the expected value for the quantity of interest we use (15) and (16) to write

$$\Psi(\eta, \vec{x}) = \sum_k \frac{sU(k)}{k^2} \sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} F(k)e^{i\vec{k} \cdot \vec{x}}. \tag{19}$$

where we have added the factor \( U(k) \) to represent the aspects of the evolution of the quantity of interest associated with the physics of period from re-heating to de-coupling, which includes among others the acoustic oscillations of the plasma.
After some algebra we obtain

$$\alpha_{lm} = s \sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} \sum \mathbf{k} \frac{U(k) \sqrt{k}}{k^2} F(\mathbf{k}) 4\pi i j_l(\mathbf{R}_D) Y_{lm}(\mathbf{k}),$$  \hspace{1cm} (20)

where \( \mathbf{k} \) indicates the direction of the vector \( \mathbf{R}_D \). It is in this expression that the justification for the use of statistics becomes clear. The quantity we are in fact considering is the result of the combined contributions of an ensemble of harmonic oscillators each one contributing with a complex number to the sum, leading to what is in effect a 2 dimensional random walk whose total displacement corresponds to the observational quantity.

To proceed further we must evaluate the most likely value for such total displacement. This we do with the help of the imaginary ensemble of universes, and the identification of the most likely value with the ensemble mean vale. Now we compute the expected magnitude of this quantity. After taking the continuum limit we find,

$$|\alpha_{lm}|^2_{M.L.} = s^2 \hbar \frac{2\pi a^2}{C(k) k^4 j_l^2(||\mathbf{R}_D||)k^3 dk},$$  \hspace{1cm} (21)

where

$$C(k) \equiv 1 + (2/z_k^2) \sin(\Delta_k)^2 + (1/z_k) \sin(2\Delta_k).$$  \hspace{1cm} (22)

The last expression can be made more useful by changing the variables of integration to \( x = kR_D \) leading to

$$|\alpha_{lm}|^2_{M.L.} = s^2 \hbar \frac{2\pi a^2}{C(x/R_D) x^4 j_l^2(x)x^3 dx},$$  \hspace{1cm} (23)

which in the exponential expansion regime where \( \mu \) vanishes and in the limit \( z_k \to -\infty \) where \( C = 1 \), and taking for simplicity \( U(k) = U_0 \) to be independent of \( k \), (neglecting for instance the physics that gives rise to the acoustic peaks), we find:

$$|\alpha_{lm}|^2_{M.L.} = s^2 U_0^2 \hbar \frac{1}{2a^2 \mu(l+1)}.$$  \hspace{1cm} (24)

Now, since this does not depend on \( m \) it is clear that the expectation of \( C_l = (2l + 1)^{-1} \sum_m |\alpha_{lm}|^2 \) is just \( |\alpha_{lm}|^2 \) and thus the observational quantity \( OB_l = l(l+1)C_l = \frac{s^2 U_0^2 \hbar}{2a^2} \) independent of \( l \) and in agreement with the scale invariant spectrum obtained in ordinary treatments and in the observational studies.

Thus, the predicted value for the \( OB_l \) is,

$$OB_l = \frac{(\pi/3)G\hbar}{V} \frac{(V')^2}{U_0^2} = \frac{(2\pi/3)\epsilon\tilde{V}U_0^2},$$  \hspace{1cm} (25)

where we have used the standard definition of the dimensionless slow roll parameter \( \epsilon \equiv (1/2)(M_{Pl}^2/\hbar)(V'/V)^2 \) which is normally expected to be rather small and the dimensionless potential \( \tilde{V} \equiv V \hbar^3/M_{Pl}^2 \). Thus, if \( U \) could be prevented from becoming too large
during re-heating, the quantity of interest would be proportional to $\epsilon$ a possibility that was not uncovered in the standard treatments. That is, the present analysis offers a path to get rid of the “fine tuning problem” for the inflationary potential, i.e. even if $V \sim M_{Pl}^4$, the temperature fluctuations in the CMB could remain rather small (at the level of $10^{-5}$ as observed in the CMB).

Now, let us focus on the effect of the finite value of times of collapse $\eta_k$. That is, we consider the general functional form of $C(k)$. The first thing we note is that in order to get a reasonable spectrum there seems to be only one simple option: That $z_k$ be essentially independent of $k$ that is the time of collapse of the different modes should depend on the mode’s frequency according to $\eta_k = z/k$. This is a rather strong conclusion which could represent relevant information about whatever the mechanism of collapse is.

Let us turn next to examine a simple proposal about the collapse mechanism which following Penrose’s ideas is assumed to be tied to Quantum Gravity, and examine it with the above results in mind.

3 A version of ‘Penrose’s mechanism’ for collapse in the cosmological setting

Penrose has for a long time advocated that the collapse of quantum mechanical wave functions might be a dynamical process independent of observation, and that the underlying mechanism might be related to gravitational interaction. More precisely, according to this suggestion, collapse into one of two quantum mechanical alternatives would take place when the gravitational interaction energy between the alternatives exceeds a certain threshold. In fact, much of the initial motivation for the present work came from Penrose’s ideas and his questions regarding the quantum history of the universe.

A very naive realization of Penrose’s ideas in the present setting could be obtained as follows: Each mode would collapse by the action of the gravitational interaction between it’s own possible realizations. In our case, one could estimate the interaction energy $E_I(k, \eta)$ by considering two representatives of the possible collapsed states on opposite sides of the Gaussian associated with the vacuum. Clearly, we must interpret $\Psi$ as the Newtonian potential and consequently the right hand side of Eq. (5), (after a rescaling by $a^{-2}$ to replace the laplacian expressed in the comoving coordinates $x$ to a laplacian associated with coordinates measuring physical lenght ) should be identified with matter density $\rho$. Therefore, $\rho = a^{-2}\dot{\phi}_0\Gamma$, with $\Gamma = \pi y/a = \dot{\delta}\phi$. Then we have:

$$E_I(\eta) = \int \Psi^{(1)}(x, \eta)\dot{\rho}^{(2)}(x, \eta)a^3d^3x = a\int \Psi^{(1)}(x, \eta)\dot{\phi}_0\Gamma^{(2)}(x, \eta)d^3x.$$  \hspace{1cm} (26)

Note that in this section we are ignoring the overall sign of this energy which being a gravitational binding energy would naturally be negative. We next express this energy
in terms of the Fourier expansion leading to:

\[ E(\eta) = (a/L^6)\Sigma_{k,k'}\Psi_k^{(1)}(\eta)\hat{\phi}_0\Gamma_k^{(2)}(\eta) \int e^{ix(k-k')}d^3x = (a/L^3)\hat{\phi}_0\Sigma_k\Psi_k^{(1)}(\eta)\Gamma_k^{(2)}(\eta), \quad (27) \]

where (1), (2) refer to the two different realizations chosen. Recalling that \( \Psi_k = (s/k^2)\Gamma_k \), with \( s = 4\pi G\dot{\phi}_0 \), we find

\[ E(\eta) = 4\pi G(a/L)^3\hat{\phi}_0^2\Sigma_k(1/k^2)\Gamma_k^{(1)}(\eta)\Gamma_k^{(2)}(\eta), \quad (28) \]

Using equation (11), we estimate \( \Gamma_k^{(1)}(\eta)\Gamma_k^{(2)}(\eta) \) by

\[ \langle |\Gamma_k|^2 \rangle = h(kL_3)^3(1/2a)^2, \]

and thus we find

\[ E_I(\eta) = \Sigma_k((\pi hG/ak)(\dot{\phi}_0)^2. \quad (29) \]

which can be interpreted as the sum of the contributions of each mode to the interaction energy of different alternatives. According to all the considerations we have made, we view each mode’s collapse as occurring independently, so the trigger for the collapse of mode \( k \) would be, in accordance to Penrose’s ideas, the condition that this energy \( E_I(k, \eta) = (\pi \hbar G/ak)(\dot{\phi}_0)^2 \) reaches the ‘one-graviton level’, namely, that it equals the value of the Planck Mass \( M_p \). Now we use the specific expressions for the scale factor \( a = \frac{1}{\eta H_I} \) and the slow rolling of the background scalar field \( \dot{\phi}_0 = (1/3)(\ddot{a}/a^3)V' \) to find

\[ E_I(k, \eta) = \frac{\pi \hbar G}{9H_I^2}(a/k)(V')^2. \quad (30) \]

Thus the condition determining the time of collapse \( \eta_k^c \) of the mode \( k \) is

\[ z_k = \eta_k^c = \frac{\pi}{9}(hV')^2(H_I M_p)^{-3} = \frac{\epsilon}{8\sqrt{6\pi}}(\tilde{V})^{1/2} \equiv z^c, \quad (31) \]

which is independent of \( k \), and thus, leads to a roughly scale invariant spectrum of fluctuations in accordance with observations. Note that the energy of mode \( k \) in Eq. (30) is an increasing function of conformal time \( \eta \) during the slow roll regime.

We can look closer into this issue and ask when do the relevant modes collapse?. In order to do this we use the value for \( z^c \) and recall that the time of collapse is determined by \( \eta_k^c = z^c/k \), and thus the scale factor at the time of collapse of the modes with wave number \( k \) was

\[ a_k^c = (H_I \eta_k^c)^{-1} = (12/\epsilon)k l_p (V')^{-1} \quad (32) \]

where \( l_p \) stands for the Planck length. As the value of the scale factor \( a \) at the last scattering surface was \( a \approx 10^{-4} \) (recall that the scale factor \( a \) has been set so its value today is 1), the modes that are relevant to say scales of order \( 10^{-3} \) of the size of the surface of last scattering (corresponding to a fraction of a degree in today’s sky) have \( k \approx 10^{-10}l_y^{-1} \).

Thus, taking \( \epsilon \times \tilde{V} \) to be of order \( 10^{-5} \), we have for those modes \( a_k^c \approx 10^{-45} \) corresponding to \( N_e = 103 \) e-folds of total expansion, or something like 80 e-folds before the
end of inflation in standard type of inflationary scenarios. Thus in this scheme inflation must have at least 90 e-folds for it to include the complete description of the regime we are considering and to account also for the collapse of the modes that are of the order of magnitude of the surface of last scattering itself. The usual requirements of inflation put the lowest bound at something like 60 e-folds of inflation so the present requirement is not substantially stronger.

This result can be directly compared with the so called, time of “Horizon crossing” \( \eta_k^H \) for mode \( k \), corresponding to the physical wavelength reaching the Hubble distance \( H_I^{-1} \). Therefore these latter time is determined from:

\[
\eta_k^H \equiv a(\eta_k^H) = k/(2\pi H_I) = kl_p(3/32\pi^3)^{1/2}(\dot{V})^{-1/2}. \quad (33)
\]

Thus the ratio of scale factors at collapse and at horizon crossing for a given mode is

\[
a_k^c/a_k^H = (16/\epsilon)(6\pi^3)^{1/2}(\dot{V})^{-1/2},
\]

which would ordinarily be a very large number, indicating that the collapse time would be much later than the time of “Horizon exiting” or crossing out, of the corresponding mode.

Thus we find that a naive realization of Penrose’s ideas seems, at first sight, to be a good candidate to supply the element that we argued is missing in the standard accounts of the emergence of the seeds of cosmic structure from quantum fluctuations during the inflationary regime in the early universe. However more research along these lines is necessary to find out, for instance, whether the scheme would imply a second collapse of modes already collapsed, and whether such secondary collapse could disrupt in a substantial way the observational spectrum.

### 4 An Alternative Collapse Scheme and the Fine Tuning problem

This section should be considered even more speculative that the others because the ideas here proposed have not yet undergone much substantial checking. Nevertheless, it seems worthwhile to present it here because it illustrates the power of the new way of looking at some of the relevant issues. We have considered one collapse scheme that seemed very natural. However there is another scheme that could be considered even more natural in light of the point view explored in the previous section, that the uncertainties in the matter sources of the gravitational field are the triggers of the collapse. We note that it is only the conjugate momentum to the field \( \pi_k(\eta) \) that acts as source of the “Newtonian potential” in Eq. (5) and contributes to the gravitational interaction energy in Eq. (26), thus it seems natural to assume that it is only this quantity what is subject to the collapse (i.e is only this operator that is subjected to a “self induced measurement”) while the field \( y_k(\eta) \) itself is not. In this case, the analysis is almost identical: The collapse is defined by

\[
\langle \hat{y}_k^{R.I}(\eta_c) \rangle_{\Theta} = 0, \quad \langle \hat{\pi}_k^{(y)R.I}(\eta_c^k) \rangle_{\Theta} = x^R_{k} \sqrt{(\Delta \hat{\pi}_k^{(y)R.I})_0^2} = x_{k,2} |g_k(\eta_c^k)| \sqrt{\hbar L^3/2}, \quad (34)
\]
where $x_k$ are selected randomly from within a Gaussian distribution centered at zero with spread one. Again from these equations we solve for $d_k^{R,I}$, and proceed as before. The only difference so far is that the function $C(k)$ containing information about the collapse changes slightly (see [4]) to:

$$C'(k) = 1 + (1 - 1/z_k^2) \sin(\Delta_k)^2 - (1/z_k) \sin(2\Delta_k).$$  \hspace{1cm} (35)

Compare the above expression with that corresponding to the first collapse scheme Eq. \hspace{1cm} (23). However the point we want to make is that this scheme seems to be a rather serious candidate to alleviate the fine tuning problem, that, as we have mentioned in the discussion around Eq. \hspace{1cm} (25), seems to affect most inflationary scenarios. The point is that according to quite general analysis, \hspace{1cm} [8] the quantity\hspace{1cm}

$$\zeta \equiv \Psi + aH\delta\phi/\dot{\phi}_0$$ \hspace{1cm} (36)

remains constant through the cosmological evolution even if the equation of state changes, so it seems natural to expect that in our context the corresponding quantity

$$\zeta \equiv \Psi + aH\langle\delta\phi\rangle_\Theta/\dot{\phi}_0 = \Psi + H\langle y\rangle_\Theta/\dot{\phi}_0$$ \hspace{1cm} (37)

would be conserved from immediately after the collapse (a process through which the classical equations would not hold) through the reheating and up to the late times associated with the observation, (we are essentially relying on Ehrenfest’s theorem). However for the collapse mode we have considered in this section the last term in the equations vanishes just after the collapse so the value of the Newtonian potential would be that of the estimation we have made before, and, as indicated in the discussion of the last part of section \hspace{1cm} [2] this indicates a substantial amelioration of the fine tuning problem. This seems a very interesting possibility, but clearly it must be investigated much more profoundly before any compelling claims in this regard could be made.

## 5 Noteworthy features of this approach

The quantities of interest $\alpha_{l,m}$ are now understood as different realizations of the random walk described by Eq. \hspace{1cm} (20), so one can study their spreading and in clear way compare the model with the observations. An interesting research issue would be to estimate how many different modes $k$ effectively contribute sum, i.e how many steps are the various the random walks made of.

Another important observation follows directly from the basic point of view adopted in this analysis: The source of the fluctuations that lead to anisotropies and inhomogeneities lies in the quantum uncertainties of the scalar field, which collapse due to some unknown quantum gravitational effect. Once collapsed, these density inhomogeneities

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5Note that in contrast with this reference the analysis here is carried out in terms of the conformal time, and thus the slight difference with the expressions in that work.
and anisotropies feed into the gravitational degrees of freedom leading to nontrivial perturbations in the metric functions, in particular the so called newtonian potential. However, the metric itself is not a source of the quantum gravitational induced collapse (in following with the equivalence principle the local metric perturbations have no energy). Therefore, as the scalar field does not act as a source for the gravitational tensor modes – at least not at the lowest order considered here – the tensor modes can not be excited. The scheme thus naturally leads to the expectation of a zero– or at least a strongly suppressed– contribution of the tensor modes to the CMB fluctuation spectrum.

As pointed out at the end of section 2 and in section 4 this approach also opens new avenues to address the fine tuning problem that affects most inflationary models, because one can follow in more detail the objects that give rise to the anisotropies and inhomogeneities, and by having the possibility to consider independently the issues relative to formation of the perturbation, and their evolution through the reheating era.

And finally and as explicitly exhibited in the previous section, this approach allows us to consider concrete proposals for the physical mechanisms that give rise to the inhomogeneities and anisotropies in the early universe, and confront them with observations.

6 Conclusions

We have reviewed a serious shortcoming of the inflationary account of the origin of cosmic structure, and have given a brief account of the proposals to deal with them which were first reported in [4]. These lines of inquiry have lead to the recognition that something else seems to be needed for the whole picture to work and that it could be pointing towards an actual manifestation quantum gravity. We have shown that not only the issues are susceptible of scientific investigation based on observations, but also that a simple account of what is needed, seems to be provided by the extrapolation of Penrose’s ideas to the cosmological setting.

The scheme exhibits several differences in the predictions as compared with the standard analyses of this problem where the metric and scalar field perturbations are quantized, in particular the suppression of the tensor modes\[6\]. These predictions can, in principle, be tested, indicating that an issue that could ab initio be considered to be essentially a philosophical problem, leads instead to truly physical matters.

In fact, it might well be, that in our frantic search for physical manifestations of new physics tied to quantum aspects of gravitation, the most dramatic such occurrence has been in front of our eyes all this time and it has just been overlooked: The cosmic structure of the Universe itself.

\[6\]See however [9] for another scheme which also leads to strong suppression of tensor modes.
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References

References


