The pre-inflationary vacuum in the cosmic microwave background

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We consider the effects on the primordial power spectrum of a period of radiation-dominated expansion prior to the inflationary era. If inflation lasts a total of only 60 e-folds or so, the boundary condition for quantum modes cannot be taken in the short-wavelength limit as in the standard perturbation calculation. Instead, the boundary condition is set by the vacuum state of the prior radiation-dominated epoch, which only corresponds to the inflationary vacuum state in the ultraviolet limit. This altered vacuum state results in a modulation of the inflationary power spectrum. We calculate the modification to a best-fit model from the WMAP3 data set, and find that power is suppressed at large scales. The modified power spectrum is favored only very weakly by the data.

INTRODUCTION

The high degree of spatial flatness, isotropy and homogeneity that we observe in the universe today is well explained by inflationary cosmology \[1\]. In order to obtain the universe that we observe, the minimum number of e-foldings of inflationary expansion must lie somewhere between \(N = 46\) and \(N = 60\), but inflation could have lasted for significantly longer. The nature of the universe prior to inflation is unknown, largely due to the fact that inflation is very good at washing out initial conditions. The existence of attractor solutions to the inflationary equations of motion make obtaining information about this earlier time is therefore required to uniquely calculate the inflationary power spectrum. Information about this earlier time is therefore required to uniquely calculate the inflationary power spectrum.

In this paper, we study the effects of a prior period of radiation-dominated expansion on the subsequent inflationary epoch in models which yield the minimum amount of inflation, which we take to be \(N = 60\) for definiteness. The transition from a radiation-dominated phase to a de Sitter universe was first studied by Vilenkin and Ford \[3\], Starobinsky \[4\], and by Linde \[5\], and the production of gravity waves during such a transition has been studied by Sahni \[6\]. In such a circumstance, the initial quantum modes are born not in the inflationary vacuum, but in the vacuum appropriate to radiation-dominated expansion. This has a profound effect on the largest-scale perturbations. The standard calculation of perturbations in inflation assumes a well-defined short-wavelength limit for quantum modes. However, a fluctuation that leaves the horizon at the very start of inflation would need to have ‘started out’ large, since no non-inflationary mechanism could have put it there. Therefore, when we set initial conditions for the fluctuations at the start of inflation, we are forced to consider quantum fluctuations far from the short-wavelength limit. As we show, fluctuations that are born at large wavelength during the radiation-dominated era affect the power spectrum very differently than fluctuations born at small wavelength during the inflationary era, resulting in a strong suppression of power on large scales. We emphasize the role of the choice of vacuum in the modulation of the power spectrum, as opposed to the pre-inflationary dynamics of the inflaton field itself, as considered by previous authors \[7\] and \[8\].

Finally, we calculate the modification to the likelihood of a WMAP3 best-fit model, and find that, despite the strong modification to the underlying power spectrum, the \(C_\ell\) spectrum is altered only modestly, with an improvement over the best-fit model of \(-2\Delta \ln(L) = 1\). Therefore, power spectra with suppressed amplitude at large scales are favored only weakly by the data.
A RADIATION DOMINATED PRE-INFLATIONARY PHASE

During inflation, quantum fluctuations in the field or fields driving the expansion are quickly redshifted to scales far greater than the causal horizon, where they manifest themselves as classical curvature perturbations. Specializing to the case of a single scalar field \( \phi \), the field fluctuations \( \delta \phi \) couple at linear order to the metric perturbation. It is therefore useful to introduce the gauge invariant Mukhanov potential,

\[
u = a \delta \phi - \frac{\phi'}{H} \mathcal{R}, \tag{1}\]

where \( a \) is the scale factor of the universe, \( H \) is the Hubble parameter, and \( \mathcal{R} \) the scalar curvature perturbation on comoving hypersurfaces. Primes denote derivatives with respect to conformal time, \( \tau \). On comoving hypersurfaces, \( \delta \phi = 0 \), and the Mukhanov potential is related to the comoving curvature perturbation,

\[
\mathcal{R} = \frac{\nu}{z}, \tag{2}\]

where \( z = \phi'/H \). The power spectrum of the curvature perturbation may then be written as a function of comoving wavenumber \( k \),

\[
P_\mathcal{R}(k) = \frac{k^3}{2\pi^2} \langle \mathcal{R}_k^2 \rangle = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2, \tag{3}\]

where the \( u_k \) are the Fourier modes of the gauge-invariant potential satisfying the equation

\[
u'' + \left( k^2 - \frac{2\nu'}{z} \right) u_k = 0. \tag{4}\]

A non-zero correlation function \( \langle \mathcal{R}_k \mathcal{R}_l \rangle \) means that there has been particle production due to the inflationary expansion. In an expanding spacetime, the initial vacuum state associated with positive frequency quantum modes will in general be associated with a mixture of both positive and negative frequency modes at later times. An observer in a state initially devoid of quanta will register a thermal bath of particles as the universe expands. When solving Eq. (4), the initial conditions on the modes correspond to a specific choice of vacuum. As an example, consider the simple case of a free scalar field fluctuation evolving in a de Sitter background, \( H = \text{const.} \). It is convenient to work with the variable \( y = k/aH \), which is the ratio of the Hubble radius to the physical wavelength of the perturbation. In a de Sitter background the conformal time is

\[
\tau = -\frac{1}{aH}, \tag{5}\]

so that \( y = -k\tau \) and Eq. (4) becomes

\[
y^2 a^2 \frac{d^2 u_k}{dy^2} + (y^2 - 2) u_k = 0. \tag{6}\]

This has the general solution

\[
u_k(y) = \frac{1}{2} \sqrt{\frac{\pi y}{k}} \left[ c_1 H^{(1)}_{3/2}(y) + c_2 H^{(2)}_{3/2}(y) \right] \tag{7}\]

where \( H^{(1,2)} \) are Hankel functions of the first and second kind, respectively. Since de Sitter space is eternal, the initial conditions must be set in the infinite past, \( y \to \infty \). In this limit Eq. (7) becomes

\[
u_k(y) = \frac{1}{\sqrt{2k}} (c_1 e^{-iy} + c_2 e^{iy}). \tag{8}\]

In order that the mode reduce to the vacuum state annihilated by positive frequency excitations, we must choose \( c_1 = 0 \), corresponding to the Bunch-Davies vacuum. Then Eq. (7) becomes

\[
u_k(y) = \frac{1}{\sqrt{2k}} (1 + i) e^{iy}. \tag{9}\]

This solution starts off as a plane wave in the infinite past, and at late times as \( y \to 0 \), the second term in parentheses comes to dominate, and particle production occurs.

In general, not all exact solutions to Eq. (4) will asymptote to plane waves at initial times. In this case, the best one can do is construct an adiabatic vacuum to some finite order in a parameter which characterizes the slowness of the expansion. The adiabatic vacuum can be thought of as the vacuum that best approximates the Minkowski vacuum in an expanding spacetime. In order for such an approximation to be sensible, the background expansion must be slow relative to the frequency of the quantum fluctuation. From Eq. (4) with \( z''/z = C(\tau) \) and \( \omega = \sqrt{k^2 - C(\tau)} \), this condition can be written

\[
\frac{d\ln(C)}{d\tau} \ll \omega. \tag{10}\]

The vacuum obtained above clearly satisfies this condition (in the infinite past the cosmological expansion \( C(\tau) \to 0 \)), and is equivalent to the lowest-order adiabatic vacuum. In de Sitter space, the adiabatic condition Eq. (10) is \( -k\tau \gg 0 \), which is not only satisfied in the infinite past, but at any later time for infinitely-large momentum modes, \( k \to \infty \). Therefore, as long as we take the initial mode momenta sufficiently large, the adiabatic vacuum is a good approximation to the exact solution. This is to be expected, since at such small length scales the quantum modes do not feel the expansion of the universe. However, modes that exit the horizon at the start of inflation will not have originated in the short wavelength limit. In order to impose the Bunch-Davies condition, we need to be able to construct an adiabatic vacuum with respect to which we can define positive frequency modes. A horizon scale fluctuation with \( k \sim H \) is of relatively low momentum and cannot be well approximated by such a construction. Obviously, information
about the preceding phase of cosmological expansion is necessary to resolve this issue. In what follows, we consider a period of radiation-domination (RD) prior to the inflationary epoch.

During a radiation-dominated era, quantum fluctuations evolve according Eq. (4) with $a(\tau) \propto \tau$. Then we have $y = k/aH = k\tau$ with $\tau > 0$, and we obtain the equation,

$$\frac{d^2u_k}{dy^2} + u_k = 0,$$

which has the plane wave solution $u_k \propto c_1 e^{-iy} + c_2 e^{iy}$. This demonstrates the well-known fact that there is no particle production during radiation-dominated expansion, since the quantum fluctuation never evolves out of its vacuum state. Since $C(\tau) = 0$, the adiabatic condition Eq. (10) is satisfied for all time and for all $k$. The vacuum is therefore well defined for all momenta. However, during a radiation-dominated phase, modes evolve under expansion from the long-wavelength limit $y \ll 1$ to the short-wavelength limit $y \gg 1$, and we cannot populate superhorizon scales with modes redshifted from the Minkowski vacuum in the ultraviolet. We nevertheless expect quantum fluctuations to exist on all scales, including superhorizon, during the radiation-dominated phase [11]. Off-shell modes may be populated acausally as long as they do not couple to classical perturbations on superhorizon scales. These modes will later be "converted" to classical perturbations during inflation.

Using these initial conditions, we calculate the effect of a pre-inflationary RD phase on the WMAP3 best-fit inflation model with no tensors, no running of the scalar spectral index and $n_s = 0.951$ [12]. We assume that the best-fit model is generated by some inflation model with sufficient inflation to ensure that all observable scales originated in the Buch-Davies limit of the inflationary vacuum. This is equivalent to specifying the observables $n_s$ and $r$ at $N = 60$. We compare this to result to the case of the same best-fit inflationary model, but with a pre-inflationary RD phase ending at $N = 60$ (Figure 1). Instead of using the Buch-Davies boundary condition for the modes, we use the RD vacuum solution to Eq. (11) as a boundary condition:

$$u_{k,i} = \sqrt{\frac{1}{2k}}[\cos(y_i) + i\sin(y_i)]$$

To calculate the modified spectrum, we solve Eq. (4) numerically mode-by-mode. The suppression of large-scale power is immediately evident in Figure 1. The scales corresponding to these fluctuations are vacuum modes that were in existence on superhorizon scales at the onset of inflation. As a result, their amplitudes are highly suppressed in relation to quantum modes that would have attained these scales as a result of inflation. Modes evolving during inflation undergo mode freezing as they cross outside of the horizon, effectively locking-in their amplitudes at relatively large values. On sub-horizon scales, $k > aH$, the modified spectrum undergoes oscillations, rapidly approaching the standard inflationary spectrum. While the inflationary Buch-Davies vacuum only exists for modes $k \gg aH$, it has the same form as Eq. (12). Therefore, as $k \to \infty$, the inflationary vacuum approaches the RD vacuum, and the mode solutions become identical. A similar spectrum was obtained in [7], in which the effects of an initially fast-rolling inflaton were studied. Burgess et al. [8] also obtain similar spectra arising from a hybrid model in which a rapidly oscillating auxiliary field leads to an era of matter domination prior to inflation. We emphasize that we are encoding the effect of the pre-inflationary physics solely in the choice of vacuum state for the inflaton fluctuations. We do not address the separate questions of how the universe achieves the necessary level of homogeneity prior to the onset of inflation, or of how the universe makes the transition from the radiation-dominated to the inflationary phase. In this sense, our analysis should be regarded as a physically motivated "toy" model which demonstrates the effect of the pre-inflationary vacuum state on the primordial power spectrum.

Using a modified version of CAMB [13], we are able to generate the $C_l$ spectrum for prior RD, Figure 2. The lack of power at large-scales is evident in the low-$l$ multipoles, but the two spectra become exactly the same at around $l \sim 11$. However, the improvement in the fit to the data is modest.
CONCLUSIONS

We have considered the effects on the primordial power spectrum of a period of radiation-dominated expansion prior to the inflationary epoch. We impose initial conditions on the quantum fluctuations at the start of inflation appropriate to the pre-inflationary era. The largest-scale modes that exit the horizon at the start of inflation are most strongly affected by the phase transition, since such modes were never in the ultraviolet limit. This forces one to consider that the initially large-scale fluctuations are vacuum fluctuations of very low momentum. These relatively low momentum modes lead to a strong suppression of the power spectrum on the largest scales.

The resulting power spectrum is shown to lower the $C_l$'s of the CMB temperature anisotropies at low $l$, particularly around the quadrupole. We find an improvement of $-2\Delta \ln L = 1$ over the best-fit WMAP model with no tensors and no running when we suppose that this model was preceded by a period of radiation domination. The phase transition must occur close to the time when the largest observable scales are leaving the horizon, or else the effects of the transition will be washed out. Hirai and Takami [10], apply a methodology very similar to that presented here to investigate the effect of pre-inflationary physics, including that from a prior radiation-dominated phase. However, instead of considering a modification to the boundary condition of the quantum scalar mode $\delta \phi$, they use a boundary condition appropriate to a hydrodynamical radiation-dominated mode, with $c_s^2 = 1/3$. This results in the apparently unphysical conclusion that modulations to the inflationary power spectrum persist at all scales, regardless of the duration of inflation. However, we argue that the dominant contribution to the post-inflationary curvature perturbation will be from those generated by fluctuations in the inflaton field itself, for which $c_s^2 = 1$ even in the pre-inflationary phase.

Finally, we draw attention to previous work on this subject, namely [7] and [8, 9], who consider a pre-inflationary fast-rolling phase and matter dominated phase, respectively. What is different in our analysis is that the suppression of power on large scales is manifestly the result of a choice of vacuum. The details of the background dynamics do not enter into the calculation, except to specify the vacuum state in the pre-inflationary phase. The similarity of the modifications to the power spectrum suggests the conclusion that the effect of pre-inflationary physics can be generically considered as an effect of vacuum definition, in a fashion similar to that for trans-Planckian modulations [13]. (This issue was considered in an effective field theory context in Ref. [14].) However, in all cases the resulting modification to the spectrum of fluctuations in the CMB results in only a modest improvement of the fit to the data. Since the observational uncertainties at these scales are dominated by cosmic variance, evidence for these effects is likely to remain inconclusive in the absence of additional observable evidence.

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