Four-Photon Interference with Asymmetric Beam Splitters

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Two experiments of four-photon interference are performed with two pairs of photons from parametric down-conversion with the help of asymmetric beam splitters. The first experiment is a generalization of the Hong-Ou-Mandel interference effect to two pairs of photons while the second one utilizes this effect to demonstrate a four-photon de Broglie wavelength of $\lambda/4$ by projection measurement.

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Quantum interference of multiple photons plays pivotal roles in quantum information sciences. Although two-photon interference has been widely studied [1] and is applied to some quantum information protocols [2], quantum interference of more than two photons has only recently been the focus of research because of its role in the fundamental study of quantum nonlocality [3, 4, 5] and the improvement in the precision of phase measurement [6, 7, 8, 9, 10].

The most well-known two-photon interference is the Hong-Ou-Mandel effect [11], where two photons enter a symmetric beam splitter (50:50) from two sides respectively (Fig.1a). Because of two-photon destructive interference, the probability for the two photons to exit at separate ports is zero. Generalization to higher photon number is not straightforward. For example, for an input state of $|2_a, 2_b\rangle$, i.e., two from each side respectively, the output state for a symmetric beam splitter is in the form

$$|\Phi_2\rangle_{\text{out}} = \sqrt{3/8} (|A, 0_B\rangle + |0_A, 4_B\rangle) - 1/2 |2_A, 2_B\rangle,$$  \hspace{1cm} (1)

where we have a nonzero probability for the $|2_A, 2_B\rangle$ state at the output, contrary to the two-photon counterpart.

Recently, however, Wang and Kobayashi [12] proposed a generalization of the Hong-Ou-Mandel effect to three photons with an asymmetric beam splitter ($T \neq R$). With a state of $|2_a, 1_b\rangle$ input at a beam splitter with $T = 2R = 2/3$, the output state becomes

$$|\Phi_3\rangle_{\text{out}} = 2/3 |3_A, 0_B\rangle + \sqrt{2}/3 |0_A, 3_B\rangle - 1/\sqrt{3} |1_A, 2_B\rangle. \hspace{1cm} (2)$$

Note that the state $|2_A, 1_B\rangle$ is missing in the output state due to three-photon interference. They went further to apply it to form a three-photon interferometer with a three-photon de Broglie wavelength of $\lambda/3$. Sanaka et al. [13] demonstrated this three-photon Hong-Ou-Mandel effect with three photons out of two pairs of photons generated in parametric down-conversion. Liu et al. [14] implemented the three-photon interferometer proposed by Wang and Kobayashi and demonstrated a three-photon de Broglie wavelength.

In this Letter, we generalize the idea of Wang and Kobayashi to four-photon case of input state of $|2_a, 2_b\rangle$ and observe a four-photon Hong-Ou-Mandel effect. Then we form an interferometer and demonstrate a four-photon de Broglie wavelength. We find that due to imperfection in a real experimental environment, the $|2_A, 2_B\rangle$ state cannot be completely cancelled from Eq.(1). The residual $|2_A, 2_B\rangle$ state will inevitably lead to a residual two-photon interference fringe on top of the four-photon interference fringe. On the other hand, such a residual two-photon effect can be compensated, which gives rise to a pure four-photon interference effect with a de Broglie wavelength of $\lambda/4$ for four photons.

When a state of $|2_a, 2_b\rangle$ enters an asymmetric beam splitter with $T \neq R$, the output state is

$$|\Psi_4\rangle_{\text{out}}^{\text{asy}} = \sqrt{6TR}(|A, 0_B\rangle + |0_A, 4_B\rangle) + \sqrt{6TR(T-T)}(|3_A, 1_B\rangle - |1_A, 3_B\rangle) + [(T-T)^2 - 2TR]|2_A, 2_B\rangle. \hspace{1cm} (3)$$

So when $(T-T)^2 - 2TR = 0$ or $T = (3 \pm \sqrt{3})/6, R = (3 \mp \sqrt{3})/6$, the $|2_A, 2_B\rangle$ term disappears from Eq.(3) and the probability of detecting two photons at each side is zero, i.e., $P_1(2_A, 2_B) = 0$. Hence, we realize a generalized Hong-Ou-Mandel effect for two pairs of photons.

FIG. 1: (a) Hong-Ou-Mandel interferometer with asymmetric beam splitter and (b) formation of an interferometer for the de Broglie wavelength of four photons.
If we follow the outputs by another symmetric beam splitter as shown in Fig.1b, the $|3, 1⟩_A$ and $|1, 3⟩_B$ states in Eq. (3) will not contribute to the probability $P_4(2C, 2D)$ of detecting four photons with two at each side due to a two-photon Hong-Ou-Mandel effect. Since the $|2, 2⟩_A$ state in Eq. (3) is cancelled out when $T = (3 ± √3)/6$, $R = (3 ± √3)/6$, only the part of $|4, 0⟩ + |0, 4⟩$ in Eq. (3) will contribute to $P_4(2C, 2D)$, leading to a four-photon interference effect with

$$P_4(2C, 2D) ∝ 1 + \cos 4\varphi,\quad (4)$$

where $\varphi$ is the single-photon phase difference between A and B. This can be easily confirmed by evaluating the four-photon detection probability $P_4(2C, 2D) ∝ \langle a, b | C D^2 D^2 C | a, b \rangle$ with

$$\begin{align*}
C &= (\hat{a} + e^{i\varphi} \hat{b})/\sqrt{2}, \\
D &= (e^{i\varphi} \hat{b} - \hat{a})/\sqrt{2},
\end{align*}$$

$$\begin{align*}
\hat{A} &= \sqrt{T} \hat{a} + \sqrt{R} \hat{b}, \\
\hat{B} &= \sqrt{T} \hat{b} - \sqrt{R} \hat{a},
\end{align*}\quad (5)$$

where $T = (3 ± √3)/6$, $R = 1 - T$. The sinusoidal dependence of $P_4(2C, 2D)$ on $4\varphi$ in Eq. (4) shows the four-photon de Broglie wavelength.

Experimental implementation is shown in Fig.2, where the four-photon state of $|2, 2⟩_A$ is produced from a type-II parametric down-conversion process pumped by 150 fsec frequency-doubled pulses from a mode-locked Ti:Sapphire laser operating at 780 nm. The 2mm long β-Barium Borate (BBO) crystal is so oriented that it produces two beam-like orthogonally polarized fields at the degenerate wavelength of 780 nm [15]. The horizontal (H) and the vertical (V) polarized fields are first coupled into single-mode fibers and are recombined with a polarization beam splitter (PBS1) into one beam before passing through an interference filter of 3nm bandwidth. The filtered field is then fed into the four-photon interferometer of Fig.1 But the beam splitters of Fig.1 are equivalently replaced by two polarization rotators (HWPs) and another polarization beam splitter (PBS2). Thus it is a polarization interferometer. A phase retarder (PS) is inserted between the two HWPs to introduce the single-photon phase shift $\varphi$ between the two orthogonal polarizations. The input four-photon state of $|2_H, 2_V⟩$ is generated via two pairs of down-converted photons.

In the first experiment, the rotation angle from HWP1 is set to zero so that it has no effect on the H- and V- polarizations except for a relative delay but that from HWP2 is set to zero so that it has no effect on the H- and V- generated via two pairs of down-converted photons. The four-photon state of $|2_H, 2_V⟩$ is inserted between the two HWPs to introduce the single-photon phase shift $\varphi$ between the two orthogonal polarizations. The input four-photon state of $|2_H, 2_V⟩$ is generated via two pairs of down-converted photons.

FIG. 2: The layout of the experiment. PBS: polarization beam splitter; HWP: half wave plate; PS: phase retarder; IF: interference filter; D1-D4: photo-detectors.

HWP2 is set at $\theta = 13.7^°$ so that $\cos^2 2\theta = (3 + √3)/6$ (angle of polarization rotation is $2\theta$). HWP2 and PBS2 together are equivalent to an asymmetric beam splitter of $T = \cos^2 2\theta$ and $R = \sin^2 2\theta$. The fiber coupler for the H-polarized photons is mounted on a micro-translation stage to introduce a delay $\Delta$ between the H- and V-polarizations. Four-photon coincidence counts are registered to measure the probability $P_4(2C, 2D)$ as a function of the delay between the H- and V-polarizations. The data is shown in Fig.3 after background subtraction. It shows the typical Hong-Ou-Mandel dip with a visibility of 88% and a full width at half height of 196μm, which are derived from a least square fit to a Gaussian (the solid curve). The less than 100% visibility is a result of imperfect temporal mode match between the two pairs of down-converted photons [16, 17, 18]. To verify that we indeed have the correct $T$ and $R$ with $\theta$ at $13.7^°$, we fix the delay $\Delta$ at the bottom of the dip in Fig.3 but change $\theta$. The measured four-photon coincidence counts after background subtraction are plotted as a function of $\theta$ in Fig.4, which shows four minima at $\theta = 13.7^°, 31.3^°, 58.7^°, 76.3^°$, corresponding to $\cos^2 2\theta = T = (3 ± √3)/6$. Again, the minimum values do not go to zero, contrary to the prediction in the coefficient of the $|2_A, 2_B⟩$ term in Eq. (4).
when $T = \cos^2 2\theta, R = \sin^2 2\theta$. The imperfection is due to temporal mode mismatch. The solid curve is a least square fit to the function \[ P_4(\theta) = C[(1 - 1.5 \sin^2 4\theta)^2 + (3 \sin^2 4\theta - 1) \times (1 - \sin^2 4\theta)(1 - E/A)/2], \] where $C$ is a scaling factor and $E/A$ is a parameter that characterizes the temporal mismatch \cite{10, 18}. Note that when $E/A = 1$, the function in Eq. (6) touches zero at the four minima.

In conclusion, we demonstrated both the generalizedHong-Ou-Mandel effect and the de Broglie wavelength of four photons with two pairs of down-converted photons in a scheme involving asymmetric beam splitters. These two effects are a result of four-photon interference.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Four-photon coincidence as a function of the phase difference $\varphi$ between H- and V-polarizations. HWP1 is set at (a) $\theta = 13.7^\circ$ and (b) $\theta = 13.1^\circ$.}
\end{figure}

\begin{align}
P_4(\varphi) &= C[(1 - 1.5 \sin^2 4\varphi)^2 + (3 \sin^2 4\varphi - 1) \times (1 - \sin^2 4\varphi)(1 - E/A)/2], \tag{6}
\end{align}

with $V_4 = 0.62$ and $V_2 = 0.39$.

Fortunately, the uneven peaks in Fig. 5a can be balanced \cite{19} by slightly adjusting HWP1 away from $13.7^\circ$ to $13.1^\circ$, as shown in Fig. 5b. The least square fit for the data in Fig. 5b to the function in Eq. (6) gives $V_4 = 0.59$ and $V_2 = -0.03$. The smallness of $V_2$ indicates a good cancellation of the $\cos 2\varphi$ term in Eq. (6).

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\begin{align}
P_4(2C, 2D) &= C(1 + V_4 \cos 4\varphi + V_2 \cos 2\varphi) \tag{7}
\end{align}

\begin{thebibliography}{99}
\bibitem{1} L. Mandel, Rev. Mod. Phys. \textbf{71}, S274 (1999).
\bibitem{14} B. Liu \textit{et al.}, \texttt{quant-ph/0610266}.
\bibitem{19} A more detailed multi-mode theory similar to Refs.\cite{16,18} will take into account the temporal mismatch between the two pairs of down-converted photons. This theory will be presented elsewhere.
\end{thebibliography}