RECENT RESULTS ON SIMULATIONS OF LATTICE QCD WITH QUENCHED FERMIONS

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I will talk about two subjects: the analysis of the hadron spectrum\(^1\) and the measurement of the string tension\(^2\). All the results have been obtained by simulating lattice QCD on a \(10^3 \times 20\) lattice, where the longest direction is defined as the time.

The generalities about the calculation of hadron masses are well known\(^3\). One looks at the correlation function at a certain space-time distance of two local operators \(O\) carrying definite quantum numbers. The behaviour of such correlations for large values of the time distance provides information on the value of the lowest-lying state coupled to the operator \(O\). In formulae:

\[
C(t) = \int d^3 x \, \langle \sigma(x) \sigma^+(0) \rangle \sim A \cdot e^{-M_\sigma t}
\]

Examples of the operators \(O\) are:

\[
\bar{\psi} \gamma_5 \psi, \quad \bar{\psi} \gamma^\mu \psi, \ldots \quad \text{for mesons}
\]

\[
\epsilon^{abc} \left[ \bar{\psi}^{(A)}(x) \gamma^\rho \psi^{(B)}(y) \right] \bar{\psi}^{(C)}(z), \ldots \quad \text{for baryons}
\]

where \(\psi\) are the quark fields.

The "quenching" approximation is made throughout the whole calculation: one neglects the diagrams where fermion loops are created by gluons. The integration over the fermion fields appearing in the average of Eq. (1) for a fixed gauge field configuration then reduces to the calculation of the quark propagator in the presence of a given external background field. The correlation function of Eq. (1) becomes a combination of two (mesons) or three (baryons) propagators suitably projected in colour and spin by the operators \(O\). Various different field configurations are produced by a Monte Carlo algorithm, with the probability distribution:

\[
d\mathcal{P}(A) = \frac{d[A]}{\int d[A]} e^{-S(A)}
\]
where $S(A)$ is the action of the gauge fields and the Wick rotation to a Euclidean space-time has been made.

The final result of Eq. (1) is obtained by averaging over the collected gauge field configurations the values of the correlation of the two operators:

$$\int dP(A) \sigma(x; A) \sigma^+(0; A) = \langle \sigma(x) \sigma^+(0) \rangle$$

(4)

Table 1 contains a summary of information on the type of action used on the lattice, on the parameters chosen$^8$) and on the techniques used for generating the sample of gauge field configurations and for calculating the fermion propagator in a given external field.

The results show a definite improvement with respect to those obtained at the same value of $\beta = 6/g^2$ on a $5^3 \times 10$ lattice$^3$; in particular:

i) the fluctuations among the values of the masses estimated from different configurations by fitting the time behaviour in Eq. (1) are drastically reduced;

ii) the results for mesons are stable against different choices of the operators $0$ (for example, $\bar{\psi} \gamma^4 \psi$ or $\bar{\psi} \gamma^5 \psi$ for the $\rho$ and $\bar{\psi} \gamma^5 \psi$ or $\bar{\psi} \gamma^5 \psi$ for the pion.

Concerning fluctuations, they are generally expected to decrease as the volume increases$^4$; this, however, could not explain the drastic difference appearing in Fig. 1, where the value of the $\rho$ mass obtained from different gauge field configurations (taken every hundred Monte Carlo sweeps) for the case of the $5^3 \times 10$ (black points) and of the $10^3 \times 20$ (open points) lattices is given. There is, in fact, an extra effect due to the use of periodic boundary conditions for fermions$^5$). These conditions allow for diagrams like the one in Fig. 2a, where one quark follows a path lying in the original lattice (O) while the second reaches the final point on the replica (R). Seen on a two-dimensional torus, the second path wraps around it as in Fig. 2b. These diagrams are spurious: in practice, their contribution is small if the phase given by the line integral of the gauge field over the wrapping path fluctuates among the different trajectories.

$^8$We have always used $\beta \equiv 6/g^2 = 6$. For the comparison of these results with those at a different $\beta$ ($\beta = 5.7$), see the talk by Wallace at this meeting.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of the lattice</td>
<td>$10^3\cdot20$</td>
</tr>
<tr>
<td>Gauge action</td>
<td>Wilson</td>
</tr>
<tr>
<td>Fermion action</td>
<td>Wilson</td>
</tr>
<tr>
<td>$\beta \equiv 6/g^2$</td>
<td>6</td>
</tr>
<tr>
<td>No. of gauge field configurations</td>
<td>27</td>
</tr>
<tr>
<td>Link updating procedure</td>
<td>Metropolis (10 hits)</td>
</tr>
<tr>
<td>No. of sweeps spent for thermalization</td>
<td>1300</td>
</tr>
<tr>
<td>Interval between gauge field configurations used for mass estimates</td>
<td>100 sweeps</td>
</tr>
<tr>
<td>Calculation of the quark propagator</td>
<td>Gauss-Seidel</td>
</tr>
<tr>
<td>Values of K used</td>
<td>0.150, 0.1535, 0.155</td>
</tr>
</tbody>
</table>
For a single trajectory, such a line integral is essentially a Polyakov loop: the magnitude of the average \( \langle L \rangle \) of such a loop oriented, say, in the \( z \) direction over the orthogonal volume (\( x, y, t \) points), governs the importance of the spurious diagrams\(^\#\)). One obtains

\[
\langle L \rangle_{5^3 \times 10} \sim 100 \cdot \langle L \rangle_{10^3 \times 20}
\]

(5)

On the large lattice such diagrams are then negligible, as are the large fluctuations induced by their presence\(^5\).

Concerning the stability against the use of different operators the values of the \( \rho \) mass in lattice units for different values of \( K \) for two different operators are reported in Table 2. The consistency between the two sets of results is a signal that the asymptotic regime of "large \( t \)" indicated in Eq. (1) has been reached. In this regime, the lowest-lying state is supposed to dominate the correlation function, and the only dependence upon the choice of the operator \( 0 \) will appear in the constant \( A \) multiplying the exponential in Eq. (1). As an extra check, one can construct the quantity:

\[
m(t) = -\mathcal{L}_m \left[ \frac{C(t+\Delta)}{C(t)} \right]
\]

(6)

If only one state dominates \( C(t) \), \( m(t) \rightarrow \text{constant} \); if higher excited states contribute, \( m(t) \) will be a decreasing function of \( t \). For mesons, the asymptotic regime is reached for \( t > 7 \); for baryons, this happens only at the lowest value of \( K \) (highest quark mass) while for higher values some contamination from higher excited states seems to persist up to \( t \sim 9^{**} \).

<table>
<thead>
<tr>
<th>( K )</th>
<th>0.150</th>
<th>0.1535</th>
<th>0.155</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\psi}_\gamma \psi )</td>
<td>0.64 ± 0.02</td>
<td>0.51 ± 0.02</td>
<td>0.46 ± 0.02</td>
</tr>
<tr>
<td>( \bar{\psi}_\sigma \psi )</td>
<td>0.66</td>
<td>0.53</td>
<td>0.49</td>
</tr>
</tbody>
</table>

\(^\#\) The expectation value of loops oriented in the time direction is used as an order parameter for the analysis of the finite temperature deconfining transition in QCD\(^6\).

\(^{**}\) The contamination from excited states seems to be the main cause of discrepancy between the \( 5^3 \times 10 \) and \( 10^3 \times 20 \) results.
All the results on mesons and baryons are summarized in Fig. 3, where the values of the hadron masses in lattice units are given as a function of the quark mass in lattice units. The full curves are fits to the points which assume a linear dependence on the quark masses of the baryon and of the square of meson masses. In this figure there are two unknowns: the value of the lattice spacing in physical units and the appropriate value of the quark mass. Two inputs are then needed to fix such quantities: if one takes the value of the masses of $\rho$ and pion, one gets the scales in GeV which are reported close to the lattice units scales corresponding to $a^{-1} \approx 2$ GeV. In addition, the value of the quark mass that gives a good fit to the $\pi-\rho$ mass splitting is $m_{q} \approx 7$ MeV, a very small value, well outside the domain where the calculation is performed. As an output, one obtains

$$m_{\text{Proton}} = [1.15 \pm 0.05] \text{ GeV}$$

$$m_{\Delta} = [1.33 \pm 0.08] \text{ GeV}$$

The proton mass turns out to be rather high.

An alternative procedure is that of using as inputs the values of masses of hadrons which are made of strange quarks. Using the $\phi$ mass and that of a pseudoscalar $\eta$ [by SU(3) mass formulae it would be $\sim 690$ MeV] one gets:

$$a^{-1} = 2.4 \text{ GeV}$$

$$m_{\pi} \approx 120 \text{ MeV} \quad \text{and} \quad M_{\Delta} = 1.65 \text{ GeV}$$

By adding the pion as an input, one can also estimate the masses of hadrons containing one light and one strange quark. The summary of inputs and outputs for the two procedures described above is given in Table 3.

The general pattern is very encouraging for mesons and for "heavy" baryons (i.e., made of strange quarks): light baryons come out too heavy!

However, for these states, the contamination from higher excited states might be more severe: a preliminary result obtained by using a different operator based on the non-relativistic SU(6) wave function for the proton, gives

$$m_{\text{Proton}} \text{ Non-Rel.} \sim 1.0 \text{ GeV}$$

\# Up and down quarks are taken to be degenerate.
Table 3

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_\pi, m_\rho)</td>
<td>(a^{-1} = (2 \pm 0.2) \text{ GeV})</td>
</tr>
<tr>
<td></td>
<td>(m_u = m_d \sim 7 \text{ MeV})</td>
</tr>
<tr>
<td></td>
<td>(m_p = (1.15 \pm 0.05) \text{ GeV})</td>
</tr>
<tr>
<td></td>
<td>(m_\Delta = (1.33 \pm 0.08) \text{ GeV})</td>
</tr>
<tr>
<td>(m_\phi, m_{\eta_S})</td>
<td>(a^{-1} = 2.1 \text{ GeV})</td>
</tr>
<tr>
<td></td>
<td>(K_S = 0.155 \rightarrow n_S = 120 \text{ MeV})</td>
</tr>
<tr>
<td></td>
<td>(M_{\Omega} = 1.65 \text{ GeV})</td>
</tr>
<tr>
<td>(m_\phi, m_{\eta_S})</td>
<td>(m_\rho = (0.82 \pm 0.04) \text{ GeV})</td>
</tr>
<tr>
<td>and (m_\pi)</td>
<td>(m_K = (0.45 \pm 0.03) \text{ GeV})</td>
</tr>
<tr>
<td></td>
<td>(m_{K^*} = (0.90 \pm 0.04) \text{ GeV})</td>
</tr>
<tr>
<td></td>
<td>(m_P = (1.2 \pm 0.1) \text{ GeV})</td>
</tr>
<tr>
<td></td>
<td>(m_\Delta = (1.35 \pm 0.15) \text{ GeV})</td>
</tr>
<tr>
<td>(K_{\text{critical}})</td>
<td>(0.1570 \pm 0.0003)</td>
</tr>
</tbody>
</table>

This value is certainly inconsistent with the previous one beyond the statistical fluctuations, and strongly suggests a systematic error due to the presence of radial excitations. A complete study of the proton correlation function using different operator is under way\(^7\). Another effect which might justify the high values obtained for baryons is the suppression, due to the quenching approximation, of diagrams like the one in Fig. 4, responsible for the corrections to the proton mass due to the emission and reabsorption of pions.

The second subject of the talk is a new measurement of the string tension \(K\) made on the same gauge field configurations from which the values of hadron masses have been extracted\(^2\). Indeed, the value of \(a^{-1} = 2 \text{ GeV}\) obtained from the hadron mass fit would lead to a value for \(A_{\text{lattice}}\), which, given the usually quoted relation between \(\sqrt{K}\) and \(A_L\) \((\sqrt{K} \approx 170 A_L)^\text{\(8\)}\), would imply \(\sqrt{K} \sim 800 \text{ MeV}\) against the expected value of 400 \(\div 450 \text{ MeV}\). On the other hand, the existing estimates of \(K\) for SU(3) have been made on symmetric and relatively small plaquettes \((4 \times 4 \text{ or } 5 \times 5)\), and may give results not directly interpretable in terms of the \(q\bar{q}\) potential.
We decided to study the correlation of two Polyakov loops as a function of the distance. The expected behaviour is

\[ \langle P_z(0,0,0) P_z(0,0,T) \rangle \sim e^{-L_z V(T) - L_z T \cdot K} \]

where \( P_z(x,y,t) \) is a loop in the direction \( z \) and of length \( L_z \) calculated at the point of co-ordinates \( x, y, t \). In our case \( L_z = 10 \) and, given the rather large area already involved for \( T \approx 2x3 \), a straightforward measurement of the quantity in Eq. (9) based on the available statistics of the order of 27 configurations (to be multiplied by two, since \( P_z \) and \( P_y \) loops are used) would be heavily affected by statistical fluctuations. We have used a new method which increases the statistics by about a factor one hundred. I will briefly describe the idea for a spin system: let us consider, in the Ising model, the value of the spontaneous magnetization \( \langle \sigma_i \rangle \)

\[ \langle \sigma_i \rangle = \frac{\int d\sigma_1 d\sigma_2 \ldots d\sigma_i \ldots \cdot \sigma_i e^{-\beta H\{\sigma\}}}{\int d\sigma_1 d\sigma_2 \ldots d\sigma_i \ldots \cdot e^{-\beta H\{\sigma\}}} \]

where \( H\{\sigma\} \) is the Hamiltonian of the system. If \( H\{\sigma\} = \sum_{ij} J_{ij} \sigma_i \sigma_j \), the integration over the \( i \)th spin \( \sigma_i \) can be performed exactly and gives:

\[ \langle \sigma_i \rangle = \langle t_{gh} \beta J_{ik} \sigma_k \rangle \]

The second operator has the same average value (it comes after an exact integration), but much less fluctuations: for example, at \( \beta = 0 \), \( \sigma_i \) still fluctuates while the second operator gives identically zero (the correct result). The method can be extended to the case of average values of more spin variables if they are statistically independent: the terms in the Hamiltonian containing one of them do not contain any of the others. This method is particularly useful in the case of the calculation of the Polyakov loops: each of the links appearing in the loop is statistically independent from the others with a Wilson-type action. This is also true for the correlations if they are taken at distance equal to or greater than two. To increase the statistics, we have measured the correlation between a point and a plane:

\[ \langle x, y, t \rangle = \frac{1}{L_x L_y L_z} \sum_{x,y,t} \langle P_z(x,y,t) P_z(x',y',t+t') \rangle \]

The resulting function of \( t \) is given in Fig. 5, where the curve represents a least squares fit to the data points of the form \( \pi(t) = A \cosh[(10-t)\eta] \). The periodicity is enforced by the use of periodic boundary conditions. The value of \( \eta \) is: \( \eta = 0.39 \pm 0.04 \) which, by using the expression in Eq. (9) with \( L_z = 10 \), gives
\[ \kappa a^2 \mid_{\beta=6} = 0.039 \pm 0.004 \]

If we use the value of \( a \) extracted from the hadron spectrum, we get \( K \approx 400 \text{ MeV} \), now in agreement with the expected value. This result confirms that the value of the lattice spacing in physical units at \( \beta = 6 \) is about 0.1 Fermi and renormalizes previous results which were converted into physical units assuming \( a_{\beta=6} \approx 0.2 \text{ Fermi} \). For example, this changes the conventionally quoted value of the \( 0^{++} \) glueball mass which now moves to \( \approx 1200 \text{ MeV} \) and, more dramatically, increases the latent heat of fusion of hadrons into gluons at the deconfinement temperature \( T_C \approx 370 \text{ MeV} \) to about 10 GeV/Fm\(^3\). This might have far-reaching consequences on the possibilities of studying such a phase transition by accumulating high energy densities in heavy ion collisions.

REFERENCES


   D. Weingarten, Indiana University preprint IUHET-82 (1982).

4) See, for example, the talk by G. Parisi at the Les Houches Workshop, 
   (March 1983) and H.W. Hamber, E. Marinari, G. Parisi and C. Rebbi, 


6) See, for example, T. Celik, J. Engels and H. Satz, Bielefeld preprint, 
   BI-TP 83/07 (1983).


Fig. 1: The fluctuations of the $\rho$ mass squared in lattice spacing units at $K = 0.1475$ on the $5^3 \times 10$ lattice (closed points) and at $K = 0.150$ on the $10^3 \times 20$ lattice (open points).

Fig. 2: a) A spurious diagram where one quark trajectory lies in the original lattice (O) and the other goes into the replica (R).

b) The diagram of Fig. 2a seen on the torus.
Fig. 3: Various hadron masses in lattice spacing and in physical units against quark masses in lattice spacing and in physical units. The lines are the fits to the data points.
**Fig. 4:** Example of a diagram for the proton propagation which is absent in the quenching approximation.

**Fig. 5:** The correlation of Polyakov loops $\pi(t)$ (see text) as a function of the distance $t$. 