We propose a generic approach to nonresonant laser cooling of atoms/molecules in a bistable optical cavity. The method exemplifies a photonic version of Sisyphus cooling, in which the matter-dressed cavity extracts energy from the particles and discharges it to the external field as a result of sudden transitions between two stable states.

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Laser cooling is an extremely successful method for creating the ultra-cold ensembles of atoms.\(^1\)-\(^3\). Doppler cooling (a crucial component of laser cooling for all but lowest temperatures) relies on the multiple absorption-emission cycles in a closed system of levels. It is limited to alkali metals and to a small number of other atomic species. This makes laser cooling of molecules very difficult, although there are a few proposals and experimental demonstrations of cooling molecules.\(^5\) Moreover, the effects such as reabsorption of the spontaneously emitted light, radiation trapping, and excited state collisions limit the final achievable phase-space density in cooling methods based on resonant light absorption.

Non-resonant laser cooling methods such as cavity cooling\(^6,\)\(^7\) and stochastic cooling\(^8\)-\(^10\) were shown to be potentially useful for cooling dense samples of the trapped particles down to the mK region. Cavity cooling is based on the generation of viscous-type friction force imposed on the atoms as they move inside a leaky cavity. Recently, single atom cavity cooling was experimentally demonstrated\(^11,\)\(^12\), and further improvements by introducing of a linear feedback were suggested\(^13\). The related subject of radiation pressure cooling of microscopic mechanical resonators, such as micromirrors\(^14,\)\(^15\) and microlever\(^16\) has attracted significant attention recently, potentially leading to the observation of the quantum ground state of a micro-mechanical system.

In this paper we propose a generic scheme for non-resonant atomic/molecular cooling in bistable optical cavities. The cooling mechanism is of Sisyphus type, in which the cavity mode (not the atom) performs sudden transitions between two stable states. Conventional cavity cooling relies on non-adiabatic effects in the cavity-atom interaction, and requires high finesse cavities. In contrast, our technique best operates in the opposite limit of “bad cavity” and slowly moving particles, so that the cooling rate does not experience deterioration at low atomic velocities.

Consider an optical resonator supporting a standing electromagnetic wave, of (bare) cavity resonant frequency \(\omega_r\) and mode function \(\cos(2\pi x/\Lambda)\) (where \(\Lambda\) is the spatial period). The resonator is excited from the outside by an incident field \(E_i \exp(-i\omega t) + c.c\) which is nearly resonant with the cavity mode. A point-like polarizable particle moving inside the resonator will contribute an effective refractive index depending on the mode function at its instantaneous position causing resonance frequency shift (away from \(\omega_r\)), depending on the particle position \(x\). As a result, the intra-cavity field becomes strongly coupled to the position of the particle.
The complementary aspect of this coupling is that the dipole force felt by the particle depends on the local intensity of the field inside the resonator. This generic interplay between the cavity field dynamics and the particle motion forms the basis for all cavity cooling schemes. The combined system dynamics is given by the following set of coupled equations for the cavity field amplitude and the particle velocity and position:

\[
\frac{dE}{d\tau} = \frac{E_i}{\sqrt{T}} - E \left[ 1 + \frac{v(\xi)}{\kappa} + i \frac{\Delta_c + U(\xi)}{\kappa} \right]
\]

\[
\frac{dv}{d\tau} = -\frac{\varepsilon_0 V}{\omega \kappa^2 L^2 m_a} \frac{dU(\xi)}{d\xi} |E|^2
\]

\[
\frac{d\xi}{d\tau} = v
\]

Here \( E \exp(-i\omega t)\cos(2\pi\xi) + c.c. \) is the field in the cavity, \( \xi \equiv x / \Lambda, \nu \equiv v/(\kappa \Lambda) \) and \( \tau \equiv \kappa t \) are dimensionless coordinate, velocity and time respectively. The decay rate of the cavity field is denoted by \( \kappa = Tc/2L \). Here \( T \) is the transmission coefficient of the cavity mirrors and \( L \) is the cavity length. In addition, \( \alpha \) is polarizability of the particle, \( \gamma(\xi) = \omega \text{Im}(\alpha)/(|\varepsilon \nu|) \cos^2(2\pi\xi) \) is the scattering rate of the atom, \( \Delta_c = \omega - \omega_c \) is the detuning of the empty cavity from the frequency of the incident field, \( U(\xi) = U_0 \cos^2(2\pi\xi) \) is the frequency shift of the cavity due to the interaction with the particle, \( U_0 = \omega \text{Re}(\alpha)/(|\varepsilon \nu|) \), \( V \) is the mode volume, \( m_a \) is the mass of the particle, \( c \) is the speed of light in vacuum and \( \varepsilon_0 \) is the permittivity of free space.

For a particle moving slowly enough (\( \nu \ll \kappa \Lambda \)), the field in the cavity adjusts itself adiabatically to the instantaneous position of the particle. The steady-state intensity of the cavity field is given by

\[
|E|^2 = \frac{|E_i|^2}{T \left[ (1 + \gamma(\xi) / \kappa)^2 + (\Delta_c + U(\xi))^2 / \kappa^2 \right]}
\]

In the adiabatic approximation, by substituting of Eq. (2) into the second equation of system Eqs. (1), one reduces the problem to the motion of particle in an effective periodic potential. Because of the conservative character of this motion, energy that is lost while climbing a potential hill is regained while sliding down on the other side. Therefore, no net energy loss is possible in the adiabatic limit, and the cooling process requires non-adiabatic effects. All previously discussed cavity cooling schemes\(^{6, 7}\) rely on high finesse cavities that ensure a retarded response of the internal field to the changing position of the particle. There is however, a notable example for a profound non-adiabatic effect in “bad” (low finesse) cavities, which is related to the phenomenon of optical bi- or multi-stability\(^{19}\). Under certain conditions (see below), an optical resonator has two co-existing stable steady states. A sharp transition between these states may happen even at infinitely slow variation of the resonator parameters, leading to a rapid passage from high to low cavity field (or vice versa). In what follows, we demonstrate a novel scheme for cooling using a bistable cavity.

Consider a resonantly pumped cavity (\( \Delta_c = 0 \)), with the pump frequency far detuned from any atomic transition, so that scattering losses \( \gamma(\xi) \) may be neglected. We introduce an external
feedback circuit (to be discussed later) such that the external pump field intensity \( I_t = |E_t|^2 \) depends on the field intensity \( I = |E|^2 \) inside the cavity. In this case, expression (2) becomes an equation from which the steady-state intra-cavity intensity should be found:

\[
I \left[ 1 + U^2 (\xi) / \kappa^2 \right] = \frac{I_t(I)}{T}
\]

(3)

For a given dependence \( I_t(I) \), equation (3) may be solved for \( I \), e.g. graphically (see Fig. 1a) by considering the intersection of a dashed line (LHS in Eq. (3)) and the feedback curve (RHS in Eq. (3)). We assume the latter to have a sigmoid shape (solid line in Fig. 1a). The slope of the straight line depends on the particle position and varies from 1 to \( \left[ 1 + U^2 / \kappa^2 \right] \) (and back) as the particle moves. Depending on the parameter values, Eq. (3) may have one or three solutions. In the latter case, only two of them (shown by solid circles in Fig. 1a) are stable.

Fig 1. (a) Solutions of Eq. (3) corresponding to different particle positions. Stable solutions are shown by solid circles, unstable one is shown by an empty circle. Grey circles (numbered 1 and 2) show the critical points at which the solution jumps from one stable branch to another. (b) Intra-cavity intensity versus particle position. The solid (blue online) curve shows the solution corresponding to the lower branch of the feedback curve in figure (a), the dashed (red online) curve corresponds to the upper branch. Dotted lines show the intensity switching.

Let us follow the motion of a particle starting from the field’s node \( \xi = 0.25 \Lambda \) at Fig. 1b). When the particle is at the node, the cavity is tuned to resonance, and the internal field is high. As the particle moves toward the antinode of the standing wave mode, the slope of the straight line in Fig. 1a increases and the steady-state solution follows the upper branch of the sigmoid curve. At a certain particle position, the straight line detaches from the sigmoid feedback curve, and the intra-cavity intensity drops down to the solution at the lower branch of this curve. When the particle reaches the antinode, the slope takes the maximal value \( \left[ 1 + U^2 / \kappa^2 \right] \) and it starts decreasing back. The steady state solution follows the lower branch of the feedback curve until the multiple intersections disappear, and the intra-cavity intensity jumps to its high value. As follows from Fig. 1a, the field switches up and down at non-equivalent particle positions (hysteresis effect that is well known for bistable resonators). Such a pair of sudden jumps occurs on every period of the standing wave mode. When the particle moves adiabatically over the standing wave, it gains kinetic energy when sliding down the effective potential and loses it when climbing up. However, in the present case, the loss and gain are not equal because of the
hysteresis effect. By proper control of the relevant parameters, the particle can be programmed
to either lose or gain energy on average. Moreover, the particle loses the same amount of energy
on passing every single spatial period of the standing wave until it is trapped by the optical
potential. The average stopping force acting on the particle is, therefore, constant and does not
depend on the velocity. It is noteworthy that while in other cavity cooling schemes\textsuperscript{6,7,18} the
resonator introduces viscous-type friction (proportional to the particle velocity), our bistable
resonator accommodates a “dry friction” force that is velocity independent. Hence, this scheme
promises a more efficient slowing the particle.

To estimate this force, we consider a simple model in which the sigmoid feedback curve is
replaced by a step-like function: $|E|_n^2 = I_1$ for $|E|^2 < I_{sw}$ and $|E|_n^2 = I_2$ for $|E|^2 \geq I_{sw}$, where $I_{sw}$
is the cavity intensity value at which the intensity of the incident field switches from $I_1$ to $I_2$.
The steady-state intra-cavity intensity is then double-valued: $|E|_n^2(\xi) = \kappa^2 I_n \left[ T \left( \kappa^2 + U^2(\xi) \right) \right]$, $(n = 1,2)$. Under these conditions, the effective potential energy felt by the particle has two
branches as well:

$$W_n(\xi) = \int d\xi' E_0 V \frac{dU(\xi')}{d\xi'} |E|_n^2(\xi), \quad (n = 1,2).$$ (4)

For a slowly moving particle, the energy loss per half a period of the mode function is given by,

$$\Delta E = -\frac{V E_0 \kappa (I_2 - I_1)}{\omega T} \left[ \frac{\kappa}{\kappa + \omega T} - \frac{I_2}{I_1} \right]$$ (5)

where $U_1$, $(U_2)$ are the critical interaction-induced frequency shifts at which the straight lines
touch the feedback curve: $(U_{1,2}/\kappa) = (I_{1,2} - IT_{sw})/(TI_{sw})$. If the parameters are tuned such
that the intra-cavity intensity switches up when the particle is in the field node, and the intensity
goes down when the particle approaches the anti-node, the net energy loss is

$$\Delta E = -V E_0 (I_2 - I_1) \kappa \arctan \left( \frac{U_0}{\kappa} \right) / (\omega T)$$

and the average stopping force is:

$$F_{\text{stop}} = \frac{2V E_0 (I_2 - I_1) \kappa \arctan \left( \frac{U_0}{\kappa} \right)}{\Lambda \omega T}.$$ (6)

Fig. 2a depicts the simulated temporal evolution of the velocity of a single particle moving
in a bistable cavity (solid line) and compares it to cavity cooling without the feedback (dashed
line). In this simulation, the sigmoid feedback step function was represented by a smooth
continuous function:

$$I_i = I_0 + (\Delta I/2) \tanh \left[ a (I - I_{sw}) T / I_0 \right],$$

$$I_0 = (I_1 + I_2)/2, \quad \Delta I = I_2 - I_1$$ (7)

The parameter $a$ controls the steepness of the curve step (cavity cooling without feedback
corresponds to $\Delta I = 0$). The dimensionless simulation parameters were chosen as $a = 10$,\n$\Delta I / I_0 = (U_0/\kappa)^2 = 0.5$, $TI_{sw}/I_0 = 1$, and $2\pi \Re(\alpha) I_0 / (\kappa^2 \Lambda^2 T m_a) = 10^{-5}$. The latter parameter is
actually proportional to the ratio of the polarization energy and a typical kinetic energy of a
particle moving with the velocity $v = \kappa \Lambda$. If the particle is initially rather fast ($v \geq \kappa \Lambda$), the
non-adiabatic effects are prevailing and the velocity of the particle decreases in a way similar to the conventional cavity cooling. As the particle slows down so that the condition $\nu \ll \kappa \Lambda$ is fulfilled, the bistability shows up, and the velocity starts decreasing linearly with time (see Fig. 2a). The numerical value of the decelerating force is in good agreement with the analytical estimate, Eq. (6). Insert in Fig. 2a shows the time dependence of the cavity field in the domain of constant deceleration. Jump-like changes in the field intensity are clearly seen as discussed above (compare with Fig. 1b). The deceleration stops as the particle becomes trapped by the optical potential. For comparison, in Fig. 2b we analyze deceleration process in a bistable cavity (black line) under the conditions close to those reported in the experiment by Maunz et al. where cooling of a single atom in a cavity was observed. The process of ‘conventional’ cavity cooling of a single atom is simulated (grey line), with the following parameters: $(U_0/\kappa)^2 = 10^{-2}$ and $2\pi \text{Re}(\alpha) I_0 / (\kappa^2 \Lambda^2 T m_p) = 10^{-4}$, that corresponds to a particle having mass of Rb atom moving in the cavity of 120 $\mu$m length with mode diameter 30 $\mu$m and $\kappa = 10^7$ Hz, pumped by 10 pW light of 1 $\mu$m wavelength. The black curve is for bistable feedback cooling with the same parameters and the feedback function is the same as in a previous example. The initial velocity of the particle is $\sim 50$ times larger than the recoil velocity, $v_{\text{rec}}$ of the rubidium atom. Fig. 2b demonstrates again an efficient and rapid particle deceleration due to the bistable character of the cavity. The particle is finally trapped by the cavity mode when the velocity reaches the value of $\sim 10 v_{\text{rec}}$. Further cooling will require adaptive manipulation of the laser power, to be discussed in the future.

We have also analyzed cooling of an ensemble of $N$ particles coupled to a single mode of a bistable resonator. Fig. 3a presents the results of a direct simulation of cavity cooling for $N = 5$ particles that are initially randomly dispersed inside the cavity with a Gaussian distribution in velocity. In Fig. 3a, the evolution of the velocity variance as a function of time is depicted for both conventional (dashed) and bistable (solid) cavity cooling. In the bistable regime, the
average kinetic energy decreases in a series of sharp random steps. Each step is correlated with a “jump” in the intra-cavity field intensity. As appears from Fig. 3a, bistable cavity provides more efficient cooling compared to the regular one. The difference in the cooling rate is even more evident in Fig. 3b, where the case of \( N = 25 \) particles is presented. A detailed analytical and numerical study of ensemble cooling in a bistable cavity, its dependence on the external parameters and scaling with \( N \) is currently under study and will be reported elsewhere.

Fig 3. (a) Evolution of the velocity variance of the ensemble of \( N = 5 \) particles during cooling in bistable cavity (solid line) and in conventional cavity cooling (dashed line). (b) The same for \( N = 25 \).

In conclusion, we have presented a new approach to non-resonant laser cooling of atoms and molecules based on their interaction with a bistable cavity. The cooling mechanism presents a photonic version of Sisyphus cooling, in which the conservative motion of the particles (atoms or molecules) is interrupted by sudden transitions between two stable states of the cavity mode. The mechanical energy is extracted due to the hysteretic nature of those transitions. The bistable character of the cavity may be achieved by an external feedback loop (like the one considered in the present paper) or by means of additional nonlinear intracavity optical elements (saturable absorber, etc.). In contrast to the conventional cavity cooling\(^6,7\), in which atoms experience a viscous-type force, bistable cavity cooling imitates “dry friction”, and stops atoms much faster. Our technique operates in the “bad cavity” limit and preserves its efficiency at low particle velocities. Classically, the limit to this cooling mechanism is set by the trapping of particles in the optical potential. Quantum effects, which are not included in the present discussion, will certainly limit the cooling near the recoil limit, and are the subject of the ongoing studies. Finally, we expect that in analogy to the methodology presented here, bistable cavity cooling may also be advantageous for cooling of micro-mechanical resonators\(^14\text{–}16\).

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