Fine structure in the gamma-ray sky and the origin of UHECR

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The EGRET results for gamma ray intensities in and near the Galactic Plane have been analysed in some detail. Attention has been concentrated on energies above 1 GeV and the individual intensities in a 4° longitude bin have been determined and compared with the large scale mean found from a nine-degree polynomial fit.

Comparison has been made of the observed standard deviation for the ratio of these intensities with that expected from variants of our model. The basic model adopts cosmic ray origin from supernova remnants, the particles then diffusing through the Galaxy with our usual ‘anomalous diffusion’. The variants involve the clustering of SN, a frequency distribution for supernova explosion energies, and ‘normal’, rather than ‘anomalous’ diffusion.

It is found that for supernovae of unique energy, and our usual anomalous diffusion, clustering is necessary, particularly in the Inner Galaxy. An alternative, and preferred, situation is to adopt the model with a frequency distribution of supernova energies. The results for the Outer Galaxy are such that no clustering is required.

If their explosion energies are distributed then supernovae can be the origin of UHECR.

1. INTRODUCTION

Although supernova remnants (SNR) are often invoked as the source of cosmic rays (CR), of energy below the ‘knee’ at about 3 PeV, \cite{12}, there are still a number of imponderables. Here, we concentrate on two aspects:

(i) SN energies are not unique, but have a certain frequency distribution involving some ‘supersupernovae’ \cite{3}. We refer to this model as SSNR.

(ii) SNR in the Galaxy are not distributed at random, but rather in clusters, in both space and time and, preferentially, in spiral arms \cite{4}. This is NSNR model (Normal SNR).

In our previous work \cite{5} we used the acceleration model of \cite{2}, which assumed that particles are trapped for up to $8 \times 10^4 y$ after the SN explosion and that SN occurred randomly in time in the Galaxy (at an average rate of $10^{-2} y^{-1}$).

The distribution in space was drawn from the usual average radial distribution of SN surface density, with a peak at a Galactocentric distance of D=4 kpc and falling rapidly with increasing D in the vicinity of the sun (at D=8.5 kpc). Here, we go on to make allowance for the clustering of the SN - and hence the SNR - in various ways, and, separately, consider a frequency distribution of SN energies: the SSNR model.

The quantity used as an indicator of CR intensity in the Galaxy is the gamma ray intensity \cite{6} which has been measured as a function of longitude, latitude and gamma ray energy. Most of the present work relates to gamma rays above 1 GeV which are considered to be produced predominantly by CR nuclei (mainly protons) of median energy $\sim 40$ GeV \cite{7}. In zeroth order of accuracy, the mean CR intensity along a particular line of sight ($\ell,b$) is simply proportional to the measured gamma ray intensity divided by the column density of target gas. In the actual analysis we use a more elaborate method but the basis is the same.

The method is to study the ratio of the observed gamma ray intensity to the large scale average and use that as an indicator of the small scale variability of CR intensity from place to place. The variability can then be compared with that predicted by models involving SN, clustered and unclustered, normal and with different energies, using the same technique of comparing the
intensity with the average.

2. THE BASIC DATA

Figure 1 gives an example of the data (from \[6\]); it refers to gamma rays with energy above 1 GeV and having \(|b| < 2^\circ\), i.e., it relates to the Galactic Plane. Identified discrete sources have been removed and the workers estimate that less than 10% of the remaining flux is due to unresolved sources.

Figure 1. Profile of the gamma ray intensity vs longitude, for \(E_\gamma > 1\) GeV and \(|b| < 2^\circ\). The results are from \[6\]. The standard deviations on the points due to Poissonian fluctuations are typically \(\pm 2\)\% in the Inner Galaxy and \(\pm 4.8\%\) in the Outer Galaxy. The experimental data are averaged over the longitude bin \(\Delta \ell = 1^\circ\). The smooth line is the best fit of the profile by the 9-degree polynomial.

We have used the basic data for the numbers of gamma rays to evaluate the statistical uncertainty in the individual intensities.

The data have been divided into `Inner' (\(\ell : 270^\circ - 350^\circ, 10^\circ - 90^\circ\)) and `Outer' (\(\ell : 90^\circ - 180^\circ - 270^\circ\)), the innermost region (\(|\ell| < 10^\circ\)) being omitted because of confusion there.

Two latitude ranges are considered: \(|b| < 2.5^\circ\) and \(|b| : 2.5^\circ/5^\circ\). The gamma rays from the individual \(b\)- and \(l\)-ranges are derived from regions of the Galaxy distributed along the line of sight but we can identify a median \(D\)-value for each range. Restricting attention to gamma ray production in molecular clouds with the scale height of \(\sim 70\) pc, the corresponding distances are \(\sim 3\) kpc and 1 kpc from the sun, for \(|b| < 2.5^\circ\) and \(|b| = 2.5^\circ/5^\circ\) respectively. Allowing for production in \(HI\), with its greater scale height, the \(D\)-values will be somewhat different (but by not much because more of the gas-associated fluctuations arise from the clumpy \(H_2\) rather than \(HI\)).

3. THE ANALYSIS

3.1. The Measured Fluctuations

The `figure of merit' adopted is the ratio of the observed gamma ray intensity to the smoothed line through the points (9-degree polynomial, equivalent, approximately, to a smoothing over a 20\(^\circ\) longitudinal bin for the Inner and Outer Galaxy regions, separately). The shape of the line is determined essentially by the kiloparsec-scale spatial variations of the cosmic ray intensity and the target gas. Figure 1 gives the smoothed curve for \(|b| < 2^\circ\).

The ratios have been determined for various bins of longitude, \(\Delta \ell\), specifically, 1\(^\circ\), 2\(^\circ\), 4\(^\circ\) and 8\(^\circ\). Figure 2 shows the corresponding frequency distributions of the differences between the EGRET data and the fit. The numbers of entries, the means and the rms (\(\sigma\)-) values are given in Table 1. It is with the latter rms values that we are mainly concerned.

The appropriate \(\sigma\)-value to be used is a compromise between the \(\Delta \ell\) over which intensity changes are expected (all values but with a bias towards degree scales) and the need to get away from small \(\Delta \ell\)-values, where bin-to-bin correlations are serious, because of the finite range of latitudes (\(|b| < 2.5^\circ\)) over which the averaging is made.

We consider that \(\Delta \ell = 4^\circ\) is most appropriate and this will be used in the model calculations.

Figure 3 gives the values of \(\sigma (4^\circ)\) for the observed data (after correction for noise) and
3.2. Variations in the column density of gas

In all our models the radial distribution of gas has axial symmetry. It means that for a smooth non-fluctuating CR distribution the longitudinal distribution of the gamma-ray intensity has also to be smooth. However, we know that the ISM is highly non-uniform and this non-uniformity will contribute to the fine structure of the gamma-ray intensity profile.

A number of facts are relevant, as follows:
(i) The large scale dependence of $N(H)$ on $l$ is automatically included through the radial distribution.
(ii) The 9-degree polynomial fit causes changes on scales of about $\delta l = 20^\circ$ (and above) to be allowed for.
(iii) Much of the clumpiness of the gas in the ISM comes from molecular clouds and we allow for correlation between the SNR and the clouds in the analysis.
(iv) Results of the type shown in Figure 2 show a dependence of $\sigma$ on $\Delta l$ in the range $\Delta l: 1^\circ - 8^\circ$ which is very similar for all the models and the observed values.

Although the implication of the above is that the effect of the fluctuations in $N(H)$ (both atomic and molecular) is not likely to be dramatic, it is certainly finite. Its magnitude will be greatest in the Inner Galaxy at low latitudes, $|b| < 2.5^\circ$, where contributions from the ‘far’ Inner Galaxy are important.

Estimate for both the Inner and Outer regions have been made. For $N(HI)$ the estimated value of the logarithmic standard deviation is 0.027 using the well known column densities of $HI$ [8]. For $N(H_2)$, data [9] give a value 0.026 referred to the whole column density ($N(HI) + N(H_2)$), leading to an overall value of $\sigma_{gas} = 0.037$. Comparison can be made with the observed value of $\sigma_{obs}$ for this $l,b$ region: 0.080. Subtraction in quadrature leads to $\sigma = 0.071$, i.e. a small, but finite reduction.

Support for our analysis comes from a study of the EGRET observations and analysis published prior to the latest observations used by us [86]. In this work, predictions were made for a model in which the column densities of gas
were used together with an inferred CR intensity distribution which was correlated with the volume density of gas (derived from the column density and assumed rotation curve) but ‘smoothed’ spatially at the kpc level. The majority of the excursions in predicted intensity were thus due to the gas column density variations alone. The fluctuations of the observed intensities ($E > 1\text{GeV}, |b| < 2^\circ, |l| : \pm 90^\circ$) about the prediction have $\sigma = 0.06 \pm 0.015$. This value is quite consistent with ours.

Turning to regions away from the Galactic Plane in the Inner Galaxy ($|b| = 2.5^\circ - 5^\circ$) the corresponding value for HI and $H_2$ is $\sigma_{\text{gas}} = 0.04$. In the Outer Galaxy the values are $\sigma = 0.025$ for $|b| < 2.5^\circ$ and 0.035 for $|b| = 2.5^\circ/5^\circ$. It will be noticed that in all cases the corrections will be small.

Our contention is that the major cause of the fluctuations in gamma-ray intensity on the $4^\circ$ scale is the stochastic nature of the SNR in space and time and, probably, in energy output. The column density of gas undoubtedly fluctuates with $l$ but the observed gamma-ray intensity fluctuations are much greater because of the CR intensity variations along the line of sight.

3.3. The model predictions

Our ‘usual’ model [25] comprises CR production by SNR, as described in paragraph 1, and propagation with ‘anomalous diffusion’. The SNR were assumed to be all identical and independent in time and space (i.e. NSNR), within the constraints of the dependence of the mean SNR density on Galactocentric radius. We denote this as NC (non-clustering), and the results for this situation are shown in Figure 3. This is the NSNR model, but with no clustering.

The SSNR model has the same mean energy content per SNR as in the previous case but a distribution of energies about the mean, with a frequency distribution of log energy having a log standard deviation of 1.2, following [2]. The results for this type of super-supernovae are now indicated as SSNR in Figure 3.

Turning to the possibility of the clustering of NSN, our clustering model has the following idealised form. At any one place there are SN in sets of 10, distributed at random, in time, over $10^8\text{y}$. The results for this model ‘C’ (clustered), are also shown in Figure 3. Here, the SN are identical.

Finally, calculations have been made for the same situation, as indicated, for ‘normal diffusion’ (ND) rather than our standard: ‘anomalous diffusion’. 

![Figure 3. Summary of standard deviations ($\sigma$-values) of gamma ray intensities about the best-fit: (a) - normal diffusion, (b) - anomalous diffusion. Small downward corrections have been applied to the OBS (observed) -values to allow for experimental noise. Model simulations are: NC - non-clustering, C - clustering, SSNR - with a frequency distribution of energies. Mean standard deviations are obtained by averaging 4-8 independent samples. Errors are not indicated for clarity of the figures, but the distribution of $\sigma$ for different samples is broad due to the stochastic nature of SN explosions. Typical errors for anomalous diffusion are 0.002-(NC), 0.0015-(C) and 0.014-(SSNR) for the Inner Galaxy, 0.004-(NC), 0.004-(C) and 0.02-(SSNR) for the Outer Galaxy. For normal diffusion the errors are by 2-5 times less. The Galactocentric distances are the means from which much of the gamma ray intensities are derived.](image-url)
3.4. Comparison of observations with the models

3.4.1. General remarks

It is apparent that the observed $\sigma$ values are higher than expected for the NSN model without clustering - i.e. non-clustering (NC). For normal diffusion NC(ND) the discrepancy is bigger still. Below we examine various factors which influence our model predictions.

3.4.2. Mixed primary mass composition

The model predictions shown in Figure 3 relate to primary protons. However, the observed gamma rays are produced by all CR nuclei, i.e. by the mixed primary mass composition. Primary nuclei are more efficient in producing gamma rays than protons of the same energy per particle due to the higher interaction cross-section and higher number of interacting nucleons in the collision. At the same time fluctuations of the gamma-ray intensity created by CR nuclei are lower than those for protons, therefore our model predictions should be reduced for the mixed primary mass composition. We calculated the fluctuations for the mixed composition with relative abundances equal to 0.416 for protons, 0.265 for helium, 0.170 for CNO group, 0.149 for iron group of nuclei \[10\]. The result is that we have to multiply our predicted $\sigma$-values by 0.93, 0.86, 0.85, 0.88 respectively for the increasing galactocentric distances indicated in Figure 3. The uncertainty of this correction is within the range of 0.005 - 0.019. As a result of the correction the discrepancy between observed and expected $\sigma$-values for the non-clustering case (NC) with a mixed primary mass composition is bigger still.

3.4.3. Mode of diffusion

It is relevant to examine the question of ‘normal or anomalous diffusion ?’. The mode of diffusion has obvious implications for the present work. Calculations have been made for all cases, i.e. non-clustering, clustering and SSNR models using normal diffusion, instead of anomalous. The result is that the fluctuations and $\sigma$-values are lower for normal diffusion in all cases (see Figure 3a compared with Figure 3b ), thus the difference between the non-clustering case and observations will be even larger.

3.4.4. Super-supernovae

Examination of Figure 3 shows that the $\sigma$-values for SSNR with a distribution of explosion energies is higher than for clustering (C) and for the observed fluctuations (OBS). It is due to the basic assumption about the fluctuations of the explosion energy. Added to the fluctuations caused by the stochastic distribution of SN in space and time these fluctuations result in a rise of the total fluctuations. Inevitably, it is possible to 'dilute' the SSNR values by assuming that only a fraction of the SNR are of such high energy. According to \[3\] the fraction of such SSNR is equal to 0.2 for SNIbc and 0.1 for SNIIn, but we regard it as a quantity 'to be determined'.

3.4.5. Clustering of supernovae and the presence of SSNR

Although at first sight the case for some clustering of SNR at all distances is strong there is a complication that must be allowed for. This concerns the correlation of SNR with target gas, particularly in molecular form (principally $H_2$). This correlation is on linear scales smaller than the kpc-scale already allowed for. Our 4° bin of longitude would correspond to a linear scale of 280 pc at a distance of 4 kpc (the typical distance to the production region in the Inner Galaxy for $|b| < 2.5^\circ$). Such a distance would embrace likely distances of molecular clouds (MC) from some, at least, of the SNR.

The magnitude of this correlation is not clear but some progress can be made. The list of known SNR \[11\] has only 3% of the entries mentioning adjacent MC but this fraction will increase with increasing SNR-MC distance. Inspection of the Galactic maps of MC \[9\] in the Inner Galaxy shows a mean separation of about 500pc: thus a correlation factor of about 0.5 would appear to be indicated. For the general case, it appears that the relation between the various quantities for the situation where there is partial clustering of the SN (coefficient $f$) and a partial correlation between SNR and molecular gas (coefficient 0.5) is:
\[ \sigma_{\text{obs}}^2 - \sigma_{\text{gas}}^2 = (1 - g)\{(1 - \delta)[f(R_c \sigma_c)^2 + (1 - f)(R_{nc} \sigma_{nc}^2)] + \delta(R_{ssnr} \sigma_{ssnr})^2\} + g(1 - \delta)\frac{R_c \sigma_c^2 + R_{nc} \sigma_{nc}^2}{2} + \delta(R_{ssnr} \sigma_{ssnr})^2 \] (1)

where \( \sigma_{\text{obs}} \) is the total, observed (corrected) standard deviation, \( \sigma_{\text{gas}} \) characterizes the fluctuations of the column density of the gas (as derived in §3.2), \( \sigma_{nc}, \sigma_c \) and \( \sigma_{ssnr} \) are the expectations for non-clustering, clustering and SSNR models respectively and the mixed primary mass composition, \( R_{nc}, R_c \) and \( R_{ssnr} \) - ratios of the model to the observed gamma-ray intensities for the same three models, \( g \) is the fraction of gas in molecular form and \( \delta \) is the fraction of SSNR among the total SNR. The first term on the rhs of equation (1) relates to SNR in the ordinary interstellar medium, the second term - to SNR inside MC. The equation satisfies the lower limit \( \sigma_{\text{obs}} = \sigma_{nc} \) when \( f = \delta = g = 0 \). In the calculations it is assumed that SSNR are so rare that they are not clustered, whereas half of the ordinary SNR (NC) are clustered in MC. The values of \( g \) are, for the approximate median D-values relating to Figure 3: 0.5, 0.3, 0.1 and 0.1. In fact, different models give different absolute gamma-ray intensities, but they can be easily normalised to the experimental data multiplying the explosion energy by a constant. The value of relative fluctuations \( \sigma \) in this operation remains unchanged. Therefore, we adopted all \( R_{nc} = R_c = R_{ssnr} = 1 \).

The fraction \( f \) is therefore the only unknown variable and it can be calculated from equation (1). The results are shown in Figure 4.

4. DISCUSSION

The situations for the two models can be considered in turn.

For the NSNR model, the model used by us in much of our work, considerable clustering is needed in the Inner Galaxy. Indeed, with \( f > 1 \) the clustering must exceed that adopted by us, viz. 10 SN per \( 10^6 \)y coincident in position. Although not impossible, it is unlikely that the necessary higher number (20-30?) is present, even in the congested region of the Giant Molecular Ring at \( D \approx 4kpc \).

Instead, we incline towards the SSNR model. Inspection of Figures 4b and 4c shows that in the important Inner Galaxy region, with \( \delta = 0.2 \), the bulk of the work is done by SSNR and little by SNR. A value of \( \delta \) in the range 0.15 to 0.2 is indicated, with 0.15 being more physical. In the Outer Galaxy, the likely range is 0.0 to 0.15.

Certainly, the values of \( f \) shown in Figure 4 have big systematic and statistical errors due
mainly to the simplified model of clustering and big fluctuations caused by the stochastic nature of SN explosions. The estimates of these errors are indicated in Figure 4.

5. CONCLUSIONS

The fluctuations in the intensity of gamma rays above 1 GeV in and near the Galactic Plane give information about the mode of production of cosmic rays in their sources, and their mode of propagation.

We have adopted alternative models of both sources (supernovae of a variety of energies and SN of unique energy but with varying degrees of 'clustering') and diffusion (our 'standard' anomalous diffusion, and normal). For 'normal diffusion', although at $D \sim 5.5$ and 11.5 kpc it is possible to achieve 'reasonable' results, for $7 < D < 11$ kpc (the region nearest the sun) this is not possible. Whatever the value of $\delta$, the fractions are unreasonably high. Perhaps this argument can be used to point against 'normal' diffusion being applicable for CR of the energies in question?

Bearing in mind our general arguments that clustering is more common in the Inner Galaxy it would appear that $\delta \sim 0.15$ is favoured (Figure 4b). We would then have a constant fraction of SN of the SSN variety, with none of them clustered, but the normal SN clustered only in the solar vicinity and the Inner Galaxy.

It seems that the solution put forward is astrophysically reasonable, for the following reasons. Concerning standard SNR, these come from stars of modest mass produced in Molecular Clouds. These clouds are composed of individual clumps in which star production (and subsequent SN) occurs. In the Inner Galaxy, where the overall MC masses are bigger [12] there will be more clusters than in the Outer. For SSN, the fractional number per clump of the required massive progenitor stars is probably the same in the Inner and Outer Galaxy. In view of continuing problems with theories of star formation [T.J.Millar, private communication] even this conclusion cannot be regarded as completely firm.

As for the UHECR connection Figure 5 shows the energy spectra of CR created by SSNR. The distribution of their explosion energies and maximum energies of accelerated CR, taken from [3] and used in our analysis, gives the possibility to accelerate particles up to ultra-high energies of about $10^9$ GeV. So our preferred explanation of large fluctuations for the gamma-ray intensity has an additional advantage to include the possible origin of UHECR.

![Figure 5. Energy spectra of CR accelerated by two types of SN - SNbc and SNIn: (a) the summed spectra of 10000 SNbc and 5000 SNIn; (b) the mean spectrum and its irregularity for samples shown in (a); The irregularity is the standard deviation from the mean CR intensity for different samples of the spectrum indicated by vertical lines. The curve obs($P$) is our estimate of the proton spectrum (or the rigidity spectrum for heavier nuclei) needed to fit the experimental data [13].](image-url)

Acknowledgments
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Table 1

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Table 1. Results for the distribution of the differences between the EGRET intensities and the 9-degree polynomial fit: nos. of entries, means and RMS (σ-) values. All values relate to the logarithm of the intensity. Latitude range, |b| < 2.5°, Eγ > 1 GeV. The histograms are given in Figure 2.

REFERENCES