Towards absolute neutrino masses

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Various ways of determining the absolute neutrino masses are briefly reviewed and their sensitivities compared. The apparent tension between the announced but unconfirmed observation of the 0νββ decay and the neutrino mass upper limit based on observational cosmology is used as an example of what could happen eventually. The possibility of a “nonstandard” mechanism of the 0νββ decay is stressed and the ways of deciding which of the possible mechanisms is actually operational are described. The importance of the 0νββ nuclear matrix elements is discussed and their uncertainty estimated.

1. Generalities

Thanks to the recent triumphs of neutrino physics we know that neutrinos are massive fermions and that they are mixed, i.e., that the neutrino flavor (νe, νμ, ντ) is not a conserved quantity. We also know, with a reasonable accuracy (some better than other), the three mixing angles and the magnitudes of mass square differences \( \Delta m^2_{ij} = m_i^2 - m_j^2 \).

These discoveries represent the first deviations from the Standard Model of particle physics that postulated massless neutrinos and conservation of the individual as well as the total lepton numbers. Thus, a new all encompassing theory, sometimes called the “New Standard Model”, should be formulated. In order to delineate a path to it, several additional questions ought to be answered. Among them, two are the topic of this talk: “Are neutrinos Majorana or Dirac fermions?” and “What is the absolute neutrino mass scale?”

The list below summarizes the methods currently used for neutrino mass determination and their estimated sensitivities.

- **Neutrino oscillations**: Only mass squared differences, sometimes only their absolute value, \( \Delta m^2_{ij} = m_i^2 - m_j^2 \), are determined. The two different \( \Delta m^2 \) values are \( |\Delta m^2_{\text{atm}}| = (1.9 - 3.0) \times 10^{-3} \text{ eV}^2 \) and \( \Delta m^2_{\text{sol}} = 8.0^{+0.4}_{-0.3} \times 10^{-5} \text{ eV}^2 \). This range and indicated error bars show the present sensitivity. The accuracy will undoubtedly improve soon, particularly for \( \Delta m^2_{\text{atm}} \). This mass determination is independent on the charge conjugation properties of neutrinos.

- **Ordinary beta decay**: The quantity determined or constrained is \( \langle m_3 \rangle^2 = \sum_i m_i^2 |U_{ei}|^2 \). Present limit on \( \langle m_3 \rangle \) is \( \sim 2 \text{ eV} \). The ultimate sensitivity appears to be \( \sim 0.2 \text{ eV} \). Again, independent on the Majorana or Dirac nature of neutrinos.

- **Observational cosmology**: The quantity determined or constrained is \( M = \sum_i m_i \). The sensitivity is at present model dependent, but probably will eventually reach \( \sim 0.1 \text{ eV} \). Again, independent on the Majorana or Dirac nature of neutrinos.

- **Double beta decay**: The quantity determined or constrained is \( \langle m_3 \rangle = |\sum_i m_i |U_{ei}|^2 e^{i\alpha_i}| \), where the Majorana phases \( \alpha_i \) are at present totally unknown. The sensitivity of the method in the near term is \( \sim 0.1 \text{ eV} \), and in a longer term (next ten years or so) \( \sim 0.01 \text{ eV} \). The 0\( \nu \beta \beta \) decay exists only for Majorana neutrinos.

Note that other, sometimes conceptually simpler, methods of neutrino mass determination
cannot reach competitive sensitivities. For example, the time-of-flight would use the time delay of massive neutrinos, compared to massless particles, traveling a distance \( D \), \( \Delta t(E) = 0.514(m/E)^2 D \) s, where \( m \) is in eV, \( E \) in MeV, and \( D \) in units of 10 kpc. For a galactic supernova various analyzes suggest sensitivity \( \sim 10^{-20} \) eV for this method.

The two body weak decays, like decay of a pion at rest, \( \pi^+ \rightarrow \mu^+ + \nu_\mu \), can be also used if the muon energy is determined. In that case \( m_\nu^2 = m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu \). However, since this is a difference of two very large numbers, the present sensitivity is only \( \sim 170 \) keV with little hope for a substantial improvement.

The various neutrino mass dependent quantities are related, as shown in Fig.1. Note that a determination of \( \langle m_{\beta\beta} \rangle \), even when combined with the knowledge of \( M \) and/or \( \langle m_\beta \rangle \) does not allow, in general, to distinguish between the normal and inverted mass orderings. This is a consequence of the fact that the Majorana phases are unknown. In regions in Fig. 1 where the two hatched bands overlap it is clear that two solutions with the same \( \langle m_{\beta\beta} \rangle \) and the same \( M \) (or the same \( \langle m_\beta \rangle \)) exist and cannot be distinguished. On the other hand, obviously, if one can determine that \( \langle m_{\beta\beta} \rangle \geq 0.1 \) eV we would conclude that the mass pattern is degenerate. And in the so far hypothetical case that one could show that \( \langle m_{\beta\beta} \rangle \leq 0.01 - 0.02 \) eV, but nonvanishing nevertheless, the normal hierarchy would be established.

2. Current situation

Figure 1. Dependence of \( \langle m_{\beta\beta} \rangle \) on the mass of the lightest neutrino \( m_{\min} \), and on \( M \) and \( \langle m_\beta \rangle \). The irreducible width of the hatched areas is due to the unknown Majorana phases. The lines take into account the current uncertainties in the oscillation parameters; they will shrink as the accuracy improves. The two sets of curves correspond to the normal and inverted hierarchies.

Figure 2. Apparent tension between the claim of the 0νββ decay discovery and the upper limit on the sum \( \Sigma \) of the neutrino masses based on the observational cosmology (from [4]). I.H. and N.H. mean inverted and normal hierarchies.

At present, some information exists on the
degenerate mass region. I use this as an avenue to discuss what can eventually happen, and what it might mean. The results of the WMAP mission\(^1\) (3 years data), combined with other observations\(^2\) (Sloan Survey and in particular the Lyman-α forest analysis) restrict the sum of the neutrino masses to about \(\Sigma m_i \leq 0.2\) eV if all the data are combined. On the other hand, a recently claimed (and as yet unconfirmed) discovery of the \(0\nu\beta\beta\) decay\(^3\) would indicate that \(\langle m_{\beta\beta} \rangle \geq 0.4\) eV. Putting these two indications together\(^4\) suggests an inconsistency as shown in Fig.2.

Leaving aside the all important question whether the \(0\nu\beta\beta\) decay experimental evidence will withstand further scrutiny and whether the cosmological constraints are reliable and model independent, let us discuss possible scenarios suggested by the comparison illustrated in Fig.2.

What can happen once all evidence becomes available:

1. Both neutrino mass determinations will yield a positive and consistent result, i.e., both results will intersect at the allowed band and both will suggest the degenerate neutrino mass pattern. Such results will be relatively readily accepted, even though many theorists do not expect the degenerate scenario.

2. Future \(0\nu\beta\beta\) decay experiments will not find a positive evidence (i.e., the present claim will be shown to be incorrect), but the observational cosmology or and the study of tritium \(\beta\) decay will find evidence for the degenerate mass pattern. This is the situation exactly opposite to the one depicted in Fig.2. This will be also, albeit reluctantly, accepted and would indicate that neutrinos are Dirac, not Majorana particles.

3. The depicted situation is confirmed. The positive evidence of \(0\nu\beta\beta\) decay is confronted with a lack of confirmation from observational cosmology. What then? Is there a possible scenario that would accommodate this situation?

3. **Mechanism of \(0\nu\beta\beta\) decay**

   The answer is yes and deserves a more detailed explanation. In fact, this can happen for two reasons. Possibility 1): The \(0\nu\beta\beta\) decay is not caused by the exchange of a light Majorana neutrino but by another mechanism. Hence the extraction of \(\langle m_{\beta\beta} \rangle\) from the lifetime is not possible. Possibility 2): Even though the \(0\nu\beta\beta\) decay is caused by the exchange of a light Majorana neutrino the relation between the lifetime and \(\langle m_{\beta\beta} \rangle\) is different than used so far, since the nuclear matrix elements are highly uncertain.

   In order to further discuss the point 1) above, note that besides the exchange of a light Majorana neutrino \(0\nu\beta\beta\) decay can be caused by the exchange of various hypothetical heavy particles in particle physics models that contain Lepton Number Violation (LNV). It turns out that the confusion about the possible mechanism can occur if the scale of such heavy particles is \(\Lambda \sim 1\) TeV. (Smaller scales are already excluded, much larger ones lead to unobservably long lifetimes.) If the \(0\nu\beta\beta\) decay is observed, how can we tell which mechanism is responsible?

   Generally, observation of the \(0\nu\beta\beta\) decay, even of the single electron spectrum and/or the angular distribution of the electrons, does not allow one to determine the mechanism responsible for the decay. It has been suggested in Ref.\(^5\) that the observation of the Lepton Flavor Violation (LFV) could be used as a “diagnostic tool” for that purpose.

   The discussion is concerned mainly with the branching ratios \(B_{\mu\rightarrow e\gamma} = \Gamma(\mu \rightarrow e\gamma)/\Gamma_\mu(0)\) and \(B_{\mu\rightarrow e} = \Gamma_{\text{conv}}/\Gamma_{\text{capt}}, \) where \(\mu \rightarrow e\gamma\) is normalized to the standard muon decay rate \(\Gamma_\mu(0) = (G_F^2 m_\mu^5)/(192\pi^3), \) while \(\mu \rightarrow e\) conversion for the \(\mu^-\) on an atomic orbit of a nucleus is normalized to the corresponding capture rate \(\Gamma_{\text{capt}}.\)

   The main diagnostic tool in the analysis is the ratio

   \[
   R = B_{\mu\rightarrow e}/B_{\mu\rightarrow e\gamma},
   \]

   and the relevance of our observation relies on the potential for LFV discovery in the forthcoming experiments MEG \(^6\) (\(\mu \rightarrow e\gamma\)) and MECO \(^7\) (\(\mu \rightarrow e\) conversion)\(^8\).

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\(^2\)Even though MECO experiment was recently canceled,
As explained in [5] if the ratio $R$ is $\sim 10^{-2(2-3)}$ one expects that the $0\nu\beta\beta$ decay is caused by the exchange of a light Majorana neutrino and hence the decay rate is proportional to $\langle m_{\beta\beta} \rangle^2$. On the other hand, observation of $R \gg 10^{-2}$ could signal non-trivial LNV dynamics at the TeV scale, whose effect on $0\nu\beta\beta$ has to be analyzed on a case by case basis. Therefore, in this scenario no definite conclusion can be drawn based on LFV rates. In addition, non-observation of LFV in muon processes in forthcoming experiments would likely imply that the scale of non-trivial LFV and LNV is above a few TeV, and thus $\Gamma_{0\nu\beta\beta} \sim \langle m_{\beta\beta} \rangle^2$.

The conclusion above, with some important caveats, was reached in [3] by analyzing two phenomenologically viable models that incorporate LNV and LFV at low scale, the left-right symmetric model and the R-parity violating supersymmetry. However, it is likely that the basic mechanism at work in these illustrative cases is generic: low scale LNV interactions ($\Delta L = \pm 1$ and/or $\Delta L = \pm 2$), which in general contribute to $0\nu\beta\beta$, also generate sizable contributions to $\mu \rightarrow e$ conversion, thus enhancing this process over $\mu \rightarrow e\gamma$.

4. Nuclear matrix elements

If indeed the exchange of a light Majorana neutrino is responsible for the $0\nu\beta\beta$ decay, the half-life and the effective mass are related by

$$\frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(Q, Z) |M^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2,$$  \hspace{1cm} (2)

where $G^{0\nu}(Q, Z)$ is a phase space factor that depends on the transition $Q$ value and through the Coulomb effect on the emitted electrons on the nuclear charge $Z$ and that can be easily and accurately calculated, and $M^{0\nu}$ is the nuclear matrix element that can be evaluated in principle, although with a considerable uncertainty.

It follows from eq.2 that (i) values of the nuclear matrix elements $M^{0\nu}$ are needed in order to extract the effective neutrino mass from the measured $0\nu\beta\beta$ decay rate, and (ii) any uncertainty in $M^{0\nu}$ causes a corresponding and equally large uncertainty in the extracted $\langle m_{\beta\beta} \rangle$ value. Thus, the issue of an accurate evaluation of the nuclear matrix elements attracts considerable attention and in its extreme form can explain the situation depicted in Fig.2.

Common to all methods of calculating $M^{0\nu}$ is the description of the nucleus as a system of nucleons bound in the mean field and interacting by an effective residual interaction. The used methods differ as to the number of nucleon orbits (or shells and subshells) included in the calculations and the complexity of the configurations of the nucleons in these orbits. The two basic approaches used so far for the evaluation of the nuclear matrix elements for both the $2\nu$ and $0\nu\beta\beta$ decay modes are the Quasiparticle Random Phase Approximation (QRPA) and the nuclear shell model (NSM). They are in some sense complementary; QRPA uses a larger set of orbits, while NSM can include only a rather small set of orbits but includes essentially all possible configurations. NSM also can be tested in a considerable detail by comparing to the nuclear spectroscopy data; in QRPA such comparisons are much more limited.

Since the calculations using QRPA are much simpler, an overwhelming majority of the published calculations uses that method. There are suggestions to use the spread of these published values of $M^{0\nu}$ as a measure of uncertainty[8]. Following this, one would conclude that the uncertainty is quite large, a factor of three or as much as five. But that way of assigning the uncertainty is questionable. Using all or most of the published values of $M^{0\nu}$ means that one includes calculations of uneven quality. Some of them were devoted to the tests of various approximations, and concluded that they are not applicable. Some insist that other data, like the $M^{2\nu}$, are correctly reproduced, others do not pay any attention to such test. Also, different forms of the transition operator $G^{0\nu}$ are used, in particular some works include approximately the effect of the short range nucleon-nucleon repulsion, while others neglect it.

In contrast, in Ref. [7] an assessment of uncertainties in the matrix elements $M^{0\nu}$ inherent in the QRPA was made, and it was concluded that
with a consistent treatment the uncertainties are much less, perhaps only about 30% (see Fig. 3). That calculation uses the known $2\nu$ matrix elements in order to adjust the most important free parameter, the effective proton-neutron interaction constant. There is a lively debate in the nuclear structure theory community, beyond the scope of this talk, about this conclusion.

![Figure 3](image.png)

Figure 3. Nuclear matrix elements and their variance for the indicated approximations (see Ref. [9]).

It is of interest also to compare the resulting matrix elements of Rodin et al. [9] based on QRPA and its generalizations, and those of the available most recent NSM evaluation [10]. Note that the operators used in NSM evaluation do not include the induced nucleon currents that in QRPA reduce the matrix element by about 30%. The QRPA [9] and NSM [10] $M^{0\nu}$ are compared in Table 1. In the last column the NSM values are reduced by 1.3, to account approximately for the effects of the induced nucleon currents.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>QRPA</th>
<th>NSM</th>
<th>NSM/1.3</th>
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</thead>
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<tr>
<td>$^{76}$Ge</td>
<td>2.3-2.4</td>
<td>2.35</td>
<td>1.80</td>
</tr>
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<td>1.9-2.1</td>
<td>2.26</td>
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<tr>
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<td>1.1-1.2</td>
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<td>$^{130}$Te</td>
<td>1.3</td>
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<td>1.77</td>
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</tr>
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</table>

Table 1: Comparison of the calculated nuclear matrix elements $M^{0\nu}$ using the QRPA method [9] and the NSM [10]. In the last column the NSM values are reduced, divided by 1.3, to account approximately for the effects of the induced nucleon currents.

Once the nuclear matrix elements are fixed (by choosing your favorite set of results), they can be combined with the phase space factors (a complete list is available, e.g. in the monograph [11]) to obtain a half-life prediction for any value of the effective mass $\langle m_{\beta\beta} \rangle$. It turns out that for a fixed $\langle m_{\beta\beta} \rangle$ the half-lives of different candidate nuclei do not differ very much from each other (not more than by factors $\sim 3$ or so) and, for example, the boundary between the degenerate and inverted hierarchy mass regions corresponds to half-lives $\sim 10^{27}$ years. Thus, the next generation of experiments should reach this region using several candidate nuclei, making the corresponding conclusions less nuclear model dependent.

5. Summary

In this talk I discussed the status of neutrino mass determination, in particular the role of the double beta decay. I have shown that if one makes the minimum assumption that the light neutrinos familiar from the oscillation experiments, which are interacting only by the left-handed weak current, are Majorana particles, then the rate of the $0\nu\beta\beta$ decay can be related to the absolute scale of the neutrino mass in a straightforward way.

On the other hand, it is also possible that the $0\nu\beta\beta$ decay is mediated by the exchange of heavy particles. I explained that if the corresponding
mass scale of such hypothetical particles is $\sim 1$ TeV, the corresponding $0\nu$ decay rate could be comparable to the decay rate associated with the exchange of a light neutrino. I further argued that the study of the lepton flavor violation involving $\mu \rightarrow e$ conversion and $\mu \rightarrow e + \gamma$ decay may be used as a “diagnostic tool” that could help to decide which of the possible mechanisms of the $0\nu$ decay is dominant.

Further, I have shown that the range of the effective masses $\langle m_{\beta\beta} \rangle$ can be roughly divided into three regions of interest, each corresponding to a different neutrino mass pattern. The region of $\langle m_{\beta\beta} \rangle \geq 0.1$ eV corresponds to the degenerate mass pattern. Its exploration is well advanced, and one can rather confidently expect that it will be explored by several $\beta\beta$ decay experiments in the next 3-5 years. This region of neutrino masses (or most of it) is also accessible to studies using the ordinary $\beta$ decay and/or the observational cosmology. Thus, if the nature is kind enough to choose this mass pattern, we will have a multiple ways of exploring it.

The region of $0.01 \leq \langle m_{\beta\beta} \rangle \leq 0.1$ eV is often called the “inverted mass hierarchy” region. In fact, both the inverted and the quasi-degenerate but normal mass orderings are possible in this case, and experimentally indistinguishable. Realistic plans to explore this region using the $0\nu\beta\beta$ decay exist, but correspond to a longer time scale of about 10 years. They require much larger, $\sim$ ton size $\beta\beta$ sources and correspondingly even more stringent background suppression.

Intimately related to the extraction of $\langle m_{\beta\beta} \rangle$ from the decay rates is the problem of nuclear matrix elements. At present, there is no consensus among the nuclear theorists about their correct values, and the corresponding uncertainty. I argued that the uncertainty is less than some suggest, and that the closeness of the Quasiparticle Random Phase Approximation (QRPA) and Shell Model (NSM) results are encouraging. But this is still a problem that requires further improvements.

REFERENCES