Coupling constants of $D^*D_sK$ and $D_s^*DK$ processes

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We calculate the coupling constants of $D^*D_sK$ and $D_s^*DK$ vertices using the QCD sum rules technique. We compare results obtained in the limit of SU(4) symmetry and found that the symmetry is broken on the order of 40%.

The knowledge of coupling constants in hadronic vertices is crucial to estimate cross sections when hadronic degrees of freedom are used. For example, absorption of charmonium by kaons will be one of the indicatives of the formation of the quark-gluon plasma (QGP) in relativistic heavy ions collisions$^1$. The kaon is one of the commovers light mesons that can annihilate the charmonium in medium given as result of D mesons. The quantitative measure of this cross section is an essential tool to understand the formation of QGP, but it is not an easy task even for the experimentalists. The processes of absorption of $J/\Psi$ by kaons can be visualized in the Figure 1. To obtain theoretical values of the cross section, in the case of charmonium absorption, one of the more used methods is based on effective SU(4) Lagrangians$^2, 4$. The effective Lagrangians that described the processes represented in Fig.1 are:

$$\mathcal{L}_{D_sD^*K} = i\mathcal{G}_{D_sD^*K}D^*_\mu(D_j\bar{\gamma}_\mu K - (\bar{\partial}_\mu D_j)K) + H.c.,$$

(1)

$$\mathcal{L}_{D_s^*DK} = i\mathcal{G}_{D_s^*DK}D^*_\mu(D_j\bar{\gamma}_\mu \bar{K} - (\bar{\partial}_\mu D_j)\bar{K}) + H.c..$$

(2)

In this formalism it is necessary to have the detailed knowledge of the form factors and coupling constants in the hadronic vertices to obtain the cross section. With the use of the form factors or not, the cross section may change by a factor of two$^2$. Also the values of the coupling constants used are calculated by SU(4) exact symmetry and in the case the coupling constants are derivated putting the masses of the quarks $u, d$, identical to the $s$ and $c$ quarks. In this way the values of the coupling constants for the two vertex of the right side in the processes, in Fig. 1, are identical:

$$\mathcal{G}_{D_sD^*K} = \mathcal{G}_{D_s^*DK} = \frac{g}{2\sqrt{2}}$$

(3)

In this work we study the $D^*D_sK$ and $D_s^*DK$ vertices using the QCD Sum Rules technique$^5$, to obtain the form factors and to infer the coupling constants.

We have been working on the problem of compute coupling constants for others processes and have a consistent method for this$^6, 7, 8, 9, 10, 11, 12, 13, 14$.

Following the QCDSR formalism described in our previous works$^6, 7, 8, 9, 10, 11, 12, 13, 14$, we write the three-point correlation function associated with the $D^*D_sK$ vertex, which is given by

$$\Gamma_{\mu}^{(K)}(p,p') = \int d^4x d^4y e^{ip\cdot x} e^{-i(p'-p)\cdot y}$$

$$\langle 0 | T \{ j_\mu^D(x) j_\mu^{K\dagger}(y) j_{D_s}\dagger(0) \} | 0 \rangle$$

(4)

for $K$ meson off-shell, where the interpolating currents are $j_\mu^D = \bar{c}\gamma_\mu d$, $j_\mu^K = \bar{s}\gamma_\mu s$, and $j_{D_s} = \bar{c}\gamma_\mu s$, and

$$\Gamma_{\mu\nu}^{(D_s)}(p,p') = \int d^4x d^4y e^{ip\cdot x} e^{-i(p'-p)\cdot y}$$

$$\langle 0 | T \{ j_\mu^K(x) j_{D_s}\dagger(y) j_{D_s}\dagger(0) \} | 0 \rangle$$

(5)

for $D_s$ meson off-shell, with the interpolating currents $j_\mu^D = \bar{\mu}\gamma_\mu \gamma_5 s$, $j_{D_s} = \bar{c}\gamma_\mu c$, with $u, d, s$ and $c$ being the up, down, strange and charm quark fields respectively. In both cases, each one of these currents has the same quantum numbers as the corresponding mesons.

Using the above currents to evaluate the correlation functions$^6$ and$^8$, the theoretical or QCD side is obtained. The framework to calculate the correlators in the QCD side is the Wilson operator product expansion (OPE). The Cutkosky’s rule allows us to obtain the double discontinuity of the correlation function for each one of the Dirac structures appearing in the correlation function. Then we use spectral representation over the virtualities $p^2$ and $p'^2$, holding $Q^2 = -q^2$ fixed. The amplitudes receive contributions from all terms in the OPE. The leading contribution comes from the perturbative term.

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FIG. 1: Annihilation of $J/\Psi$ by kaons given $D_s$, $D^*$, $D_s^*$ and $D$ mesons production.
The phenomenological side of the sum rule, which is written in terms of the mesonic degrees of freedom, is parametrized in terms of the form factors, meson decay constants and meson masses. We introduce the meson decay constants $f_K$, $f_D$, and $f_{D^*}$, which are defined by the following matrix elements

$$\langle 0 | J^K | K \rangle = \frac{m^2_K f_K}{m_s + m_q},$$

(6)

$$\langle 0 | J^D_s | D_s \rangle = \frac{m^2_{D_s} f_{D_s}}{m_c + m_s},$$

(7)

and

$$\langle 0 | J_{D^*} | D^* \rangle = m_{D^*} f_{D^*} \epsilon^*_\nu,$$

(8)

where $\epsilon^*_\nu$ is the polarization vector of the $D^*$ meson. The QCD sum rule is obtained by matching both representations, using the universality principle. The matching is improved by performing a double Borel transform on both sides. The perturbative contribution for both Eqs. (6) and (7) is given in details in ref. [14]. We chosen one structure that appear in both sides and also must have a stability that guarantees a good match between the two sides of the sum rule. The structures that obey these two points are $p'_\mu$, in the case $K$ off-shell, and $p'_\mu, p'_\nu$ in the case $D_s$ off-shell.

The Borel transformation [15] in the variables $P^2 = -p^2 \rightarrow M^2$ and $P^2 = -p^2 \rightarrow M^2$ allows to get the final form of the sum rule, which allow us to obtain the form factors $g^{(M)}_{D^*D_sK}(Q^2)$ where $M$ stands for the off-shell meson.

We use Borel masses satisfying the constraint $M^2/M^2 = m_{in}/m_{out}$, where $m_{in}$ and $m_{out}$ are the masses of the incoming and outgoing meson respectively. The values of the parameters used in the calculation of the vertices are depicted in Table I and in Table II.

The continuum thresholds $s_0$ and $t_0$ are the two parameters that are including in the QCDSR, and are important to control the pole contribution. Using $\Delta_s = \Delta_u = 0.5$ GeV for the continuum thresholds and fixing $Q^2 = 1$ GeV$^2$, we found a good stability of the form factor $g^{(K)}_{D^*D_sK}$ as a function of the Borel mass $M^2$, in the interval $3 < M^2 < 5$ GeV$^2$. In the case of the form factor $g^{(D_s)}_{D^*D_sK}$ the interval for stability of the sum rule is $2 < M^2 < 5$ GeV$^2$.

Fixing $\Delta_s = \Delta_u = 0.5$ GeV and $M^2 = 3$ GeV$^2$, we evaluate the momentum dependence of both form factors. The results are shown in Fig. 2 where the squares corresponds to the $g^{(K)}_{D^*D_sK}(Q^2)$ form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the $g^{(D_s)}_{D^*D_sK}(Q^2)$ form factor.

In the case of the $K$ meson off-shell, our numerical results can be parametrized by an exponential function (dotted line in Fig. 2):

$$g^{(K)}_{D^*D_sK}(Q^2) = 2.83 e^{-\frac{Q^2}{5 GeV^2}} \rightarrow g^{(K)}_{D^*D_sK} = 3.01,$$

(9)

where the define coupling constant, $g^{(K)}_{D^*D_sK}$ is chosen as the value of the form factor at $Q^2 = -m^2_K$.

In the case when the $D_s$ meson is off-shell, our sum rule results can be parametrized by a monopole formula (solid line in Fig. 2):

$$g^{(D_s)}_{D^*D_sK}(Q^2) = \frac{9.01}{Q^2 + 6.86} \rightarrow g^{(D_s)}_{D^*D_sK} = 3.02,$$

(10)

where $g^{(D_s)}_{D^*D_sK}$ is the coupling constant is chosen as the value of the form factor at $Q^2 = -m^2_{D_s}$.

Comparing the results in Eqs. (9) and (10) we see that the method used to extrapolate the QCDSR results in both cases, $K$ and $D_s$ off-shell, allows us to extract values for the coupling constant which are in very good agreement with each other.

In order to study the dependence of this results with the continuum threshold, we vary $\Delta_s = \Delta_u$ in the interval $0.4 \leq \Delta_s = \Delta_u \leq 0.6$ GeV. This procedure give us uncertainties in such a way that the final results for the couplings in each case are:

$$g^{(K)}_{D^*D_sK} = 3.02 \pm 0.15$$

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<th>$m_{c}$</th>
<th>$m_{s}$</th>
<th>$m_{c}$</th>
<th>$m_{s}$</th>
<th>$m_{D}$</th>
<th>$m_{D}$</th>
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TABLE I: Masses of quarks and mesons used in the calculation of the QCD sum rule. All quantities are in GeV.

<table>
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<th>$f_K$</th>
<th>$f_D$</th>
<th>$f_{D^*}$</th>
<th>$f_D$</th>
<th>$f_{D^*}$</th>
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<tbody>
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<td>0.280</td>
<td>0.240</td>
<td>0.200</td>
<td>0.330</td>
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TABLE II: Decay constant used in the calculation of the QCD sum rule. All quantities are in GeV.
and

\[ g_{D^*D,DK}^{(D)} = 3.03 \pm 0.14. \]

Now we study the \( D^*_sDK \) vertex. The treatment is similar to the previous case. For details of the calculation see reference [14]. The correlation functions are

\[ \Gamma_{\mu}^{(K)}(p, p') = \int \frac{d^4x d^4y}{e^{ip'x} e^{-i(p'-p)y}} \langle 0|T\{j_{\mu}^D(x)j^{K\dagger}(y)j^{D\dagger}(0)\}|0\rangle \] (11)

for \( K \) meson off-shell, where the interpolating currents are \( j_{\mu}^D = \bar{c}\gamma_\mu s \), \( j^K = i\bar{u}\gamma_5 s \) and \( j^{D\dagger} = i\bar{c}\gamma_5 s \), and

\[ \Gamma_{\mu}^{(D)}(p, p') = \int \frac{d^4x d^4y}{e^{ip'x} e^{-i(p'-p)y}} \langle 0|T\{j_{\mu}^K(x)j^{D\dagger}(y)j^{D\dagger}(0)\}|0\rangle \] (12)

for \( D \) meson off-shell, with the interpolating currents \( j_{\mu}^D = \bar{\psi}_0 t s \), \( j^D = \bar{c}\gamma_5 s \) and \( j^{D\dagger} = i\bar{c}\gamma_5 s \). We introduce the decay constants \( f_D \) and \( f_{D^*} \), which are defined by the following matrix elements:

\[ \langle 0|j^D|D\rangle = \frac{m_D^2}{m_e + m_q} f_D, \] (13)

\[ \langle 0|j^{D\dagger}|D^*\rangle = m_{D^*} f_D^* \varepsilon^\nu, \] (14)

where \( \varepsilon^\nu \) is the polarization vector of the \( D^*_s \) meson.

In Fig. 3 the squares corresponds to the \( g_{D^*_sDK}^{(K)}(Q^2) \) form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the \( g_{D^*_sDK}^{(D)}(Q^2) \) form factor.

In the case when the \( K \) meson is off-shell, our numerical results can be parametrized by an exponential function (dashed curve in Fig. 3) and the coupling constant is extracted at the value of the form factor at \( Q^2 = -m_K^2 \).

\[ g_{D^*_sDK}^{(K)}(Q^2) = 2.69 e^{-\frac{Q^2}{2.87}} \rightarrow g_{D^*_sDK}^{(K)} = 2.87. \] (15)

In the case when the \( D \) meson is off-shell, the sum rule results are represented by the triangles in Fig 3 and they can be parametrized by a monopole formula (solid line in the figure) and the coupling constant is the value of the form factor at \( Q^2 = -m_D^2 \).

\[ g_{D^*_sDK}^{(D)}(Q^2) = \frac{7.78}{Q^2 + 6.34} \rightarrow g_{D^*_sDK}^{(D)} = 2.72. \] (16)

Studying the dependence of our results with the continuum threshold, for \( \Delta_{\mu, s} \) varying in the interval \( 0.4 \leq \Delta_{\mu, s} \leq 0.6 \text{ GeV} \), we obtain the following values, with errors, for the couplings in each case:

\[ g_{D^*_sDK}^{(K)} = 2.87 \pm 0.19 \]

and

\[ g_{D^*_sDK}^{(D)} = 2.72 \pm 0.31. \]

Concluding, we have studied the form factors and coupling constants of \( D^*DK \) and \( D^*_sDK \) vertices in a QCD sum rule calculation. For each case we have considered two particles off-shell, the lightest and one of the heavy ones: the \( K \) and \( D_s \) mesons for the \( D^*DK \) vertex, and the \( K \) and \( D \) mesons for the \( D^*_sDK \) vertex. In the two situations, the off-shell particles give compatible results for the coupling constant in each vertex. The results are:

\[ g_{D^*DK} = 3.02 \pm 0.14 \] (17)

and

\[ g_{D^*_sDK} = 2.84 \pm 0.31. \] (18)

We can compare our result with the prediction of the exact SU(4) symmetry [4], which would give the following relation among these numbers [4]: \( g_{D^*DK} = g_{D^*_sDK} = g_{DK} \). Eqs. (17) and (18) shows that the coupling constants in the vertices \( D^*DK \) and \( D^*_sDK \) are consistent one with the other, but that they are relatively far from the value given by the SU(4) symmetry in the cited reference. Therefore, we conclude that the SU(4) symmetry is broken by approximately 40% in the calculation performed here. This is expected because the coupling constant obtain by the exact SU(4) symmetry put the masses of \( u, d \) quarks are the same values that the \( s \) and \( c \) quarks. In this case there is not experimental value to compare our result but the values of the coupling constants were obtained by two different way at the same vertex and give compatible results.