I. INTRODUCTION

The pion electromagnetic form factor, $F_{\pi}(Q^2)$ in vacuum shows that the physical $\rho$ meson is not a pure isospin eigenstate and it can mix with the $\omega$ meson \cite{1}. In vacuum this mixing amplitude can be determined by measuring $F_{\pi}(Q^2)$ which, although dominated by the $\rho^0$ pole, shows a kink near the $\omega$ meson mass. Such mixing (after electromagnetic correction) implies that the charge symmetry is broken at the most fundamental level in strong interaction through the small mass difference between up and down quarks in the QCD Lagrangian. Consequently, the physical $\rho$ and $\omega$ mesons that we deal with are admixtures of the corresponding isospin eigenstates. At the hadronic level this mixing can be understood in terms of neutron-proton mass difference in effective models \cite{2}.

$\rho-\omega$ mixing has important and interesting consequences. It plays an important role in generating contributions to few body charge symmetry violating observables \cite{3}. The $\rho-\omega$ mixing amplitude is determined from $e^+e^- \rightarrow \pi^+\pi^-$ by measuring the pion form factor in the interference region \cite{1}. With the extracted value of the mixing amplitude one is able to explain a number of observables namely, the non-Coulombic binding energy difference (Nolen Schiffer anomaly \cite{4,5}) of $A = 3$ (mirror) nuclei, significant contributions to the $np$ asymmetry at 183 MeV and non-negligible contributions to the difference of $nn$ and $pp$ scattering lengths \cite{6}.

The mixing of different isospin states will be modified in matter. Such medium effects have recently been investigated by several authors \cite{3,8,9}. Unlike neutron-proton mass difference, which is responsible for $\rho-\omega$ mixing in free space, the mixing in matter can be induced if the neutron-proton densities are different. This happens even if the Hamiltonian preserves the isospin symmetry i.e., if $M_n = M_p$, akin to the ‘spontaneous symmetry breaking’ driven by the $n \rightarrow p$ asymmetric ground state. It has been shown in ref.\cite{7} that the density dependent mixing is of similar magnitude as the usual vacuum mixing at normal nuclear matter density. Subsequently, Brodsky and Lann shown that both the shape and the pole position of the pion form factor in dense asymmetric nuclear matter is different from its vacuum counterpart with $\rho-\omega$ mixing. This is due to the density and asymmetry dependent $\rho-\omega$ mixing which could even dominate over its vacuum counterpart in matter. Effect of the in-medium pion factor on experimental observables e.g., invariant mass distribution of lepton pairs has been demonstrated.

II. FORMALISM

The Lagrangian describing $\rho-\omega$ meson-nucleon interaction is given by,

$$\mathcal{L} = \bar{\psi}(\tau_3\Gamma_{\rho,\mu}\rho^0)\psi + \bar{\psi}(\Gamma_{\omega,\mu}\omega^0)\psi,$$

$$\Gamma_{\nu,\mu} = g_\nu \left[ \gamma_\mu - \frac{n_\nu}{2M} \sigma_{\mu\nu} \partial^\nu \right].$$

Here $\tilde{M} = (M_p + M_n)/2$ where the subscript $p$ ($n$) stands for proton (neutron), $\rho$ meson couples to neutron with a negative sign in contrast to the $\omega$ meson while both couple with the proton. This is the basic formalism of the calculation.
where \( T_{\mu\nu} \) represents relevant traces as in ref.\[13\].

The \( \Pi_{\mu\nu}^{D}(q) = \Pi_{\mu\nu}^{w} + \Pi_{\mu\nu}^{\text{int}} \) functions in this case are as follows

\[
\Pi_{\mu\nu}^{w}(k, M^*, q_0, |q|) = \frac{g_\rho g_\omega}{\pi^3} \int_0^{k F} \frac{d^3 k}{E_k} \frac{Q_\mu Q_\nu}{Q^4 - 4(K \cdot Q)^2} \tag{7}
\]

\[
\Pi_{\mu\nu}^{\text{int}}(k, M^*, q_0, |q|) = -\frac{g_\rho g_\omega}{\pi^3} \frac{(k_F^2)}{8M} 2Q^4 Q_{\mu\nu} \\
\times \int_0^{k F} \frac{d^3 k}{E_k} \frac{1}{Q^4 - 4(K \cdot Q)^2} \tag{8}
\]

where \( K_{\mu\nu} = (K_\mu - \frac{K \cdot Q}{Q^2} Q_\mu)(K_\nu - \frac{K \cdot Q}{Q^2} Q_\nu) \), \( Q_{\mu\nu} = (-g_\mu + i\frac{Q_\mu Q_\nu}{Q^2}) \) and \( E_k = \sqrt{k^2 + M^2} \). In the above expressions, for proton (neutron) loop we substitute \( M_p \) (\( M_n \)) and \( k_F^p \) (\( k_F^n \)) respectively. Moreover, at this point it might be recalled that for a vector meson moving in nuclear matter the longitudinal (\( L \)) and transverse (\( T \)) polarization tensors are different because of \( K_{\mu\nu} \), unlike the vacuum part which is proportional to \( Q_{\mu\nu} \). The \( L \) and \( T \) modes are constructed as \( L_1 = -L_0 + L_{22} \) and \( T = L_{22} + L_{11} \), where the meson momentum \( Q = (q_0, 0, 0, |q|) \) (see Appendix for details).

The mixing amplitude is characterized by \( \Pi_{\lambda,T}^\omega \) which involves scattering from the neutron and proton Fermi spheres :

\[
\Pi_{\lambda,T}^\omega = \Pi_{L,T}^\omega(k_F, M^*, q_0, |q|) - \Pi_{L,T}^\omega(k_F^p, M^*, q_0, |q|). \tag{9}
\]

The negative sign arises because of the \( \tau_3 \) in the \( \rho-NN \) interaction.

The pure part of the polarization can be obtained by taking appropriate vertex factor like \( g_\omega \) or \( g_\rho \), for both the vertices. Accordingly, the total is given by a sum over the neutron and proton loops instead of the difference.

\[
\Pi_{L,T}^{\rho}\Pi_{L,T}^{\rho}(k_F, M^*, q_0, |q|) + \Pi_{L,T}^{\rho}(k_F^p, M^*, q_0, |q|) \times 10
\]

and similarly \( \rho \rightarrow \omega \) gives results for the \( \omega \) meson polarization function. We take \( g_\omega = 10.1, \kappa_\omega = 0 \) and \( g_\rho = 2.63, \kappa_\rho = 6.0 \) \[13\] in numerical computations.

### A. Density dependent part

The density dependent piece of the mixed polarization (\( \rho-\omega \)) due to \( p-p \) or \( n-n \) excitations is generically given by

\[
\Pi_{\mu\nu}^{D}(k, M^*, q_0, |q|) = \frac{g_\rho g_\omega}{(2\pi)^4} \int_0^{k F} \frac{d^3 K}{E_k} \delta(k^0 - E_k) \]

where

\[
\theta(k_F - |\vec{k}|) \times \left[ \frac{T_{\mu\nu}(K - Q, K)}{(K - Q)^2 - M^2} \right]_{\text{vac}} \tag{10}
\]

### B. Free part

The vacuum part will also give rise to mixing which is same as ref.\[2\] with \( n \) and \( p \) mass \( M_{n,p} \) replaced by the in-medium masses, \( M_{n,p}^\text{int} \). In Walecka model this is determined from the following self-consistent condition \[12\].

\[
\Pi_{\mu\nu}^{D}(k, M^*, q_0, |q|) = \Pi_{\mu\nu}^{w} + \Pi_{\mu\nu}^{\text{int}} \]

where \( \Pi_{\mu\nu}^{D}(k) = \Pi_{\mu\nu}^{w} + \Pi_{\mu\nu}^{\text{int}} \) functions in this case are as follows

\[
\Pi_{\mu\nu}^{w}(k, M^*, q_0, |q|) = \frac{g_\rho g_\omega}{\pi^3} \int_0^{k F} \frac{d^3 k}{E_k} \frac{Q_\mu Q_\nu}{Q^4 - 4(K \cdot Q)^2} \tag{7}
\]

\[
\Pi_{\mu\nu}^{\text{int}}(k, M^*, q_0, |q|) = -\frac{g_\rho g_\omega}{\pi^3} \frac{(k_F^2)}{8M} 2Q^4 Q_{\mu\nu} \\
\times \int_0^{k F} \frac{d^3 k}{E_k} \frac{1}{Q^4 - 4(K \cdot Q)^2} \tag{8}
\]

where \( K_{\mu\nu} = (K_\mu - \frac{K \cdot Q}{Q^2} Q_\mu)(K_\nu - \frac{K \cdot Q}{Q^2} Q_\nu) \), \( Q_{\mu\nu} = (-g_\mu + i\frac{Q_\mu Q_\nu}{Q^2}) \) and \( E_k = \sqrt{k^2 + M^2} \). In the above expressions, for proton (neutron) loop we substitute \( M_p \) (\( M_n \)) and \( k_F^p \) (\( k_F^n \)) respectively. Moreover, at this point it might be recalled that for a vector meson moving in nuclear matter the longitudinal (\( L \)) and transverse (\( T \)) polarization tensors are different because of \( K_{\mu\nu} \), unlike the vacuum part which is proportional to \( Q_{\mu\nu} \). The \( L \) and \( T \) modes are constructed as \( L_1 = -L_0 + L_{22} \) and \( T = L_{22} + L_{11} \), where the meson momentum \( Q = (q_0, 0, 0, |q|) \) (see Appendix for details).

The mixing amplitude is characterized by \( \Pi_{\lambda,T}^\omega \) which involves scattering from the neutron and proton Fermi spheres :

\[
\Pi_{\lambda,T}^\omega = \Pi_{L,T}^\omega(k_F, M^*, q_0, |q|) - \Pi_{L,T}^\omega(k_F, M^*, q_0, |q|). \tag{9}
\]

The negative sign arises because of the \( \tau_3 \) in the \( \rho-NN \) interaction.

The pure part of the polarization can be obtained by taking appropriate vertex factor like \( g_\omega \) or \( g_\rho \), for both the vertices. Accordingly, the total is given by a sum over the neutron and proton loops instead of the difference.

\[
\Pi_{L,T}^{\rho}\Pi_{L,T}^{\rho}(k_F, M^*, q_0, |q|) + \Pi_{L,T}^{\rho}(k_F^p, M^*, q_0, |q|) \times 10
\]

and similarly \( \rho \rightarrow \omega \) gives results for the \( \omega \) meson polarization function. We take \( g_\omega = 10.1, \kappa_\omega = 0 \) and \( g_\rho = 2.63, \kappa_\rho = 6.0 \) \[13\] in numerical computations.
where $\rho_i^s$ ($i = p, n$) represent scalar densities given by

$$\rho_i^s = \frac{M_i^*}{2\pi^2} \left[ E_i^2 k_F - M_i^* \ln \left( \frac{E_i^2 + k_F^2}{M_i^*} \right) \right].$$

(12)

The free part of the polarization tensor can be written as,

$$\Pi_F^{\mu\nu} = (-g^{\mu\nu} + Q^\mu Q^\nu/Q^2)\Pi_F(Q^2).$$

(13)

The mixing contributions to $\Pi_F$ are given by:

$$\Pi_F^{\mu\nu}_{p,n} = \frac{g_\rho g_\omega}{2\pi^2} \frac{Q^2}{M^*} \int_0^1 dz \frac{z(1-z)}{\ln \left( \frac{M_p^2 - Q^2 z(1-z)}{M^2 - Q^2 z(1-z)} \right)} - (p \rightarrow n),$$

(14)

$$\Pi_F^{\mu\nu}_{v,t} = \frac{g_\rho g_\omega}{2\pi^2} \frac{Q^2}{M^*} \int_0^1 dz \ln \left( \frac{M_p^2 - Q^2 z(1-z)}{M^2 - Q^2 z(1-z)} \right) - (p \rightarrow n).$$

(15)

The pure $\rho$ ($\omega$) meson self-energies for the vector-vector, vector-tensor and tensor-tensor parts are given by

$$\Pi_F^{\rho(\omega)} = \frac{g_\rho^2 Q^2}{2\pi^2} \sum_{i=p,n} \left[ I_1^{(i)} + \frac{K_{\rho(\omega)}^2 M_i^*}{2 M_i^*} I_2^{(i)} \right] + \frac{1}{2} \left[ \frac{K_{\rho(\omega)}^2}{2 M_i^*} \right]^2 \left( Q^2 I_1^{(i)} + M_i^* I_2^{(i)} \right),$$

(16)

where

$$I_1^{(i)} = \int_0^1 dz \frac{z(1-z)}{\ln \left( \frac{M_p^2 - Q^2 z(1-z)}{M_i^* - Q^2 z(1-z)} \right)},$$

(17)

$$I_2^{(i)} = \int_0^1 dz \ln \left( \frac{M_p^2 - Q^2 z(1-z)}{M_i^* - Q^2 z(1-z)} \right).$$

(18)

It is to be noted that the free part $\Pi_F^{\rho \pi}$ vanishes in the limit $M_\pi = M_p$. We extract the real part of the vacuum mixing amplitude (with free nucleon mass) to be $\sim -3447\text{MeV}^2$ at the omega pole and this is consistent with that of ref. [10]. When in-medium nucleon masses are included $\Pi_F^{\rho(\omega)}(M_\pi^2)$ is equal to $-3716\text{MeV}^2$ and $-4675\text{MeV}^2$ at $\rho_0$ and $2\rho_0$ respectively in symmetric nuclear matter.

III. PION FORM FACTOR AND $\pi^+\pi^- \rightarrow e^+e^-$ CROSS SECTION

The $\rho$-dominated (unmixed) pion electromagnetic form factor is given by:

$$F_\pi(Q^2) = 1 - \frac{g_\rho\pi}{s_\rho} \frac{Q^2}{s_\rho g_\omega} \epsilon \frac{Q^2}{s_\omega}$$

(20)

where $\epsilon = \frac{\Pi_{\rho\pi}}{s_\rho g_\omega}$ with $s_{\rho(\omega)} = Q^2 - m_{\rho(\omega)}^2 + i m_{\rho(\omega)}(\Pi_{\rho(\omega)})$. Here the coupling of the physical $\rho$ and $\omega$ states to the photon is considered. In this form the $\omega \rightarrow \pi^+\pi^-$ decay is understood to proceed exactly like the $\rho$ but modified by the mixing factor $\epsilon$.

The coupling constants used are $g_{\rho\pi}\pi^2/4\pi \sim 2.9$, $g_\rho^2/4\pi \sim 2.0$ [1] and

$$\frac{g_\omega}{g_\rho} = \frac{m_\omega \Gamma(\rho \rightarrow e^+e^-)}{m_\rho \Gamma(\omega \rightarrow e^+e^-)} = 3.5 \pm 0.18.$$ (21)

The cross section for dilepton production from pion annihilation is intimately connected to the density dependent pion form factor which is given by:

$$\sigma(q_0,|q|,\rho_0,\alpha) = \frac{4}{3} \frac{\alpha_m^2}{Q^2} \sqrt{1 - 4m_n^2/Q^2} |F_\pi(q_0,|q|)|^2.$$ (22)

It is to be noted that unlike in vacuum, the pion form factor in nuclear medium depends both on $q_0$ and $|q|$. The dilepton emission rate in terms of the above cross section is given by

$$\frac{dR}{dM} = \frac{\sigma(q_0,|q|,\rho_0,\alpha)}{(2\pi)^3} M^4 T K_1(M/T) \left( 1 - 4m_n^2/M^2 \right)$$ (23)

where $K_1$ is the modified Bessel function of second kind.

IV. RESULTS

In Fig. 2 the in-medium pion form factor for $|q| = 200$ MeV together with its vacuum counterpart is shown. In matter the pole position shifts towards lower invariant mass indicating dropping $\rho$ and $\omega$ meson masses in matter. It might be mentioned that in medium, the mass modification is caused by two different mechanisms, viz. the scattering from Fermi sphere and excitation of the Dirac vacuum. While the former gives rise to an increase of their masses, the latter dominates resulting in an overall reduction. Near the pole, the mixing amplitude increases by large factors depending upon the value of the asymmetry parameter $\alpha$. Clearly the density and asymmetry parameter dependent mixing is much larger than the mixing due to $n-p$ mass difference. In Ref. [11] no
FIG. 2: Pion form factor as a function of invariant mass ($M = \sqrt{Q^2}$) at normal nuclear matter density with mean field including both vacuum and Fermi sea contributions. The solid (dashed) line represents pion form factor for $\alpha = 0.7(0.3)$. The dot-dashed line denotes vacuum mixing ($\alpha = 0$) due to only the $n-p$ mass difference, while the dotted line depicts the result without any mixing.

FIG. 3: Same as Fig.2 with $q = 0.6$ GeV. Substantial which can be attributed to the tensor interaction. Fig. 8 shows results for $|q| = 600$ MeV. Though the qualitative features remain similar, one can see that the mixing amplitude depends strongly on the three momentum of the moving meson, in fact, the mixing is stronger for a slower vector meson.

FIG. 4: Same as Fig.3 for $\rho = 2\rho_0$.

FIG. 5: The cross section for $\pi^+\pi^- \rightarrow e^+e^-$ for various cases is shown.

As an application of the density and asymmetry parameter dependent pion form factor, we calculate the pion annihilation cross section in nuclear matter. The dilepton production cross section is directly proportional to the square of the pion form factor and bears similar qualitative features. This is shown in Fig. 5. For completeness, we also present results for the dilepton production rates for various combinations of density and asymmetry parameter at $T = 100$ MeV in Fig. 6. Evidently the medium modified pion form factor leads to enhanced production of dileptons in the low invariant mass region.

In Fig 4 we present results for various values of the asymmetry parameter which might be realized in heavy ion collisions viz. at GSI energies. The effect of mixing is seen to be more pronounced at higher densities near the pole. Moreover comparing Figs. 8 and 4 we observe that for a given asymmetry and momentum the mixing amplitude increases with density.
as well as yield is enhanced due to dropping of vector meson mass
nihilation in matter. In the low mass region the dilepton
also calculated dilepton production rate due to pion an-
factor through dilepton measurements. Hence we have
unique opportunity to probe the in-medium pion form
ativistic heavy ion collisions at GSI energies offers the
to two times the normal nuclear matter density.
FIG. 6: Dilepton production rate with and without mixing at
of relativistic mean field theory . It is known that the
or in asymmetric nuclear matter within the frame
raturally influences the pion annililation cross section
to the presence of scalar mean field leading to the re-
ing aspect is the shift of pole position. This is related
by the ground state due to the difference of neutron and
In this paper, we have discussed the possible enhance-
ω
meson indicating isospin symmetry violation.
VI. APPENDIX
In this appendix the mathematical expressions for po-

\begin{align*}
A_{\mu\nu} &= Q^2 \int \frac{d^3k}{E_k^*} \frac{K_{\mu\nu}}{Q^4 - 4(K \cdot Q)^2} \\
&= Q^2 \int \frac{d^3k}{E_k^*} \frac{K_{\mu}K_{\nu}}{Q^4 - 4(K \cdot Q)^2} + \frac{Q_{\mu}Q_{\nu}}{4Q^2} (\bar{B} - \alpha(k_f)) \\
&\quad - \frac{1}{4} \int \frac{d^3k}{E_k^*} (Q_{\mu}K_{\nu} + Q_{\nu}K_{\mu}) \left[ \frac{1}{Q^2 - 2K \cdot Q} \\
&\quad - \frac{1}{Q^2 + 2K \cdot Q} \right] \\
B_{\mu\nu} &= Q^4 \int \frac{d^3k}{E_k^*} \frac{Q_{\mu\nu}}{Q^4 - 4(K \cdot Q)^2} \\
C_{\mu\nu} &= \int \frac{d^3k}{E_k^*} \frac{(K \cdot Q)^2}{Q^4 - 4(K \cdot Q)^2} \\
A_T &= Q^2 \int \frac{d^3k}{E_k^*} \frac{k_2^2}{Q^4 - 4(K \cdot Q)^2} \\
&= \frac{\pi}{4q} \int \frac{k^3dk}{E_k^*} I_2 \\
&\quad - \frac{\pi}{16q^3} \int \frac{kdk}{E_k^*} \left[ (Q^4 + 4E_k^* \omega^2)I_2 - 4Q^2E_k^*\omega I_1 \\
&\quad - 8Q^2kq \right] \\
A_L &= Q^2 \int \frac{d^3k}{E_k^*} \frac{k_2^2 + E_k^*}{Q^4 - 4(K \cdot Q)^2} - \frac{1}{4} (\bar{B} - \alpha(k_f)) \\
&\quad + \frac{1}{2} \int \frac{d^3k}{E_k^*} (\omega E_k^* - q_\kappa k_\kappa) \left[ \frac{1}{Q^2 - 2K \cdot Q} - \frac{1}{Q^2 + 2K \cdot Q} \right] \\
&= \frac{\pi}{8q^3} \int \frac{kdk}{E_k^*} \left[ (Q^4 + 4E_k^* \omega^2)I_2 - 4Q^2E_k^*\omega I_1 - 8Q^2kq \right] \\
&\quad - \frac{\pi}{2q} \int kdk E_k^* I_2 + \frac{\pi}{4q} \int \frac{kdk}{E_k^*} [Q^2 I_2 - 8kq] \\
&\quad - \frac{1}{4} (\bar{B} - \alpha(k_f)) \\
B_T &= B_L = B, C_T = C_L = C \\
B &= Q^4 \int \frac{d^3k}{E_k^*} \frac{1}{Q^4 - 4(K \cdot Q)^2} \\
&= \frac{\pi Q^2}{2q} \int \frac{kdk}{E_k^*} I_2 \\
C &= \int \frac{d^3k}{E_k^*} \frac{(K \cdot Q)^2}{Q^4 - 4(K \cdot Q)^2} \\
&= \frac{\pi}{8q} \int \frac{kdk}{E_k^*} [Q^2 I_2 - 8kq] \\
&= \frac{\pi}{2q} \int \frac{kdk}{E_k^*} I_2 + \frac{\pi}{4q} \int \frac{kdk}{E_k^*} [Q^2 I_2 - 8kq] \\
&= \frac{\pi}{2q} \int \frac{kdk}{E_k^*} I_2
\end{align*}

VI. APPENDIX
In this appendix the mathematical expressions for po-

\begin{align*}
\Pi_{\mu\nu}^{\gamma} &= \frac{g_\gamma g_\rho}{\pi^3} [A_{\mu\nu} - C_{\mu\nu}] \\
\Pi_{\mu\nu}^{\rho\tau} &= \frac{g_\rho g_\tau}{\pi^3} [(\kappa_\mu + \kappa_\nu)M^* - B_{\mu\nu}]
\end{align*}
\[= 2\pi \left[ k_f\epsilon_f - M^* \ln \frac{k_f + \epsilon_f}{M^*} \right] \]
\[
\tilde{B} = \frac{\pi Q^2}{2q} \int \frac{kdk}{E_k^*} I_1 \quad \text{(29)}
\]

\[I_2 = \ln \left( \frac{(Q^2 + 2kq)^2 - 4E^*_k\omega^2}{(Q^2 - 2kq)^2 - 4E^*_k\omega^2} \right) \quad \text{(30)}\]

and
\[I_1 = \ln \frac{Q^4 - 4(E^*_k\omega - kq)^2}{Q^4 - 4(E^*_k\omega + kq)^2} \]