We calculate the constraints on the primordial curvature perturbation at the end of inflation from the present day abundance of Primordial Black Holes (PBHs), as a function of the reheat temperature $T_{\text{RH}}$. We first extend recent work on the formation of PBHs on scales which remain within the horizon during inflation and calculate the resulting constraints on the curvature perturbation. We then evaluate the constraint from PBHs that form, more conventionally, from super-horizon perturbations. The constraints apply for $T_{\text{RH}} < 10^8\text{GeV}$ and the inclusion of sub-horizon PBHs leads to a limit which is roughly three times tighter than the bound from super-horizon PBHs.

PACS numbers: 98.80.Cq

I. INTRODUCTION

The spectrum of the primordial curvature perturbation is accurately measured on large scales by observations of the Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) surveys. However, on small scales the only constraints come from avoiding the overproduction of Primordial Black Holes (PBHs).

Typically PBHs are light black holes which may form in the early Universe, in particular via the collapse of large density perturbations \cite{1, 2}. If the density perturbation at horizon entry in a given region exceeds a threshold value (which is of order unity) then the region will collapse to form a PBH with mass roughly equal to the horizon mass. There are tight limits on the initial abundance of PBHs from the consequences of their evaporation via Hawking radiation and their present day abundance (for a review see Ref. \cite{3}). These limits can in turn be used to constrain the amplitude of the primordial perturbations on small scales and hence constrain models of inflation over a far wider range of scales than those constrained by CMB and LSS observations \cite{4, 5, 6, 7, 8, 9}. To date these calculations have been carried out in terms of the density perturbation. In this paper we calculate, for the first time, the constraints in terms of the amplitude of the primordial curvature perturbation power spectrum.

The standard PBH formation calculation applies to scales which are well outside the horizon at the end of inflation, with PBH formation subsequently occurring on a given scale, if the fluctuations are sufficiently large, when that scale re-enters the horizon after inflation. Lyth et al. \cite{10} extended the calculation to smaller scales on which the perturbations do not exit the horizon during inflation, and therefore never become classical. We use the sub-horizon PBH formation calculation of Ref. \cite{10} to calculate the constraint on the primordial curvature perturbation from the present day abundance of sub-horizon PBHs and compare this with the conventional constraint from scales which exit the horizon during inflation.

Throughout this work we use ‘super-horizon’ (‘sub-horizon’) to refer to scales which do (do not) exit the horizon during inflation. Note, however, that in the super-horizon case PBH formation occurs when the scale re-enters the horizon after inflation. In Sec. II we outline the calculation of the evolution of the Bardeen potential after the end of inflation during radiation domination from the initial conditions set during the inflationary epoch, and the formation of PBHs on sub-horizon scales. In Sec. III we calculate the constraints on the primordial curvature perturbation from the present day density of both super- and sub-horizon PBHs before finally discussing our results in Sec. IV.

II. PBH FORMATION ON SUB-HORIZON SCALES

A. Evolution of the Bardeen potential

During inflation the vacuum fluctuations of any light scalar field are promoted to a classical perturbation after horizon exit \cite{11, 12}. The fluctuations on sub-horizon scales never exit the horizon, and hence never become classical.

---

1 Here and in the following by “small scales” we mean scales that are smaller than the horizon size at decoupling.
However, if the perturbations are sufficiently large then PBH formation could still occur at the end of inflation on sub-horizon scales. In Ref. [10] it was shown that sub-horizon PBH formation should be formulated in terms of the Bardeen potential $\Phi$, rather than using, for example, the density contrast. To find the abundance of sub-horizon PBHs we therefore need to calculate the Bardeen potential immediately after inflation, for scales which remain sub-horizon during inflation. This was done in Ref. [10]. We summarise here the essential details of the calculation.

We write the metric with scalar perturbations in the longitudinal (or Newtonian) gauge as [13]

$$ ds^2 = a^2(\tau) \left[ -(1 + 2\Psi) d\tau^2 + (1 + 2\Phi) \delta_{ij} dx^i dx^j \right]. $$

(2.1)

In this gauge the metric perturbations coincide with the gauge invariant Bardeen potentials defined in [14], $\Psi = \Phi_A$ and $\Phi = \Phi_H$. For a scalar field the anisotropic stress is zero so that the Bardeen potentials are related by $\Phi = -\Psi$ and $\Psi$ is related to $\delta$, the comoving density contrast by [13] [14]

$$ \delta_k = \frac{2}{3} \left( \frac{k}{aH} \right)^2 \Psi_k, $$

(2.2)

where $a$ is the scale factor, $k$ the comoving wave number, $H$ the Hubble parameter defined as $H = \dot{a}/a$ where the dot denotes differentiation with respect to coordinate time $t$, and we denote Fourier components by a subscript “$k$”.

If inflation is driven by a light slowly rolling field $\phi$ then, to leading order in the slow roll approximation, the field equation for field fluctuations, $\delta \phi$, on flat slices lives in unperturbed spacetime. Defining $\psi = a\delta \phi$, the Fourier components of the field perturbations obey

$$ \frac{d^2 \psi(k, \tau)}{d\tau^2} + \left( k^2 - \frac{2}{\tau^2} \right) \psi(k, \tau) = 0, $$

(2.3)

where $\tau = -(aH)^{-1}$ is conformal time (it is assumed that inflation is exponential). The solution of this equation, with initial condition corresponding to the flat space-time mode function $\psi(k, \tau) = \exp(-ik\tau)/\sqrt{2k}$, is

$$ \psi(k, \tau) = \frac{1}{\sqrt{2k}} (1 + ik\tau) \frac{1}{ik\tau} \exp(-ik\tau). $$

(2.4)

The Fourier components of the comoving curvature perturbation $R$ are related to the field fluctuations on flat slices $\delta \phi$ by

$$ R_k = -\frac{H}{\phi} \delta \phi_k. $$

(2.5)

For a definition of $R$ in terms of the metric perturbations and its relation to curvature perturbations defined on different hypersurfaces see for example Ref. [15]. On sub-horizon scales therefore

$$ R_k(t) = \frac{1}{\sqrt{2k}} \left( \frac{H^2}{k} \left( i + \frac{k}{aH} \right) \exp \left( i\frac{k}{aH} \right) \right). $$

(2.6)

The Bardeen potential $\Psi$ is related to $R$ by

$$ \frac{2}{3H} \Psi_k + \frac{(5 + 3w)}{3} \Psi_k = -(1 + w) R_k, $$

(2.7)

where $w = p/\rho$, with $p$ and $\rho$ being the pressure and the energy density, respectively. Under slow-roll conditions during inflation $w \approx -1$, and therefore $\Psi$ is practically zero.

We need to set the initial conditions for the radiation dominated epoch after the end of inflation. We assume for simplicity that the slow-roll conditions hold at the end inflation and that reheating is rapid. This is not necessarily the case; the decay and thermalization of the inflaton may be slow and in single field models slow-roll has to be violated at the end of inflation. Our assumptions are valid in, for instance, hybrid inflation models where the secondary, waterfall, field has a large mass so that the slow-roll conditions hold at the end of inflation, and the inflaton field rolls to the minimum of its effective potential at the end of the inflationary stage almost instantaneously [16].

Under these conditions, we can smoothly match the solutions of the Bardeen potential between inflation and radiation domination by requiring the intrinsic metric and the extrinsic curvature to be continuous on comoving hypersurfaces [17, 18, 19]. Then $\Psi$ during the radiation dominated epoch is given by [10]

$$ \Psi_k(\tau) = \frac{3(1 + w) R_k(\tau_*)}{2\pi^2} \left[ (x - x_e) \cos (x - x_e) - (1 + xx_e) \sin (x - x_e) \right], $$

(2.8)
if \( R \) in PBHs with a mass similar to or larger than the smoothing scale exceed a certain threshold value are assumed to form PBHs with mass greater than \( M \). The fraction of the Universe in this application the Bardeen potential \( \Psi \) is smoothed on a mass scale which is often used in large-scale structure studies \[20\]. The quantity of interest (usually the density field, but in We discuss the consequences for the constraint on the amplitude of the curvature perturbation of these two limits in Sec. [III]

\[ x \equiv c_s k \tau, \] (2.9)

the subscript “\( e \)” denotes the end of inflation, and \( x_e \equiv c_s k \tau_e = c_s k/a_e H_e \). On super-horizon scales, \( k \ll a_e H_e \), and Eq. (2.8) reduces to the standard relation. On sub-horizon scales the Bardeen potential undergoes damped oscillations, reaching a maximum value \( \Psi(x_e) \) at \( x = x_e \) during the first oscillation. The mass variance of \( \Psi \) is approximately given by \( \sigma^2_{\Psi}(M) \approx \mathcal{P}_\Psi(M) \) so that

\[ \sigma^2_{\Psi}(M, t) = \frac{4\mathcal{P}_\Psi(x_e)}{x^6} \left[(x - x_e) \cos(x - x_e) - (1 + x x_e) \sin(x - x_e)\right]^2, \] (2.10)

where, for \( x_e \geq c_s \),

\[ \mathcal{P}_\Psi(x_e) = A_R \left[1 + \frac{x^2_e}{c^2_s}\right], \] (2.11)

defining \( A_R \equiv \left[(H^2_e/\phi_e)/2\pi\right]^2 \), the amplitude of the curvature perturbation power spectrum at the end of inflation on super-horizon scales.

In Ref. [10], it was argued that Eq. (2.8) is no longer valid when the amplitude of the curvature perturbation \( R \) becomes greater than one. Since \( R \) increases with \((k/a H)\), there would then be a scale below which it appears that linear theory breaks down. At the end of inflation this scale is given by \((k/a_e H_e) \sim 1/R_{HC} \), where \( R_{HC} \) is the amplitude of \( R \) at horizon crossing. However this is a conservative limit because well inside the horizon gravity should be less important, and therefore it is the scalar field perturbation becoming larger than unity which signals the breakdown of linear theory. An estimate of the critical wavenumber can be found by equating the energy density of the field perturbation and the background energy density. This gives \((k/a_e)_{\text{crit}} \sim \sqrt{M_P H_e} \), which can be written in terms of the reheating temperature as

\[ \left(\frac{k}{a_e H_e}\right)_{\text{crit}} \sim 5 \times 10^{10} \times \left(\frac{10^7 \text{GeV}}{T_{RH}}\right), \] (2.12)

The critical wavenumber decreases with increasing reheating temperature. The minimum value, which occurs for a reheating temperature of \( 10^{16} \text{GeV} \), is the same order of magnitude as the critical wavenumber corresponding to \( R \sim 1 \) if \( R_{HC} \sim 0.01 - 0.1 \). For smaller reheating temperatures this criterion gives a larger critical wavenumber than \( R \sim 1 \). We discuss the consequences for the constraint on the amplitude of the curvature perturbation of these two limits in Sec. [III]

### B. PBH abundance

In the standard super-horizon calculation of the abundance of PBHs one applies the Press-Schechter formalism, which is often used in large-scale structure studies \[20\]. The quantity of interest (usually the density field, but in this application the Bardeen potential) is smoothed on a mass scale \( M \), and the regions where the field perturbations exceed a certain threshold value are assumed to form PBHs with mass greater than \( M \). The fraction of the Universe in PBHs with a mass similar to or larger than the smoothing scale\(^2\), is given by Press-Schechter theory, initially, as

\[ \Omega_{\text{PBH}}(> M) \equiv F(M) = 2 \int_{|\Psi_e|}^{\infty} P(\Psi(M)) \, d\Psi(M) = \text{erfc} \left( \frac{\Psi_e}{\sqrt{2} \sigma^*_{\Psi}(M)} \right), \] (2.13)

where \( \sigma^*_{\Psi}(M) \) is the maximum mass variance of \( \Psi \), and the final equality follows from the fact that the smoothed Bardeen potential \( \Psi(M) \) has a Gaussian probability distribution. We have followed the usual Press-Schechter practise of including a factor of “\( 2 \)” in Eq. (2.13). The collapse criterion in terms of this variable is \(|\Psi| \geq |\Psi_e| = 1/2 \), which is

\(^2\) We assume throughout that the PBH mass is equal to the smoothing scale mass, which in the standard, super-horizon case corresponds to the horizon mass \( M_H \). This is not strictly true, and numerical simulations indicate that the PBH mass depends on the size and shape of the perturbations \[21, 22, 23\]; however, this uncertainty is not important when applying PBH abundance constraints, due to their relatively weak mass dependence.
outside the horizon, perturbations due to the exponential dependence of the PBH abundance on the size of the perturbations. However this factor of two improvement has an essentially negligible effect on the constraints on the primordial CDM.

In the short wave-length limit, \( x_e \gg 1 \), \( \Psi_e \) behaves like a damped sinusoidal function, and therefore the maximum occurs at \( x_e = x_0 + (\pi/2) \). In general there is no simple analytic solution for \( x_0 \). The difference \( B(x_e) = (x_e - x_0) \) converges very rapidly to the value \( \pi/2 \) inside the sound horizon. We used this fact in Ref. [10] to give an estimate of the mass fraction of PBHs. Here in order to extend the calculation to scales of the order of the horizon scale at the end of inflation, we calculate \( B(x_e) \) numerically. This calculation breaks down when the first maximum occurs outside the horizon, \( x_0 < c_e \). The PBH mass is related to \( x_e \) by

\[
M = \frac{4\pi}{3} \rho_e \frac{a_{\Psi}}{k} = \frac{4\pi}{3} \rho_e \left( \frac{a_{\Psi}}{k} \right)^3 \frac{x_e}{x_e + B(x_e)} = M_e \frac{c^3}{x_e^2(\pi/2)} \left( \frac{x_e}{x_e + B(x_e)} \right),
\]

where \( M_e \) is the horizon mass at the end of inflation,

\[
M_e = \frac{4\pi}{3} \rho_e (H_e^{-1})^3 = M_{eq} \left( \frac{g_{eff}}{g_{eff,i}} \right)^{1/3} \left( \frac{a_{eq}}{a_{i}} \right)^2 = 1.2 \times 10^{17} g \left( \frac{10^{7} \text{GeV}}{T_{RH}} \right)^2,
\]

with \( T_{RH} \) the reheat temperature at the end of inflation and \( g_{eff} \) the number of relativistic degrees of freedom; \( g_{eff} \approx 3 \) and at high temperatures \( g_{eff} \approx 100 \).

### III. CONSTRAINTS

PBHs with \( M_{PBH} \gtrsim 5 \times 10^{14}\text{g} \) will not have evaporated by the present day (their lifetime is longer than the age of the Universe) and their density can not be too large. Traditionally ‘not too large’ was interpreted as \( \Omega_{PBH,0} \equiv \rho_{PBH,0}/\rho_{\text{tot},0} < 1 \), where the subscript ‘0’ denotes the present epoch. This constraint can be improved to \( \Omega_{PBH,0} < 1/\rho_{\text{tot},0} \approx 0.47 \), where the numerical value is the 3-σ upper limit on the CDM density extracted from WMAP [24], however this factor of two improvement has an essentially negligible effect on the constraints on the primordial perturbations due to the exponential dependence of the PBH abundance on the size of the perturbations.

Since \( \rho_{PBH} \propto a^{-3} \) and \( \rho_{\text{tot}} \propto a^{-3} \) \( (\rho_{\text{tot}} \propto a^{-4}) \) during matter (radiation) domination, \( \Omega_{PBH,0} = \Omega_{PBH,eq} \) and

\[
\rho_{PBH,eq} = \int_{5 \times 10^{14}g}^{\infty} M \frac{dn_{eq}}{dM} dM = \int_{5 \times 10^{14}g}^{\infty} M \frac{dn}{dM} \left( \frac{a_{eq}(M)}{a_{i}} \right)^3 dM,
\]

where the initial PBH number density \( dn/dM \) is given by

\[
\frac{dn}{dM} = \frac{\rho_i (M) dF(M)}{M dM}.
\]

The initial total (radiation) density is given by

\[
\rho_i = \frac{\pi^2}{30} g_{eff} T_i^4 = \rho_{rad,eq} \left( \frac{g_{eff}}{g_{eq}} \right) \left( \frac{T_i}{T_{eq}} \right)^4 = \rho_{\text{tot,eq}} \left( \frac{g_{eq}}{g_{eff}} \right) \left( \frac{T_i}{T_{eq}} \right)^4.
\]

Combining Eqs. (3.2), (3.1) and (3.3), and using \( a \propto g_{eff}^{-1/3} T_i^{-1} \), the present day mass fraction of sub-horizon PBHs becomes

\[
\Omega_{PBH,0} = \frac{1}{2} \left( \frac{g_{eq}}{g_{eff}} \right)^{1/3} \int_{5 \times 10^{14}g}^{\infty} \left( \frac{a_{eq}}{a_i} \right) \frac{dF}{dM} dM.
\]

The upper mass limit of the integral in Eq. (3.4) should correspond to a mass approximately given by the horizon mass at the end of inflation. However, the mass function decreases so rapidly for large masses that in practise the upper limit can be safely taken to infinity. Using Eqs. (2.15) and (2.10) the present day PBH mass fraction, can be expressed in terms of \( x_e \) as

\[
\Omega_{PBH,0} = 7.2 \times 10^8 \left( \frac{T_{RH}}{\text{GeV}} \right) \int_0^{x_0} \frac{x_e}{x_e + B(x_e)} \frac{dF}{dx_e} dx_e,
\]
The constraint is negligible for the reheating temperature for which our present day abundance constraint is applicable.

The distribution is sharply peaked at small $x_e$, demonstrating that the majority of PBHs form at essentially the same time.

where $x_0$ is the value of $x_e$ corresponding to $M = 5 \times 10^{14}$ g which can be found from Eq. (2.15). For small reheating temperatures, $T_{RH} \ll 10^8$ GeV, we have $B(x_0) \ll x_0$, and $x_0$ is given by

$$x_0^3 \approx 46.2 \times \left( \frac{10^7 \text{GeV}}{T_{RH}} \right)^2.$$  \hspace{1cm} (3.6)

For larger reheating temperatures $B(x_0)$ can not be neglected, and therefore we calculate $x_0$ and hence $B(x_0)$ iteratively using Eq. (2.15).

In Fig. 1 we plot the present day differential mass fraction of sub-horizon PBHs, $d\Omega_{PBH,0}/dx_e$ from Eq. (3.5), as a function of $x_e$ for $A_R^{1/2} = 0.02$ and $T_{RH} = 1$ GeV. The PBH differential mass fraction is relatively sharply peaked with the majority of PBHs forming at essentially the same time. The present day PBH mass fraction, Eq. (3.5) is therefore given, to a good approximation, by

$$\Omega_{PBH,0} \approx 7.2 \times 10^8 \left( \frac{T_{RH}}{\text{GeV}} \right) \int_0^{x_0} \frac{dF}{dx_e} dx_e \approx 7.2 \times 10^8 \left( \frac{T_{RH}}{\text{GeV}} \right) \left[ \text{erfc} \left( \frac{\Psi_c}{\sqrt{2}\sigma_\Psi}(x_0) \right) \right],$$  \hspace{1cm} (3.7)

for $x_0 \gg 1$, and by

$$\Omega_{PBH,0} \approx 7.2 \times 10^8 \left( \frac{T_{RH}}{\text{GeV}} \right) \left( \frac{x_0}{x_\ast} \right) \text{erfc} \left( \frac{\Psi_c}{\sqrt{2}\sigma_\Psi}(x_0) \right),$$  \hspace{1cm} (3.8)

for $x_0 \lesssim 1$ where, in both cases, $\sigma_\Psi(x_0)$ is the maximum value of the mass variance at $x_e = x_0$. For a range of reheating temperature values, $T_{RH}$, we calculate the constraint on $A_R$ by first calculating $x_0$, the value of $x_e$ corresponding to PBHs with $M = 5 \times 10^{14}$ g (the lightest PBHs which will not have evaporated by the present day). We then find the mass variance of the Bardeen potential for this value of $x_0$, $\sigma_\Psi(x_0)$, using Eq. (2.8). Finally, we use either Eq. (3.7) or (3.8), as appropriate, to calculate the constraint on $A_R$ from the requirement $\Omega_{PBH,0} < 0.47$. The results are plotted in Fig. 2. As the reheat-temperature increases the constraint initially becomes tighter as the duration of the radiation dominated era, during which the fraction of the energy density in the form of PBHs grows as $\rho_{PBH}/\rho_{tot} \propto a$, increases. As the reheat temperature increases the horizon mass at the end of inflation decreases and for $10^8$ GeV $< T_{RH} < 10^8$ GeV only a fraction of the sub-horizon PBHs have $M > 5 \times 10^{14}$ g. For $T_{RH} > 10^8$ GeV, $M_e \sim 5 \times 10^{14}$ g and none of the sub-horizon PBHs are massive enough to last to the present day. Up to this point we have ignored the fact, as pointed out at the end of Sec. II A, that the calculation assumes that the linear theory holds. If we ignore PBHs formed on scales where $\mathcal{R} > 1$ at the end of inflation, we see in Fig. 2 that this weakens the constraints on $A_R^{1/2}$ for $T_{RH} < 10^5$ GeV, but the change is small (at most 20%). This is because the PBH mass function decreases rapidly for large $x_e$. Taking the linear limit for the field perturbations in Eq. (2.12) the change in the constraint is negligible for the reheating temperatures for which our present day abundance constraint is applicable ($T_{RH} < 10^8$ GeV).
FIG. 2: The constraint on the amplitude of the curvature perturbation power spectrum at the end of inflation from the present day abundance of PBHs as a function of reheat temperature \( T_{RH} \). The solid and dotted lines are for sub-horizon PBHs. The solid line assumes that the linear theory calculation is still valid on scales on which the comoving curvature perturbation becomes larger than one before the end of inflation, while the dotted line ignores PBH formation on these scales (and hence provides a conservative evaluation of the constraint). The long and short dashed lines are for super-horizon PBHs using a density contrast threshold \( \delta_c = 1/3 \) and 0.45 respectively.

We now compare the sub-horizon constraints with the conventional super-horizon constraints. A full calculation would require the complete curvature perturbation power spectrum, or equivalently the specification of a concrete inflation model. In order to be as general as possible we simply consider the amplitude of the super-horizon curvature perturbation power spectrum, \( A_R \), as a free parameter. On cosmological scales \( A_R = 2.4 \times 10^{-9} \) \cite{24}, therefore the production of an interesting (i.e. non-negligible) density of PBHs requires the curvature perturbations to be significantly larger on small scales. Assuming that the power spectrum grows monotonically with increasing wavenumber the abundance of super-horizon PBHs will be dominated by PBHs which form immediately after inflation at \( T_{RH} \).

The present day abundance of super-horizon PBHs is then given by

\[
\Omega_{PBH,0}(M > 5 \times 10^{14} \text{g}) = 7.2 \times 10^8 \frac{T_{RH}}{\text{GeV}} \text{erfc} \left( \frac{\delta_c}{\sqrt{2}} \right) \text{erfc} \left( \frac{\delta_c}{\sqrt{2}} \right). \tag{3.9}
\]

The Fourier components of the density contrast and the curvature perturbation are related by \cite{25}

\[
\delta_k(t) = \frac{2(1 + w)}{5 + 3w} \left( \frac{k}{aH} \right)^2 R_k, \tag{3.10}
\]

so that, at horizon crossing, \( \mathcal{P}_\delta = (4/9)^2 \mathcal{P}_R = (4/9)^2 A_R \). Using \( \sigma_8(M) \approx \mathcal{P}_\delta(M) \) again, then \( \sigma_8(M) = (4/9)^2 A_R \).

The threshold density has historically been taken as \( \delta_c = w \), where \( w = p/\rho \) is the equation of state \cite{2}, so that for formation during radiation domination (which is usually the case of interest) \( \delta_c = 1/3 \). More recently Green et al. \cite{26} carried out a new calculation using peaks theory and a formation criterion derived by Shibata and Sasaki \cite{21} which refers to the central value of the metric perturbation. They found that the standard Press-Schechter calculation agrees with the peaks theory calculation if the density threshold contrast is in the range 0.3 to 0.5, while numerical simulations by Musco et al. found a threshold \( \delta_c = 0.45 \) \cite{23}. We therefore consider two density thresholds, \( \delta_c = 1/3 \) and 0.45.

The constraints on the curvature perturbation at the end of inflation, as a function of reheat temperature, are plotted in Fig. 2. The super-horizon constraints are a factor of \( \sim 3 - 4 \) weaker than the sub-horizon constraints, and the factor of \( \sim 1.4 \) difference in the density contrast thresholds we consider gives, as would naively be expected, a similar difference in the constraints (with the larger threshold leading to a weaker constraint). The super-horizon constraint stops at \( T_{RH} \sim 10^8 \text{GeV} \) as then \( M_e < 5 \times 10^{14} \text{g} \) and the PBHs which form immediately after inflation have evaporated by the present day. PBHs with \( M > 5 \times 10^{14} \text{g} \) could form later, at \( T < T_{RH} \), though their abundance would depend on the precise scale dependence of the power spectrum.
IV. DISCUSSION

Observational constraints on the abundance of PBHs can be used to constrain primordial perturbations and hence models of inflation. These calculations are usually carried out for scales which exit the horizon during inflation (with PBH formation occurring after horizon re-entry during the subsequent radiation dominated era). Lyth et al. [10] recently studied the formation of PBHs on smaller scales on which the perturbations do not exit the horizon during inflation, and therefore never become classical. We have extended this work by calculating the constraints on the amplitude of the primordial curvature perturbation power spectrum at the end of inflation, $A_R$, from the present day abundance of these sub-horizon PBHs, as a function of the reheat temperature after inflation. For $T_{RH} < 10^6$ GeV most of the sub-horizon PBHs have $M > 5 \times 10^{14}$ g and hence have lifetimes longer than the age of the Universe. The requirement that their present-day density does not exceed the upper limit on the cold dark matter density leads to the constraint $A_R \lesssim 0.25 - 0.3$ with a weak dependence on the reheat-temperature (the higher the reheat-temperature, the longer the duration of the radiation dominated epoch during which the fraction of the energy density in PBHs grows). As the reheat temperature increases the horizon mass at the end of inflation decreases. For $10^6$ GeV $< T_{RH} < 10^8$ GeV the fraction of the sub-horizon PBHs which have $M > 5 \times 10^{14}$ g decreases rapidly with increasing $T_{RH}$ and hence the constraint on $A_R$ is weakened. For $T_{RH} > 10^8$ GeV, $M_c \sim 5 \times 10^{14}$ g so that none of the sub-horizon PBHs are massive enough to last to the present day and hence there is no constraint on $A_R$ from the present day abundance of sub-horizon PBHs.

We also calculated the constraint from the present day abundance of PBHs which form on larger scales, that do exit the horizon during inflation. This is a factor of $\sim 3 - 4$ weaker than the constraint from sub-horizon scales. For higher reheat temperatures there will be constraints from the effects of the PBH evaporation products on the successful predictions of nucleosynthesis (see e.g. Ref. [3]). These constraints are however model dependent, depending on the PBH mass function and hence the scale-dependence of the primordial power spectrum, which is beyond the scope of this paper.

There may also be constraints from the present day density of relics which may be the end point of PBH evaporation and the emission of long-lived massive particles. The applicability, or otherwise, of these constraints depends on the details of physics beyond the standard model of particle physics and are hence also beyond the remit of this paper.

Acknowledgments

The authors would like to thank David Lyth for useful discussions and comments. AMG and KAM were supported by PPARC. MS was supported by JSPS Grant-in-Aid for Scientific Research (S) No. 14102004, (B) No. 17340075, and (A) No. 18204024.