Equivalence principle and experimental tests of gravitational spin

effects

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Abstract

We study the possibility of experimental testing the manifestations of equivalence principle in
spin-gravity interactions. We reconsider the earlier experimental data and get the first experimental
bound on anomalous gravitomagnetic moment. The spin coupling to the Earth’s rotation may be
also explored at the extensions of neutron EDM and g−2 experiments. The spin coupling to the
terrestrial gravity produces a considerable effect which may be discovered at the planned deuteron
EDM experiment. The Earth’s rotation should be also taken into account in optical experiments
on a search for axion-like particles.

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I. INTRODUCTION

Equivalence principle is known to be one of the basic postulates of the modern physics, constituting the cornerstone of General Relativity. Its simplest and well-known "Newtonian" counterpart corresponds to the equality of inertial and gravitational mass and is tested with good accuracy. Another, "post-Newtonian", manifestation of equivalence principle corresponds to the interaction of spin with gravity and starts with the seminal paper of Kobzarev and Okun [1]. It means the absence of gravitational analogs of electric dipole and anomalous magnetic moments. It may be derived as a low energy theorem due to the conservation of momentum and orbital angular momentum [2].

Another formulation of the post-Newtonian equivalence principle (PNEP) consists in the equal frequencies of precession of quantum (spin) and classical (orbital) angular momenta and the preservation of helicity of Dirac particle in "gravitomagnetic" field (see Ref. [3] and references therein). These properties of spin-gravity interaction were explored in the number of theoretical papers and suggestions of experiments (see Refs. [4, 5] and references therein) although the relation to the PNEP was never mentioned. Moreover, the PNEP was implicitly used as a starting point for the most comprehensive derivation of the equations of spin motion [6]. There are also some evidences supporting the conjecture [7] that PNEP is valid separately for quarks and gluons in the nucleon, which may be related to the phenomena of confinement and spontaneous chiral symmetry breaking.

The experimental tests of PNEP are lacking and are therefore of much importance. The problem of existence of the dipole spin-gravity coupling in a static gravitational field has been discussed for a long time (see Refs. [4, 5, 8] and references therein). Evidently, this coupling given in the form of $S \cdot g$ ($g$ is the acceleration) not only contradicts to the theory [4, 6] but also violates both the PNEP and the CP invariance. Therefore, the negative result of realized experiments imposes some restrictions not only on the spin-gravity coupling but also on anomalous gravitomagnetic moments (AGMs).

In this article we carefully reanalyze the results of spin experiments with Hg atoms and get the first experimental bound on their AGMs. We also suggest to extend some experiments with spinning particles for testing the PNEP and calculate related gravitational effects.
II. SPIN COUPLING TO GRAVITY AND ROTATION

Spin rotation due to the action of the Earth’s gravity is

$$\frac{dS}{dt} = \Omega_g \times S, \quad \Omega_g = -\frac{2\gamma + 1}{\gamma(\gamma + 1)mc^2}g \times p,$$

where $S$ is the spin vector and $\gamma$ is the Lorentz factor. The maximum value of $\Omega_g$ is $2g/c$.

We studied the relativistic generalization of the pioneering results on the effect of Earth’s rotation on particle spin. We have performed the exact Foldy-Wouthuysen transformation of Dirac Hamiltonian and obtained

$$\mathcal{H}_{rot} = -\omega \cdot J, \quad J = L + S,$$

where $\omega$ is the angular frequency of the Earth’s rotation, $L = r \times p$ is the angular momentum, and $J$ is the total angular momentum. The equal coupling of rotation to orbital and spin momenta (which is not true for a magnetic field) is a manifestation of the PNEP. The form of this quantum Hamiltonian is similar to the classical one and also agrees with the earlier result. Calculation of dynamical variables following Ref. leads to coinciding the classical and quantum expressions for Coriolis and centrifugal forces.

III. EXPERIMENTS WITH ATOMS AND COLD NEUTRONS

The unique opportunity to test the PNEP is provided by experiments with atoms and cold neutrons. The experiments are performed with two kinds of atoms (or with neutrons and atoms) designated by indices 1 and 2.

Let us reconsider the earlier results as restrictions on the AGM rather than on the dipole spin-gravity coupling. Recall that latter violates not only PNEP (like AGM) but also CP invariance and may be neglected. The spin-dependent Hamiltonian for atoms in $S$-states has the form

$$\mathcal{H} = -g\mu_N B \cdot S - \zeta \hbar \omega \cdot S, \quad \zeta = 1 + \chi,$$

where $g$ is the nuclear $g$-factor, $\mu_N$ is the nuclear magneton, and $\chi$ is the AGM. The measured ratio of differences between neighboring Zeeman energy level, $R = |\nu_2|/|\nu_1|$, depends on the AGMs. The difference of these ratios for two opposite directions of magnetic field is given
by

\[ R_+ - R_- = \pm \frac{2f \cos \theta}{|\nu_1|} (\zeta_2 - \zeta_1), \quad G = \frac{g_2}{g_1}, \]  

where \( \theta \) is the angle between the directions of magnetic field and the Earth’s rotation axis, 
\( f = \omega/(2\pi) = 11.6 \, \mu \text{Hz} \) is the Earth’s rotation frequency, and \( |\nu_1| \) is the Zeeman frequency 
for atoms of the first kind. Experimental data obtained in \cite{13} for \(^{199}\text{Hg} \) and \(^{201}\text{Hg} \) atoms correspond to \( \theta \approx 0 \), \( G = -0.369139 \) and result in the following restriction:

\[ |\chi(201\text{Hg}) + 0.369\chi(199\text{Hg})| < 0.042 \quad (95\% \text{C.L.}). \]

To our best knowledge, this is the first experimental bound on the AGM, and consequently the first test of PNEP. The sensitivities of similar experiments fulfilled with deuterium \cite{14} and beryllium \cite{15} atoms are not sufficient to obtain significant restrictions.

Another experiment is fulfilled at ILL with ultracold neutrons placed in electric and magnetic fields \cite{16} and aimed to search for their electric dipole moment (EDM). There is a recent claim \cite{17} that spin-rotation coupling should be already taken into account when analyzing the data obtained. To address the problem of testing PNEP, the data for the opposite directions of magnetic field should be considered separately, while averaging over the directions of electric field should be performed. The correction for the Earth’s rotation is rather large and corresponds to the EDM of \( 1.7 \times 10^{-24} \) e·cm when \( E = 10 \) kV/cm. The expected sensitivity of this experiment to the AGM is also of order of \( 10^{-2} \). It is also possible \cite{18} to use the magnetic resonance methods for atomic and molecular beams.

Spin coupling to the Earth’s rotation may in principle be also investigated in GRANIT \cite{19,20} experiment, where quantum states of cold neutrons in the terrestrial gravitational field were observed.

Ultracold neutrons can also be used in interferometer experiments with rotating spin-flippers \cite{21} and implemented at the existing and developed interferometers at ILL and Tokai \cite{22}. It seems reasonable to have two (rather than one as suggested in \cite{21}) rotating spin-flippers. Signals should be absent if they are rotated in the same directions. In the case of rotation in opposite directions, signals should be twice larger in comparison when only one flipper rotates.
IV. COMPARISON OF SPIN-ROTATION COUPLING FOR ELECTRONS AND POSITRONS

The above mentioned experiments do not solve the important problem of the equivalence of gravitational effects for particles and antiparticles which may be tested in the storage rings. The corresponding equation of spin motion in a cylindrical coordinate system [23] with an addition of the gravitational correction is given by

\[
\frac{dS}{dt} = \omega_a \times S, \quad \omega_a = \Omega + \Omega_{EDM} + \omega + \Omega_g,
\]

where \( \Omega \) and \( \Omega_{EDM} \) are angular velocities of spin rotation caused by magnetic and electric dipole moments, respectively. The (pseudo)vectors \( \Omega \) and \( \Omega_{EDM} \) are oppositely directed for particles and antiparticles. The gravitational corrections to the angular velocity of spin rotation in Cartesian and cylindrical coordinates are \(-\omega + \Omega_g\) and \(\omega + \Omega_g\), respectively.

It is the quantity \( \omega_a \) which is measured in storage ring and Penning trap experiments. The Earth’s rotation can simulate the CPT violation because it brings a fictitious difference between \( g \) factors of electron and positron. The measurements of electron and positron \( g \)-factors in the Penning trap at the level of accuracy of order of 0.1 Hz [24, 25] were not sensitive to the Earth’s rotation.

To make the gravitational corrections observable, it is desirable to use a relatively weak magnetic field in order to decrease the spin rotation frequency as far as possible. Since this frequency is proportional to the cyclotron one, the particle trajectory should be extended and gravitational experiments should be performed in storage rings.

The best condition for the comparison of the spin-rotation coupling for electrons and positrons is perhaps provided by the use of a muon \( g-2 \) ring (namely, the 7.11 m ring of the Brookhaven National Laboratory). The electron/positron beam polarization may be measured with the methods described in Refs. [29, 30]. The frequency of spin rotation (\( g-2 \) frequency) actually measured with the accuracy of 0.16 Hz (0.7 ppm) [27] is almost the same for muons and electrons/positrons. The best sensitivity of experiment with electrons and positrons can be achieved with electric focusing and the “magic” Lorentz factor ensuring a dramatic reduction of influence of the electric field on the spin rotation and resulting in a small width of resonance line [26, 27].

The sensitivity of the proposed experiment is not affected by the systematical error in measurement of the magnetic field because it is the same for electrons and positrons and
therefore is canceled in the difference $\omega_a(e^+) - \omega_a(e^-)$. To compare the sensitivities of the proposed experiment and the muon $g-2$ one, one should take into consideration only systematical errors due to the electric field and the fitting procedure. The systematical errors caused by the horizontal and vertical coherent betatron oscillations (0.07 and 0.04 ppm, respectively, for the muons [27]) are much less important for the electrons and positrons because the decay time of these oscillations ($\sim 100 \mu s$ [27]) is very small in comparison with the beam circulation time. These systematical errors can additionally be reduced due to the fact that the electric focusing is 207 times stronger for the electrons/positrons than for the muons. The shift of the precession frequency due to the electric field depends on the momentum spread and is given by (Eqs. (17) and (21) in Ref. [27])

$$\frac{\delta \omega_a}{\omega_a} = -2\beta^2 \frac{n}{1-n} \left( \frac{p-p_0}{p_0} \right)^2,$$

(6)

where $n$ is the field index and $\beta = v/c$. The momentum spread, $(p-p_0)/p_0$, equal to 0.5% for the muons [27] can be considerably less (right up to $10^{-6}$ [28]) for the electron and positron beams providing a great reduction of the systematical error. In the muon $g-2$ experiment, this systematical error was $\sim 0.01$ Hz [27], i.e., about 10% of the electric field correction and about 1% of the linewidth $\sqrt{<(\delta \omega_a)^2>}$. We suppose that a relation between these quantities cannot be very different in the proposed experiment. If the momentum spread of the electrons/positrons is $5 \times 10^{-5}$ (two orders of magnitude less than for the muons) and $n \leq 0.2$, the linewidth is reduced $10^4$ times in comparison with the muon $g-2$ experiment. Even if the related systematical error would be $6 \div 8\%$ of the linewidth, the resulting error of frequency determination is about $10 \mu Hz$. Besides the comparison of gravitational spin-rotation coupling for particles and antiparticles, the restriction on the CPT violation would also be improved.

V. EFFECT OF EARTH’S GRAVITY ON SPIN DYNAMICS IN STORAGE RINGS

To measure the effect of the Earth’s gravity on spin dynamics, one needs to detect the spin turn around a horizontal axis. The detection can be provided if the particle spin is governed by a uniform upward magnetic field and a resonant longitudinal electric one ($E \parallel \mathbf{v}$). This field configuration corresponds to the resonant deuteron electric-dipole-moment (dEDM)
When magnetic focusing is used, the gravitational force acting on particles, \( F_g = (2\gamma^2 - 1)mg/\gamma \), defines the nonzero radial magnetic field which causes the spin turn with the average angular velocity

\[
\omega_m = \frac{(1 + a\gamma)(2\gamma^2 - 1)}{mc^2(\gamma^2 - 1)}g \times p.
\]

The resulting angular velocity \( \omega_a \) has the vertical and radial components and is given by

\[
\omega_a = \Omega_z e_z + \Omega_{EDM} + \Omega_g + \omega_m,
\]

while the average radial component of angular velocity of Earth’s rotation is zero. If we disregard terms describing systematical errors, Eq. (8) takes the form (here and below \( \hbar = c = 1 \))

\[
\omega_a = -\frac{ea}{m}B_z e_z - \frac{d}{S} \left( \frac{1}{\gamma} E + \beta \times B \right)
+ \left[ 1 + a(2\gamma^2 - 1) \right] \frac{g}{\gamma^2 - 1} \sin \Phi
+ \frac{\gamma^3}{(\gamma^2 - 1)^2} \left[ 1 - a(2\gamma^2 - 3) \right] g \times \beta,
\]

where \( d \) is the EDM and \( S \) is the spin quantum number.

In Eq. (9), the quantities \( E \) and \( \beta \) oscillate at a near-resonant frequency (see Ref. [32]).

The resulting buildup of the vertical polarization is equal to

\[
P_z = -\frac{1}{2} P_0 \Delta \beta_m \sin (\psi - \varphi_m) \left\{ \frac{d}{S} B_0 \left( 1 + a\gamma^2 \right)
+ g|\sin \Phi| \frac{\gamma^3}{\gamma^2 - 1} \left[ 1 - a(2\gamma^2 - 3) \right] \right\} t,
\]

where \( \Delta \beta_m \equiv \Delta \nu_m/c \) and \( \varphi_m \) characterize the resonant modulation of the beam velocity \( \Delta \nu_m \), \( \psi \) is the azimuthal angle of spin direction (with respect to the \( e_\rho \) axis) at zero time, \( \Phi \) is the geographic latitude, and \( P_0 \) is the polarization of the incident beam. In the planned dEDM experiment, the Earth’s gravity would bring the effect identical to that given by the deuteron EDM of \( d = 2 \times 10^{-29} \text{ e-cm} \). This effect is rather important, because the expected sensitivity of the dEDM experiment [31] is of the same order.

**VI. OPTICAL EFFECTS CAUSED BY THE EARTH’S ROTATION**

Photon being a massless spin-1 particle is significantly influenced by the Earth’s rotation. When frequencies of left-circularly and right-circularly polarized electromagnetic waves coincide in an inertial frame, they differ in a rotating frame [33]. This effect has been observed
(see Ref. [34] and references therein). The plane of polarization of a linearly polarized electromagnetic wave rotates in a stationary (but nonstatic) spacetime (Skrotskii effect [35]). This effect results in an optical rotation of electromagnetic wave in vacuum caused by the Earth’s rotation and defined by \( d\phi / dl = \omega \cdot l_0 / c \) [36], where \( l_0 \) is the unit vector pointing the wave direction and \( \omega / c = 2.43 \times 10^{-10} \) rad/km. This relatively large optical rotation has not been taken into consideration in the BFRT [37] and PVLAS [38] experiments on a search for axion-like particles.

Moreover, we checked that the much discussed effect [38] observed by PVLAS collaboration is of the same order as the effect of Earth’s rotation. The effect of the Earth’s rotation did not become apparent in the BFRT experiment because all effects independent on the angle between the plane of polarization and the magnetic field direction were eliminated [37]. The optical rotation in the Earth’s frame can be discovered and measured in a similar experiment performed without magnetic field.

VII. CONCLUSIONS

There is a number of possibilities to measure the coupling of spin to rotation and gravity and therefore to verify the PNEP. We suggest the re-interpretation of earlier experiment with atomic spin [13] leading to the first check of PNEP at few per cent level of accuracy. The straightforward extensions of experiments with (ultra)cold neutrons can also provide the important test of the PNEP. Possible gravitational experiment in the g−2 ring enables to compare the spin-rotation coupling for particles (electrons) and antiparticles (positrons). The proposed extension of the deuteron EDM experiment gives an exciting opportunity to detect the spin-gravity coupling existing only for moving particles. The Earth’s rotation should be taken into account in optical experiments on a search for axion-like particles, where observed effect [38] is of the same order as that of Earth’s rotation.
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