The Signature of Large Scale Structures on the Very High Energy Gamma-Ray Sky

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(Dated: December 21, 2006)

If the diffuse extragalactic gamma ray emission traces the large scale structures of the universe, peculiar anisotropy patterns are expected in the gamma ray sky. In particular, because of the cutoff distance introduced by the absorption of 0.1-10 TeV photons on the infrared/optical background, prominent correlations with the local structures within a range of few hundreds Mpc should be present. We provide detailed predictions of the signal based on the PSCz map of the local universe. We also use mock N-body catalogues complemented with the halo model of structures to study some statistical features of the expected signatures. The results are largely independent from cosmological details, and depend mostly on the index of correlation (or bias) of the sources with respect to the large scale distribution of galaxies. The predicted signal in the case of a quadratic correlation (as expected e.g. for a dark matter annihilation contribution to the diffuse gamma flux) differs substantially from a linear correlation case, providing a complementary tool to unveil the nature of the sources of the diffuse gamma ray emission. The chances of the present and future space and ground based observatories to measure these features are discussed.

PACS numbers: 95.85.Pw, 98.70.Vc, 95.35.+d

DSF-36/2006, FERMILAB-PUB-06-478-A

I. INTRODUCTION

Gamma ray astronomy is a flourishing field in astroparticle physics. Results have rapidly accumulated in the last decade or two: after the break-through results of the EGRET satellite, a whole series of Earth-based observatories have developed, both using the imaging air Cherenkov telescopes (ACT like WHIPPLE, HEGRA, CANGAROO, HESS, MAGIC) or surveying the sky via extensive air showers (EAS detectors, like TIBET, ARGO, MILAGRO). New projects using all these techniques (the ACT’s VERITAS, HESS II, MAGIC II, MACE, the EAS detector HAWC, the satellites AGILE and GLAST) are on the way (for a recent review of the field, see \cite{1}).

At TeV energies, a few dozen sources have been detected mainly by air Cherenkov experiments, most of which are high energy counterparts of MeV-GeV sources in the EGRET catalog. The 0.1–10 TeV range represents one of the “last” photonic windows yet to be explored at large distances. Starting from an energy of about 100 GeV (which we shall refer to as very-high energy, VHE), the absorption of high energy photons onto the extragalactic background light (EBL) via pair-production introduces an energy-loss horizon of the order of a few hundreds Mpc or smaller, well below the size of the observable Universe. This horizon drops down to \(\sim 10 \text{ kpc} \) at PeV energies, basically precluding deep space astronomy with photons above about 10 TeV. Far from being only a limitation, this phenomenon also allows the use of gamma-ray astronomy to probe the EBL, which is otherwise difficult to study directly.

Besides single sources, wide field of view instruments and satellite-based observations are sensitive to diffuse gamma-ray emissions. A particularly interesting emission is the extragalactic diffuse gamma-ray background (in the following, cosmic gamma background, or CGB). The CGB is a superposition of all unresolved sources emitting gamma-rays in the Universe and provides an interesting signature of energetic phenomena over cosmological time-scales. While a clear detection of this background has been reported by the EGRET mission \cite{2}, its origin is still uncertain, despite the fact that many models have been proposed. The most likely contribution is the one from unresolved blazars, i.e. beamed population of active galactic nuclei \cite{3}, with (probably sub-leading) components from ordinary galaxies \cite{4}, clusters of galaxies \cite{5}, and gamma ray bursts \cite{6}. However, exotic possibilities like dark matter annihilation have been proposed, that are compatible with existing data and constraints \cite{7, 8, 9, 10}. It is extremely difficult to test such models as long as the only observable is the energy spectrum. Recently, it was proposed to use the peculiar small-scale anisotropy encoded in the MeV-GeV gamma sky to probe dark matter \cite{11} or astrophysical \cite{12, 13} contributions to the CGB. In this work, we further study this topic, with particular emphasis on the large scale anisotropy in the energy range 0.1-10 TeV. The lower part of this range will be probed by the GLAST telescope \cite{14, 15}, while the energy window above the TeV is in principle accessible to EAS detectors. Different candidates to explain the CGB predict distinctive large scale features, even when similar energy spectra are expected. This is a consequence of the combined effect of a cutoff distance after which VHE gamma can travel undamped to us, and of the anisotropic distribution of matter in the local universe (i.e., within a few hundred Mpc from us). A similar anisotropy pattern in the the ultra-high energy cosmic ray sky was recently analyzed by some of the authors \cite{16} (see also \cite{17}).
main goal of this paper is to characterize various features of the CGB which may be used for diagnostics at VHE. In particular, the energy dependence and angular power spectra are analyzed in detail vs. energy for two typical diffuse background models in which underlying gamma emitters correlate linearly with large scale structures (LSS, as possibly for unresolved astrophysical sources) and quadratically with the cosmic structures density anisotropies (like in the case of dark matter annihilation emission or a strongly biased astrophysical population of sources).

The paper is structured as follows. The parametrization we use for the CGB, based on EGRET data, is summarized in Sec. III. In Sec. III we introduce our treatment of the cosmic large scale structure: a mock catalogue derived from a dark matter N-body simulation and the 3D power spectrum derived from the Halo Model of non-linear clustering. With these tools, we shall argue that in the VHE range, especially at the largest angular scales, the predictions mostly depend on the large scale structure in the local neighborhood of the Universe. We shall then use the PSCz astronomical catalogue as tracer of the real structures in the nearby universe, thus producing maps of the VHE gamma sky (Sec. IV). In Section V we analyse the perspectives for detection of these features of the forthcoming satellite GLAST, and for EAS observatories like MILAGRO. In Sec. VI we summarize our findings, and conclude. In Appendix A we provide some details on the parametrization of the EBL, and on the method we use to propagate gamma rays and calculate attenuation effects. Appendix B reviews some statistical properties of a discrete poisson process on the sphere, relevant for our estimates of the errors of the multipole maps.

II. THE COSMIC GAMMA BACKGROUND

Experimentally, the CGB is the most difficult component of the diffuse emission to study. Indeed, it is not correct to assume that the isotropic component after extracting point-like sources plus the galactic diffuse emission is entirely extragalactic: even in the pole direction, the CGB is comparable to the Galactic contribution. The deduced CGB thus depends on the adopted model of the Galactic foreground. The analysis undertaken to derive the spectrum of the CGB based on EGRET data provided the intensity spectrum

\[ I(E_{\gamma}, \hat{n}) \propto \int_0^\infty dz \frac{\rho(z, \hat{n}) g(E_{\gamma}, 1 + z)}{H(z) (1 + z)^{\delta}} e^{-\tau(E_{\gamma}, z)} , \tag{2} \]

where we assume an universal spectrum for the source, \( g(E) \), \( E_{\gamma} \) is the energy we observe today, \( \rho(z, \hat{n}) \) is the matter density in the direction \( \hat{n} \) at the redshift \( z \), where the sources are assumed to be distributed proportionally to \( \rho^4 \). The Hubble parameter is related to the present Hubble expansion rate \( H_0 \) through the matter and the cosmological constant energy density as

\[ H(z) = H_0 \sqrt{\Omega_M (1 + z)^3 + \Omega_{\Lambda}}. \]

The quantity \( \tau(E_{\gamma}, z) \) is the optical depth of photons to absorptions via pair production on the EBL (see Appendix A). For most of the following considerations the normalization in the spectrum is irrelevant. Where ever needed (e.g. to estimate the statistics which can be collected by a given experiment) we shall fix the normalization of Eq. (2) so that it matches the EGRET fit of Eq. (1) at 10 GeV.

It is worthwhile to comment that the foreground subtraction remains a delicate issue, as can be appreciated by the reanalysis of the data performed in [22], based on a revised model for the galactic propagation of cosmic rays. The deduced extragalactic spectrum is significantly lowered with respect to Eq. (1) at intermediate energies, while closer to the original result of Eq. (1) at the lowest and highest energy points. Since the removal of the isotropic galactic emission from the diffuse gamma background is an open problem, and we still lack a complete understanding of the sources of the CGB, we shall simply base our following analysis on the extrapolation of the spectrum of Eq. (1) by one to two orders of magnitude. Actually, the anisotropy pattern in the CGB sky itself may help in the foreground removal. Recently, it was proposed to use the cosmological Compton-Getting effect (whose dipole direction and amplitude are basically energy-independent) to discriminate the truly extragalactic fraction from the galactic foreground [23].

When extrapolating the EGRET flux to higher energy attenuation effects must be taken into account. It is well known that the propagation of photons in the EBL is a crucial issue for gamma astronomy in the VHE range. Absorption of VHE photons through pair-production with CMB, infrared or optical photons distorts an initial source spectrum, in particular by steepening its high energy tail. Astronomy of gamma ray emitters like blazars and active galaxies requires then an accurate modelling of the photon propagation and of the background frequency distribution. In Appendix A we provide a detailed description of the model of the EBL used, and of the treatment of absorption effects. For an overview of the knowledge of the EBL we refer the reader to the review [24]. For most of what follows it is important to realize that, while closer to the original result of Eq. (1) at the lowest and highest energy points. Since the removal of the isotropic galactic emission from the diffuse gamma background is an open problem, and we still lack a complete understanding of the sources of the CGB, we shall simply base our following analysis on the extrapolation of the spectrum of Eq. (1) by one to two orders of magnitude, it is worthwhile to comment that the foreground subtraction remains a delicate issue, as can be appreciated by the reanalysis of the data performed in [22], based on a revised model for the galactic propagation of cosmic rays. The deduced extragalactic spectrum is significantly lowered with respect to Eq. (1) at intermediate energies, while closer to the original result of Eq. (1) at the lowest and highest energy points. Since the removal of the isotropic galactic emission from the diffuse gamma background is an open problem, and we still lack a complete understanding of the sources of the CGB, we shall simply base our following analysis on the extrapolation of the spectrum of Eq. (1) by one to two orders of magnitude. Actually, the anisotropy pattern in the CGB sky itself may help in the foreground removal. Recently, it was proposed to use the cosmological Compton-Getting effect (whose dipole direction and amplitude are basically energy-independent) to discriminate the truly extragalactic fraction from the galactic foreground [23].
because of the cutoff at $z \ll 1$ existing for VHE gammas. This important property dramatically reduces the model-dependence of the following considerations, differently e.g. from the scenario considered in [11].

It is also worth commenting that the $\gamma$-rays of the CGB constitute only a tiny fraction, $f_{\gamma}$, of the cosmic ray flux. When compared with the flux of cosmic rays around the TeV ($I_{\text{CR}} = 2.582 \times 10^{-11} (E/\text{TeV})^{-2.7} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{MeV}^{-1}$) [20]) and neglecting gamma attenuation effects one gets the upper limit

$$f_{\gamma} \equiv \frac{I_{\gamma}}{I_{\text{CR}}} \lesssim 2.7 \times 10^{-5} \left( \frac{E}{\text{TeV}} \right)^{0.6}.$$  

Actually, the attenuation of $\gamma$’s on the EBL cuts the growth of $f_{\gamma}$, which never exceeds $10^{-5}$. Note finally that our extrapolation is consistent with existing observational bounds on $f_{\gamma}$ at the TeV scale [20].

### III. TRACERS OF THE LARGE SCALE STRUCTURE

Since the sources of the CGB are unknown (this is especially true for its high energy component of interest here), to predict the anisotropy pattern in the VHE sky we must start from some assumption on the distribution of the sources. In the following, we shall assume that the sources of the CGB follow the LSS distribution of the matter. Starting from Eq. (2), the integral flux above the energy $E_{\text{cut}}$ at Earth is simply written as

$$F(E_{\text{cut}}, \hat{n}) \propto \int_0^\infty \frac{\rho^3(z, \hat{n})}{H(z)(1+z)^3} W(E_{\text{cut}}, z),$$  

where we have defined the window function as

$$W(E_{\text{cut}}, z) \equiv \int_{E_{\text{cut}}}^\infty \frac{dE}{E} g(E(1+z)) e^{-\tau(E,z)}.$$  

In the limit where the effective cutoff $z_c \ll 1$, the integrals become almost independent on the cosmology, and the above expressions simplify considerably to

$$W(E_{\text{cut}}, z) \simeq \int_{E_{\text{cut}}}^\infty dE g(E) e^{-\tau(E,z)},$$  

$$F(E_{\text{cut}}, \hat{n}) \propto \int dz \rho^3(z, \hat{n}) W(E_{\text{cut}}, z),$$  

where $z$ is directly proportional to the distance, $r \simeq c z / H_0$. In particular we shall assume in the following the power-law $g(E) \propto E^{-2.1}$, which on the theoretical side is consistent with a Fermi-shock acceleration mechanism, and observationally matches both the average spectral index of blazar and the spectra of the CGB measured by EGRET, see Eq. (1).

The size of the horizon is roughly 300 Mpc $h^{-1}$ at 1 TeV (see Appendix A) allowing to use safely the existing astronomical catalogues like PSCz to predict the gamma signal. At lower energy, statistical methods like N-body simulations and the Halo model of structures are in principle required to complement existing catalogues, which are not deep and/or wide enough. We shall describe these tools in the next section and use them to derive the expected statistical properties of the anisotropies at $E < 1$ TeV. These analyses indicate that, at least at large angular scales, most of the non-statistical information contained in the surveys can still be retained, allowing one to use the existing catalogues for cross-correlation studies even at $E < 1$ TeV.

**N-Body Simulations** – Our primary tool to compute the statistical properties of the matter density distribution is a N-body catalogue. This offers the advantage compared to observations of being virtually free of most of the bias affecting astronomical catalogues (no extinction regions, no selection effects, no galaxy morphology and color bias, etc). On the other hand, to obtain reliable results one should carefully consider the types of bias introduced by the simulation, like the algorithm used, the minimum scale resolved, and the physical content and processes treated in the numerical experiment.

We have performed a set of pure dark matter N-body simulations with a standard $\Lambda$CDM universe, a periodic box size of 800 $h^{-1}$ Mpc, and resolutions of $512^3$ and $768^3$ particles using the GADGET2 code [21, 22]. The initial conditions were computed using second order Lagrangian perturbation theory [23]. Placing the observer at an arbitrary point in the box, we have computed the density on...
logarithmically spaced spherical shells. A smoothed density field $\rho(z, \hat{n})$ is reconstructed with an adaptive algorithm whose smoothing length varies depending on the local number density of dark matter particles identical to the one used in the GADGET2 code. After obtaining the window functions for different energy cuts, the sky map of the integral gamma ray flux above the cut is calculated by integration along radial lines of sight, considering a source density proportional to $\rho$ or to $\rho^2$. We have checked that the resolution used in the simulation is sufficient to suppress shot noise from undersampling and other numerical artifacts, by comparing the power spectrum extracted at different resolutions. The resulting maps $F(E_{\text{cut}}, \hat{n})$ are obtained on a Healpix grid, facilitating the use of standard tools from CMB physics to compute the final power spectrum and multipole coefficients $C_l$ \footnote{M. Colafrancesco et al. (2017).}. For the final analysis we used 2048 shells, with $12 \times 1024^2$ pixels on each shell or a density grid with almost $25 \times 10^9$ pixels. This makes it possible to reconstruct the power spectrum reliably up to $l = 3071$.

**Halo Model** – In order to check the results of the N-body simulations, we compare the $C_l$‘s obtained in the case of linear correlations with the predictions using the Halo Model \footnote{J. Novikov et al. (2017).} \footnote{J. Novikov et al. (2018).} \footnote{J. Novikov et al. (2020).}. This model is known to provide a fast and efficient semi-analytical recipe for describing the clustering of dark matter halos, and of their evolution. Comprehensive reviews of the model can be found in \footnote{J. Novikov et al. (2013).} \footnote{J. Novikov et al. (2014).}. Assuming that the gamma emitters follow the distribution of the halos, the angular power spectrum of the adimensional gamma flux anisotropy $\{F(E_{\text{cut}}, \hat{n}) - \langle F(E_{\text{cut}}) \rangle \} / \langle F(E_{\text{cut}}) \rangle$ can be calculated with the 3D matter power spectrum $P(k)$ and its normalization provided by the model. In the flat-sky limit and the Limber approximation, it simply reads

$$C_l = \frac{1}{N_w^2} \int \frac{dr}{r^2} W^2(E_{\text{cut}}, r) P \left( k = \frac{l}{r}, z(r) \right),$$

(8)

where the pre-factor $N_w \equiv \int dr W(E_{\text{cut}}, r)$ represents the contribution from the mean intensity $\langle F(E_{\text{cut}}) \rangle$.

A comparison of the Halo Model and N-body spectra for various energy cuts is shown in the top panel of Fig. 2. To properly normalize the flux resulting from the N-body catalogue we fix the monopole $C_0 = 4\pi$, where the mean flux is derived by integrating Eq. (8) over the average source density found in the simulation. We see that the agreement is pretty good at $100 \lesssim l \lesssim 1000$, while unsatisfactory at very low and high $l$. The former disagreement depends on the breakdown of the flat-sky (see e.g. Ref. \footnote{J. Novikov et al. (2013).}) and the Limber approximation, while the latter depends on the inadequacy of the Halo Model to properly take into account non-linearities for wavenumbers $k \gtrsim 10$ Mpc$^{-1}$. The biggest disagreement is at 10 TeV, for which the window function $W$ is of width of $\sim 100$ h$^{-1}$ Mpc and the non linear scales are most important. In the N-body catalogue we can only account for the flux coming from distances up to 350 h$^{-1}$ Mpc. At relatively low energies ($E_{\text{cut}} < 2$ TeV) there are essential contributions to the flux on small scales/high $l$ from distances further away than that, while negligible contributions to the flux on large scales/low $l$. At small angular scales and lower energies the simulation also suffers of residual shot noise, seeable in the break in the slope at high $l$, that makes the comparison biased.

In the lower panel, we present the angular spectrum for the hypothesis of a quadratic correlation with density. Both in the linear and the quadratic case, the three most interesting features are (i) the power law slope at low scales, which is approximately energy independent, as indicated in Fig. 2 by the dotted line, (ii) the presence of a peak (deriving from the peak of the 3D matter power spectrum), and (iii) the increasing amplitude of the anisotropies with growing energy cut. At relatively large scales, the anisotropies are contributed by the near structures, which are therefore poorly affected by the shrinking of the pair-production energy loss horizon. The isotropic or small-scale varying component of the flux has instead a significant contribution from far objects, which are cutted away at high energies. The main difference between the linear and the quadratic case is clearly in the
normalization and the slope of the spectrum. In the latter scenario the intensity of the anisotropies is enhanced, and the slope is steeper. The slope can be estimated even at large scales/low $l$, and it is a signature of the source correlation with the matter density accessible to observations. For a linear density correlation we find $l(l+1)C_l \propto l$ while for a squared density correlation we find $l(l+1)C_l \propto l^{-1.75}$. Also interesting is the prominent shift of the peak to very large $l$ in the quadratic case, although it is unlikely that this feature may be observed experimentally.

We turn now to study how shells at different distances contribute to the various multipoles. Using the predictions of the Halo Model, in Fig. 3 we plot the integrand $r^{-2} W^2(E_{cut}, r) P(k = l/r)$ versus $r$ for the two energy cuts $E_{cut} = 100$ GeV and 1 TeV at different $l$’s. Note that even for $E_{cut} = 100$ GeV, the $C_l$’s at $l = 10 - 100$ mostly depend on the contribution from structures within a radius of $\sim 600 \, h^{-1}$ Mpc. Interestingly, the dominant contribution to the first multipoles at $l = 1 - 10$ comes from within a distance of only $\sim 300 \, h^{-1}$ Mpc. Within such distances astronomical catalogues with a large field of view exist (e.g. the 2MASS and PSCz catalogues), providing the actual distribution of matter in the universe, and not only a statistical information on the density field. Thus, one may aim at the study of the pattern of the first few observables $a_{lm}$’s or, equivalently, of the anisotropy map smoothed to the appropriate resolution. Note that, the amplitude of matter fluctuation has a measured maximum at a wavenumber (scale) corresponding about to $k = 0.02 \, h \, \text{Mpc}^{-1} \, (\lambda = 300 \, h^{-1} \, \text{Mpc})$. Beyond this distance the low $l$ multipoles start to receive decreasing contributions and converge rapidly to their asymptotic value. This is a well known expectation, although only recently detailed study e.g. of the convergence of the dipole has been performed. Actually, as illustrated in Fig. 3 the presence of the window further accelerate the convergence of the low $l < 10$ sky pattern. This point is further illustrated with the help of the N-body simulation. In Fig. 4 we plot the cumulative contribution to the $C_l$’s in function of the maximum distance used in the catalogue, i.e. derived from Eq. (7) when cutting the integral defining $F(E_{cut}, \hat{n})$ at a distance $R$. The linear and quadratic cases are shown, assuming $E_{cut} = 100$ GeV (at higher energies, the effect is even more pronounced). Indeed, the simulation confirms that the convergence to the asymptotic value is faster at low $l$’s than at the higher ones. Effectively, for $l \lesssim 10 - 20$ it is meaningful to look for correlations between the gamma sky and known local structures well below the TeV scale. Of course, the
argument can be turned around: by removing structures in the nearby sky from the map, the intrinsic low-$l$ multipoles would be largely suppressed, and one may thus access more easily other signatures as the cosmological Compton-Getting dipole \cite{22}.

IV. SKY MAPS

In the recent years, modern galaxy surveys like the Sloan Digital Sky Survey \cite{36, 37} have greatly improved our knowledge of the distribution of galaxies at large scales, revealing a typical foam-like pattern of “filaments and walls” of galaxies around large cosmological voids. These very high quality data are not well suited for our analysis due to the small fraction of sky surveyed, while we need to perform comparisons with the large sky field of view of GLAST and of EAS instruments. In this respect, a fair compromise is offered by the IRAS PSCz catalogue \cite{38}.

The PSCz catalogue contains about 15,000 galaxies and related spectroscopic redshifts with a well understood completeness function out to $z \sim 0.1$. In the limit of uniform emission, above $E = 100 \text{ GeV}$ the majority of the CGB flux is expected to come from within this distance. The sky coverage of the catalogue is about 84%; the incompleteness is mainly due to the so-called zone of avoidance centered on the galactic plane and caused by the galactic extinction and to a few, narrow stripes which were not observed with enough sensitivity by the IRAS satellite. These regions are excluded from our analysis with the use of the binary mask available with the PSCz catalogue itself. However, the mask region does not represent a major limitation for present analysis, since the galactic emission at low galactic latitudes outlines the CGB anyway, prohibiting any CGB analysis in this region.

We closely follow Ref. \cite{16} for the treatment of the selection effects of the catalogue, parameterized via the selection function $\phi(z)$. Differently from Ref. \cite{16}, we do not sum source by source to obtain the final gamma sky map; instead, we use the approach of constructing a smoothed density field $\rho(z, \hat{n})$ through the same adaptive algorithm we employed in the analysis of the N-body simulation. This method has the advantage of efficiently suppressing the intrinsic catalogue shot noise at high redshifts, thus allowing us to use the catalogue sources until $z = 0.1 \pm 300 \text{ Mpc} \ h^{-1}$. We use Eq. (4), while replacing $W(E_{\text{cut}}, z) \to W(E_{\text{cut}}, z)/\phi(z)$, to take into account selection effects. To study the sensitivity to the bias of the gamma sources with respect to the baryonic density, we shall consider both values $\alpha = 1$ and $\alpha = 2$, i.e. a linear and quadratic correlations. The latter is in particular expected for dark-matter annihilation models \cite{39}.

In Fig. 5 we plot the resulting maps from the PSCz catalogue in equatorial coordinates for the linear and quadratic density correlation and for $E_{\text{cut}} = 100 \text{ GeV}$ and 1 TeV. Let’s discuss first the linear case. For the case of the map with $E_{\text{cut}} = 100 \text{ GeV}$, modulo the “hole” due to the mask the pattern is quite isotropic with some hot spots like from the Virgo and Perseus Clusters. Other structures which appear are the Shapley concentration and the Columba cluster (for a key of the local cosmological structures see \cite{16} or \cite{40}). From the color scale can be seen that the anisotropy could be also a factor of 2 in the hot spots, while is generally of the order 10% at larger angular scales as we will show in the following with a multipole decomposition analysis. Given the limited statistics of GLAST at high energies, the TeV map is of interest especially for the EAS gamma detectors like MILAGRO. We see in this case that the nearest structures, forming the Super-galactic Plane, dominate. Of course, from the Northern hemisphere (where all the present or planned EAS instruments are located) only the upper part of the map is visible. Here, the Virgo Cluster and the Perseus cluster offer the strongest anisotropy.

For the quadratic case the change is quite evident: the effect of the quadratic correlation is to give more power to the nearest structures (the Virgo and Centaurus cluster) that, in fact, are almost dominating the map with fluctuations exceeding even 10 times the average and appearing almost like point sources. It is instructive to look at the case of Shapley concentration at $z \sim 0.04$ that gives an important contribution in all the linear cases but almost disappears in the quadratic maps. The anisotropy in the quadratic case is then much more pronounced than the linear case and should be easily detectable. As an application, we just mention that the level of detected anisotropies can be used to constrain the fraction of a contribution by annihilating dark matter to the CGB.

V. THE POTENTIAL OF FORTHCOMING INSTRUMENTS

In order to estimate the chances of detection of the structures previously described by the current or next generation of instruments, one may proceed as follows. First, given the specifics of an experiment (in particular its field of view, effective area and background rejection capability) one calculates the expected number of events and of misidentified cosmic ray background events ($N_{\gamma}, N_{\text{CR}}$) under different assumptions for the $E_{\text{cut}}$ and the EBL. The events falling in the mask region must be subtracted from the ($N_{\gamma}, N_{\text{CR}}$) data used for the analysis, so that the incomplete sky coverage is taken into account. One then generates $N$ samples of $N_{\gamma}$ events from the VHE $\gamma$ map and $N_{\text{CR}}$ from an isotropic one. These $N$ realizations may then be used to perform quantitative statements, like for example the confidence level with which a given instrument is expected to distinguish between the structured sky we predict and an isotropic one, or between linear and quadratic correlation scenarios, or between scenarios characterized by a different model of CIB. This would be essentially the generalization of the approach followed in \cite{16}. Here, however, for the sake
of clarity we prefer to develop a simplified analysis in terms of the multipole coefficients, $a_{lm}$, with an analytic estimate of the expected errors due to shot noise effects (see Appendix B). Performing full Monte Carlo analyses for a few relevant cases we checked that the errors thus estimated basically agree with the ones correctly calculated. In this approach, first we fit the sky maps obtained outside the mask via the harmonic expansion

$$F_{10}(E_{\text{cut}}, \hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}).$$

As a second step, by knowing the statistics and the characteristics of the detector one can estimate the expected errors on the $a_{lm}$ and eventually on the derived coefficients $C_l$’s (see Appendix B for details).

We limit the fit to $l \leq l_{\text{max}} = 10$ for several reasons: (i) we have shown that the large scales are mainly sensitive to the local universe, for which our predictions are robust and deterministic; (ii) the lower the $l$, the higher the signal to noise ratio is, which increases the chances of detection (see below); (iii) the incomplete sky coverage due to the mask is affecting our results, and should be taken into account; however, as long as we restrict the analysis to the large structures (compared to the size of the mask cut) the bias is small. Quantitatively, the method is robust against variations in $l_{\text{max}}$ as long as the related angular scale $\theta_{\text{min}} = \pi/l_{\text{max}}$ is greater than the typical angular extension of the mask cut. More importantly, the purpose of the harmonic analysis is to reliably assess the sensitivity of a given experiment; to this aim a small bias in the $a_{lm}$’s is acceptable (we do not want to estimate the true $a_{lm}$’s, but the detectability of them, even of the biased ones). Of course, in the analysis of the real data it will be preferable to directly look for cross correlations with the full maps like the ones in Fig. 5, whose electronic version is available from the authors upon request.

In the following, we shall consider two kinds of instruments: Satellite-based missions, in particular GLAST [41], and extensive air shower experiments like TIBET [42], ARGO [43], MILAGRO [44], or planned observatories like HAWC [45]. Unfortunately, ACT instruments like HESS or MAGIC are not well suited for such kind of searches, given the small field of view, the low duty cycle, and the relatively high impact of the variability of the instrumental and atmospheric conditions on the rate of diffuse signals.
FIG. 6: The coefficients $a_{lm}$ up to $l_{\text{max}} = 10$ calculated from the PSCz gamma maps of Fig. 5. The shaded band shows the 1-$\sigma$ shot noise error given by Eq. (B6); in the right panel the inner shaded region refers to HAWC, the outer one to MILAGRO.

We report the predictions for both the linear and quadratic cases.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$A_{\text{eff}}$ ($\text{cm}^2$)</th>
<th>$\Omega_{\text{fov}}$ (sr)</th>
<th>$f_{\text{sky}}$</th>
<th>$\text{DC}$</th>
<th>$g_{\text{cut}}$</th>
<th>$h_{\text{cut}}$</th>
<th>$E_{\gamma}$ cut range</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLAST[41]</td>
<td>$10^4$</td>
<td>2.4</td>
<td>$f_m \sim 0.9$</td>
<td>$\sim 1$</td>
<td>$0.06$</td>
<td>$E \leq 0.5-1 \text{ TeV}$</td>
<td></td>
</tr>
<tr>
<td>MILAGRO[44]</td>
<td>$\gtrsim 10^7$</td>
<td>$\sim 2$</td>
<td>$\lesssim f_m \times 2\pi &gt; 0.9$</td>
<td>0.5</td>
<td>0.08</td>
<td>$E \sim 1-20 \text{ TeV}$</td>
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</tr>
<tr>
<td>HAWC[45]</td>
<td>$\sim 10^8.5$</td>
<td>$\sim 2$</td>
<td>$\lesssim f_m \times 2\pi &gt; 0.9$</td>
<td>0.5</td>
<td>0.08</td>
<td>$E \sim 0.3-10 \text{ TeV}$</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I: The characteristics of the experiments considered in our estimates. The fraction of the sky observable by a given experiment $f_{\text{sky}}$ is needed for our estimate of the errors, see Appendix B.

Denoting by $I(E_{\gamma})$ the extrapolated EGRET flux which takes into account attenuations (see section II), one can estimate the number of events, $N_{\gamma}$, above the energy $E_{\gamma}$ to be collected during the time $t$ as

$$N_{\gamma} = t \cdot g_{\text{cut}} \cdot DC \cdot \Omega_{\text{fov}} \cdot f_m \int_{E_{\gamma}}^{\infty} dE A_{\text{eff}}(E) I_{\gamma}(E), \quad (10)$$

where: $DC$ is the duty-cycle of the instrument; $\Omega_{\text{fov}}$ is the solid angle of the field of view; $f_m < 1$ is the useful fraction of the sky due to the presence of the galactic mask; $g_{\text{cut}}$ is the fraction of $\gamma$’s passing the actual cuts; $A_{\text{eff}}(E)$ is the effective collecting area of the instrument (averaged over the field of view of the instrument). In the following, we shall assume $f_m = 0.84$ due to the mask in the PSCz catalogue, but the actual value may differ. Eq. (10) assumes a quasi-isotropic $\gamma$ sky, which may be violated to some extent at the multi-ToV energies of interest for EAS detectors. Even in this case, right ascension anisotropies would not affect the estimate, and only large latitude anisotropies might affect $N_{\gamma}$ by a factor of $O(1)$. This is acceptable enough since we shall only perform a parametric study of the performances of an EAS observatory. Analogously, the CR background can be estimated as

$$N_{\text{CR}} = t \cdot h_{\text{cut}} \cdot DC \cdot \Omega_{\text{fov}} \cdot f_m \int_{E_{\gamma}}^{\infty} dE A_{\text{eff}}(E) I_{\text{CR}}(E), \quad (11)$$

where now $h_{\text{cut}}$ is the fraction of hadrons passing the cuts. Note that we consider the same area for CRs as for $\gamma$’s, although a differential performance of the instrument may be taken into account by properly rescaling the factor $h_{\text{cut}}$. The typical parameters we shall use are taken from existing literature, and reported in Tab. I. Note that GLAST is expected to have an excellent background identification, so that only cosmic rays in the amount of $\sim 6\%$ of the gamma flux pass the cuts. On the other hand, EAS experiments have a poor rejection capability (some of them like TIBET[42] have none), which increases typically the gamma content of the diffuse flux by no more than one order of magnitude. Therefore one should keep in mind that even after gamma/hadron separation, the anisotropies of the gamma sky have to be identified against a quasi-isotropic background which is $\gtrsim 10^4$ larger than the gamma flux.

In Fig. 6 we report the coefficients $a_{lm}$’s up to $l_{\text{max}} = 10$ calculated from the PSCz gamma maps of Fig. 5 with the errors estimated according to what reported in Appendix B.
The coefficients $C_l$ up to $l_{\text{max}} = 10$ calculated from the PScz gamma maps of Fig. 5. We show the level of shot noise [see Eq. (B5)] expected from four years of GLAST and one decade of HA WC. We report the predictions for both the linear and quadratic cases.

Appendix B. The GLAST mission is expected to last 5+5 years (so we plot a realistic 4 years exposure), while EAS instruments have longer run times, so we plot the expectations for a decade of collecting time by MILAGRO, or by the proposed project HA WC. GLAST should be able to detect some structures above 100 GeV at the 2$\sigma$ level, even if the correlation with matter density is only linear. For a quadratic correlation one expects a more robust detection, and possibly even hints for anisotropies at higher energies. On the contrary, instruments like MILAGRO may find hints of structures (at the 1$\sigma$ level, see gray band in the right panel of Fig. 6) only if correlations are quadratic or in any case strongly biased with overdensities. As a technical remark, note that the performances of MILAGRO above the TeV were estimated by using an effective area $A_{\text{eff}}(E_\gamma) \simeq 10^{7.1} \text{cm}^2$ (see e.g. [45]). A proper treatment should take into account the energy-dependence of the area, and calculate the expected sky maps for MILAGRO weighting accordingly the integral maps. Given the limited chances of this instrument to detect the features we have described, we consider this simple estimate sufficient to illustrate our point. It is worth to stress that for an EAS detector the error on the $a_{lm}$’s scales as $\sqrt{N_{\text{CR}}/N_{\gamma}}$. Therefore the reduction of the shot-noise error goes like $(t \cdot A_{\text{eff}})^{-1/2}$ (both $N_{\text{CR}}$ and $N_{\gamma}$ grow linearly with $t \cdot A_{\text{eff}}$), or equivalently as $\sqrt{h_{\text{cut}}/g_{\text{cut}}}$: improving the exposure is equally important as improving the gamma/hadron separation capability.

A simple inspection of Fig. 6 reveals that for a realistic detection of the features in the VHE sky one would need the improvement in effective area planned to be reached by instruments like HA WC (see inner green band in the right panel of Fig. 6). An instrument like ARGO is expected to have performances in between MILAGRO and HA WC, and may have some chance especially if a significant improvement in hadron rejection can be made. Also, note that due to their altitude HA WC and ARGO have a significant acceptance of sub-TeV events. While the gamma/hadron separation is less efficient at lower energies, the higher statistics may help in revealing these structures.

In Fig. 7 we plot the expectations for GLAST and HAWC in terms of the $C_l$ coefficients. At the large scales we are focusing on cosmic variance makes any detection of the $C_l$’s challenging even when the corresponding $a_{lm}$’s are easily detectable. This proves the importance of the deterministic nature of the expected anisotropies in the flux: for comparison, despite the larger statistics available at lower energies, the detection of the $C_l$’s at low $l$ studied in [11] is precluded by the cosmic variance.

Since our predictions are shaped by the nearby universe, for a fixed background the absolute value of the detected anisotropy in principle measures the index of the correlation of gamma sources with respect to the matter, as clearly shown by the comparison of the top and lower panels in both Fig. 6 and Fig. 7. On the other hand, a partial degeneracy exists with the intensity of the infrared background. However, both the study of single sources and of the energy spectrum of the CGB should pin down the remaining uncertainty on this quantity, and the corresponding degeneracy should eventually be broken.

1 Our estimate does not include the fraction of the CGB measured by EGRET which may be resolved by GLAST. If this is removed, our predictions should be rescaled accordingly.
Finally, let's note that these estimates are somewhat conservative: summing the power at different l's may favor the detection (see e.g. [28]), and cross-correlating directly with the maps would eventually rely on the whole information. Indeed, to some extent also the information at small scales can be exploited. In particular Fig. 6 shows that the anisotropy expected from small regions of the Virgo cluster and along the Super-Galactic plane is significantly above the average flux. Therefore, once the data on the CGB become available, reassessing the problem with a detailed study will be mandatory.

VI. SUMMARY AND CONCLUSIONS

The universe is pervaded by diffuse backgrounds of low-energy photons, of cosmological origin like the CMB or due to stellar activity, like the optical and infrared background (CIB). This extragalactic background light makes the universe opaque to energetic γ's. The most energetic part of the gamma-ray background (CGB) is thus of primary importance for high energy astroparticle physics, since it acts as a "cosmic calorimeter". Besides telling us about the integrated history of the most powerful astrophysical accelerators of the universe, it may constrain non-standard physics taking place at high energy, even much higher than the GeV–TeV scale. As an example, we remind that the most stringent limits on the decays of superheavy particles coupled at the tree-level only to neutrinos come from the observed diffuse extragalactic γ-ray flux [51].

In this work we have studied the anisotropy pattern of the CGB in particular in the very-high energy regime, beyond about 100 GeV. Due to the onset of the pair-production losses of VHE photons on the CIB, most of the flux is coming from local structures, within z ∼ 0.1. Especially at the largest angular scales, the pattern and the amplitude of the anisotropies are almost independent of the source energy spectral shape and of the cosmological model: modulo the magnitude of the CIB, the key parameter in shaping the signature is the degree of the correlation of the gamma-emitters with the known matter density field in the nearby universe. For example, dark matter annihilation models for the origin of most of the CGB predict a strong (quadratic) correlation of the flux with the matter density, which should clearly manifest in the forthcoming observations. Looking for such signatures at VHE is promising since, despite the limited statistics, the anisotropies are larger and the predictions are more robust than the ones at low energy and small scales (see e.g. [11]), which are affected by more astrophysical and particle physics uncertainties, besides the cosmic variance.

Starting from the PSCz astronomical catalogue, we produced maps of the VHE gamma sky and estimated the potential of the satellite mission GLAST or of extensive air shower observatories to detect these features.

The GLAST mission should be able to detected a significant correlation of the diffuse gamma-ray emission with the forecast maps presented in this work, providing an important complementary observable to constrain the emission models. This is especially true if an exotic contribution from dark matter annihilation is relevant, since dark matter emission correlates quadratically with overdensities. Of course, before claiming an unambiguous detection of dark matter in the CGB, detailed particle physics models and a proper foreground analysis are required. Indeed, to some extent strongly biased sources may mimick dark matter-like features. Some early investigation of this issue are however quite promising [47].

EAS experiments are instead limited by the scarce cosmic ray rejection capabilities, and only the next generation of instruments like HAWC (or maybe already ARGO) may have real chances to achieve the needed sensitivity. For EAS experiments, there is a further remark. The SK experiment has recently detected an anisotropy at the level of few×10⁻⁴ in the cosmic rays around 10 TeV, from a sample of about 2 × 10⁸ muons [49] (MILAGRO has also detected this effect, as mentioned in [47]). At similar or higher energies, the TIBET collaboration has reported the detection of several anisotropies at the ∼ 0.1% level in the cosmic ray flux, probably associated with galactic sources and/or galactic transport [50]. While the exposure needed to reveal the features we have discussed so far is within the reach of the next generation of EAS instruments, even assuming an excellent control over experimental spurious effects, the ultimate limitation in detecting these signatures comes from the understanding of the intrinsic anisotropy in the CR background. Therefore, an efficient gamma/hadron separation is not only necessary to enhance the statistical significance of the point-like or diffuse gamma ray sources observed by EAS instruments, but also to control systematics. In particular, reversing the gamma cut and thus enriching the sample in hadronic showers may help identifying and removing non-gamma anisotropies.

Acknowledgments

We thank the Danish Centre of Scientific Computing (DCSC) for granting the computer resources used. This work was also supported by the PRIN04 Fisica Astroparticellare of Italian MIUR. PS acknowledges support by the US Department of Energy and by NASA grant NAG5-10842. SH and PS thank the Max-Planck-Institut für Physik in Munich for hospitality and support during the initial phase of this work. TH thanks the DARK Cosmology Centre for hospitality during the course of this work. PS acknowledges the members of the MAGIC group in Munich and of the GLAST group at KIPAC/Stanford, and in particular Daniel Mazin and David Paneque, for useful discussions. HT thanks Manoj Kaplinghat.
APPENDIX A: GAMMA PROPAGATION

Here we report some details on the models used for the extragalactic background light (EBL), and on the technique to account for absorption effects in the propagation of photons. The main component of the EBL is the cosmic microwave background, the spectral number density of which is well known to obey a black-body spectrum

\[
n_{\text{CMB}}(\epsilon) = \frac{1}{\pi^2 (hc)^3} \frac{\epsilon^2}{\exp(\epsilon/T_{\text{CMB}}) - 1} \tag{A1}
\]

where \(T_{\text{CMB}}^0 = 2.73 \text{K} = 2.35 \times 10^{-4} \text{eV}\) is its present temperature. For the Infrared/Optical Background (CIB), which is the main source of gamma absorption, we use the simple parametrization

\[
n_{\text{CIB}}(\epsilon) = \begin{cases} 
5.42 \times 10^{11} \text{eV}^{-1} \text{cm}^{-3} \frac{\epsilon^{3.4}}{\exp(\epsilon/T_\gamma) - 1} & \lambda \in 200 \div 2000 \mu\text{m}, \\
7.4 \times 10^{-4} \text{eV}^{-1} \text{cm}^{-3} \epsilon^{-2.295} & \lambda \in 6.0 \div 200 \mu\text{m}, \\
7 \times 10^{-5} \text{eV}^{-1} \text{cm}^{-3} \epsilon^{-5} & \lambda < 6.0 \mu\text{m},
\end{cases} \tag{A2}
\]

However accurate modelling of this process suggests that most of the background formed in a burst at \(z \simeq 4 \div 5\) near the peak of star formation rate, so that for our purposes (propagation till \(z \simeq 0.2\)) simple redshifting is quite accurate \cite{24}.

In principle, the interaction of high energy photons with the EBL is a complex process in which the \(e^- e^+\) created through pair production interact again with background photons via inverse Compton scattering producing new high energy \(\gamma\)'s. We have developed our own code for calculating the evolution of gamma cascades, along the lines of Ref. \cite{52} (which we refer to for the details, including the general formalism of cascade propagation). Luckily, however, the complicated regime where the cascade of high energy \(e^+, e^-\), and \(\gamma\) has to be followed in details for calculating the final energy distribution is only attained for energies \(> 10^{15} \text{eV}\). In the TeV range relevant for gamma astronomy the development of the shower via secondary particle dynamics can be safely neglected, and the observed flux can be calculated by simply considering an energy-dependent depletion factor for the spectrum in addition to the effect of redshifting. Thus, if the spectrum at the source is of the form \(g(E)\), the observed spectral shape of the signal will be

\[
g(E_\gamma) \propto g[E_\gamma(1 + z)] P(E_\gamma, z), \tag{A4}
\]

where \(E_\gamma\) is the energy we observe today and \(P(E_\gamma, z)\) is the probability for a photon emitted at redshift \(z\) to survive without interacting till now, when it reaches us with energy \(E_\gamma\). This probability is written as

\[
P(E_\gamma, z) \equiv e^{-\tau(E_\gamma, z)}, \tag{A5}
\]

where the optical depth \(\tau\) is

\[
\tau(E_\gamma, z) = \tau(E_\gamma) \equiv \int_0^z \frac{d\tau}{1 + \frac{\lambda_0}{\lambda}} = \frac{1}{\pi} \int_0^\infty \frac{d\epsilon}{\epsilon} \frac{n_{\text{CIB}}(\epsilon)}{\epsilon} \exp(-\epsilon/T_{\gamma}),
\]

\[
\epsilon = \frac{h c \lambda_0}{m c} = \frac{h c}{m c} \lambda_0 = \frac{h c}{m c} \epsilon_0 = \frac{h c}{m c} \epsilon_0 (1 + z),
\]

\[
\frac{d\tau}{1 + \frac{\lambda_0}{\lambda}} = \frac{1}{\pi} \int_0^\infty \frac{d\epsilon}{\epsilon} \frac{n_{\text{CIB}}(\epsilon)}{\epsilon} \exp(-\epsilon/T_{\gamma}),
\]

\[
\lambda = (1 + z) \lambda_0 = \frac{h c}{m c} \epsilon_0 (1 + z) = \frac{h c}{m c} \epsilon_0 (1 + z)(1 + z) = \frac{h c}{m c} \epsilon_0 (1 + z)^2,
\]

\[
\frac{\lambda_0}{\lambda} = \frac{1}{(1 + z)^2}.
\]

\[
\tau(E_\gamma) = \frac{1}{\pi} \int_0^\infty \frac{d\epsilon}{\epsilon} \frac{n_{\text{CIB}}(\epsilon)}{\epsilon} \exp(-\epsilon/T_{\gamma}),
\]

\[
\epsilon = \frac{h c \lambda_0}{m c} = \frac{h c}{m c} \lambda_0 = \frac{h c}{m c} \epsilon_0 = \frac{h c}{m c} \epsilon_0 (1 + z),
\]

\[
\tau(E_\gamma) \equiv \frac{1}{\pi} \int_0^\infty \frac{d\epsilon}{\epsilon} \frac{n_{\text{CIB}}(\epsilon)}{\epsilon} \exp(-\epsilon/T_{\gamma}),
\]

\[
\epsilon = \frac{h c \lambda_0}{m c} = \frac{h c}{m c} \lambda_0 = \frac{h c}{m c} \epsilon_0 = \frac{h c}{m c} \epsilon_0 (1 + z),
\]
that is the rate of pair production of photons on the EBL integrated over the time during propagating from redshift $z$ to 0. The function $\sigma_{PP}(E, \epsilon, \mu)$ is the theoretically and experimentally well-known pair production cross section of a photon of energy $E$ impinging over a background photon of energy $\epsilon$ with a cosine of the impact angle given by $\mu$. In Fig. 6 we show the derived critical $\tau(E_\gamma, z) = 1$ contour that represents the gamma redshift horizon as a function of the energy. We find a satisfactory agreement of the function $\tau$ we computed with more detailed studies including the CIB distribution and evolution from simulations of star and galaxy formation [52].

**APPENDIX B: SUMMARY OF THE PROPERTIES OF NOISE**

In this Appendix we briefly review the noise properties of a discrete poisson process on the sphere. The main results can be easily obtained analytically and concern the amplitude of the noise variance and the related power spectrum. The map resulting from a random realization of $N$ equally weighted points can be written as $f(\Omega) = 4\pi/N \sum_i \delta(\Omega - \Omega_i)$, normalized to its mean value, so that to be adimensional; then the harmonic expansion coefficients $a_{lm} = \int d\Omega f Y_{lm}(\Omega)$ follow (apart the constant monopole contribution) a gaussian distribution with $\langle a_{lm} \rangle = 0$ and

$$\sigma_{a_{lm}}^2 = \langle a_{lm}^2 \rangle = \frac{4\pi}{N} \tag{B1}$$

independent from $l, m$. From this, the spectrum of the shot noise $C_l = \sum_m |a_{lm}|^2/(2l+1)$ follows a $\chi^2$ distribution with mean

$$\langle C_l \rangle = \frac{4\pi}{N} \tag{B2}$$

again independent of $l$, and variance

$$\sigma_{C_l}^2 = \langle C_l^2 \rangle - \langle C_l \rangle^2 = \left( \frac{4\pi}{N} \right)^2 \frac{2}{2l+1} \tag{B3}$$

These results surely hold in the limit of large statistics; however for our applications it is worth testing them also in the limit of very low statistics, say for $N \leq 1000$. We performed a set of Monte Carlo simulations to clarify the issue an found that the simulations and analytic results are in perfect agreement even in the completely unphysical limit of $N = 10$; the analytical results can then be safely used in all the cases of interest.

In general, in addition to the white noise $C_l^N$, one has a signal $C_l^S$. Moreover, there is normally incomplete sky coverage, and additional white noise may be present, as in our case because of the background due to cosmic rays passing the cuts. A generalization of Eq. (B3) then reads

$$\sigma_{C_l}^2 = \frac{2}{(2l+1)f_{\text{sky}}} (C_l^S + C_l^N)^2, \tag{B4}$$

where $f_{\text{sky}}$ is the fraction of the sky accessible to the experiment (assumed with uniform acceptance over this region), and the noise spectrum $C_l^N$ (including the cosmic ray background) is given by

$$C_l^N = \frac{4\pi f_{\text{sky}}}{N_\gamma} \left[ 1 + \frac{N_{\text{CR}}}{N_\gamma} \right]. \tag{B5}$$

For the $a_{lm}$’s, the variance due to the shot-noise plus the background is written as

$$\sigma_{a_{lm}}^2 = C_l^N = \frac{4\pi f_{\text{sky}}}{N_\gamma} \left( 1 + \frac{N_{\text{CR}}}{N_\gamma} \right). \tag{B6}$$

In principle, the complete formulae receive a correction at large $l$ due to the finite angular resolution of the experiment that is easily implemented performing everywhere the substitution $C_l^N \rightarrow C_l^N \exp(-l^2 \sigma_b^2/2) \sigma_b$ being the angular resolution of the experiment. However, since the experimental resolutions are better than a degree, and we limit our considerations to the most prominent signatures at large scales, this correction is unnecessary for our application.

We have used these results in estimating the errors reported in Sec. [VI]. Note that what is really measured is always the sum signal+noise: the noise is an unavoidable component in any experiment. However, being constant in $l, m$, the average level of the noise can be fitted and subtracted (this is trivial for deterministic predictions since $\langle a_{lm}\rangle = 0$). On the other hand, the error in its determination depends on the sensitivity of the experiment and thus the statistics collected, and on the level of background rejection.

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[41] See the URL: http://www-glast.slac.stanford.edu/

[42] See the URL: http://www.icrr.u-tokyo.ac.jp/em/

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2005); I. Di Mitri, at CRIS 2006, Catania, Italy.


