Massive perturbers in the galactic center

Hagai B. Perets, Clovis Hopman, and Tal Alexander

ABSTRACT

We investigate the role of massive perturbers, such as stellar clusters or giant molecular clouds, in supplying low-angular momentum stars that pass very close to the central massive black hole (MBH) or fall into it. We show that massive perturbers can play an important role in supplying both binaries and single stars to the vicinity of the MBH. We discuss possible implications for the ejection of high velocity stars; for the capture of stars on tight orbits around the MBH; for the emission of gravitational waves from low-eccentricity inspiraling stars; and for the origin of the young main sequence B stars observed very near the Galactic MBH. Massive perturbers may also enhance the the growth rate of MBHs, and may accelerate the dynamical orbital decay of coalescing binary MBHs.

1. Introduction

There is compelling evidence that massive black holes (MBHs) lie in the centers of all galaxies (e.g. (10)), including the center of our Galaxy (9; 14). The MBH affects the dynamics and evolution of the galaxy’s center as a whole and it also strongly affects individual stars or binaries that approach it. Such close encounters have been the focus of many studies and include a variety of processes such as destruction of stars by the MBH; capture and gradual inspiral of stars into the MBH, accompanied by the emission of gravitational waves (GWs); or dynamical exchange interactions in which incoming stars or binaries energetically eject a star tightly bound to the MBH and are captured in its place very near the MBH (1).

The interest in such processes is driven by their possible implications for the growth of MBHs, for the orbital decay of a MBH binary, for the detection of MBHs, for GW astronomy, as well as by observations of unusual stellar phenomena in our Galaxy, e.g. the puzzling young population of B-star very near the Galactic MBH, or the hyper-velocity B stars at the edge of the Galaxy (e.g. (4; 12, 5)), possibly ejected by 3-body interactions of a binaries with the MBH.
Here we focus on close encounters with the MBH whose ultimate outcome ("event") is the elimination of the incoming object from the system, whether on the short infall (dynamical) time, if the event is prompt (e.g. tidal disruption or 3-body exchange between a binary and the MBH), or on the longer inspiral time, if the event progresses via orbital decay (e.g. through GW emission). Such processes are effective only when the incoming object follows an almost zero angular momentum ("loss-cone") orbit with periapse closer to the MBH than some small distance $q$. To reach the MBH, or to decay to a short period orbit, both the infall and inspiral times must be much shorter than the system’s relaxation time $t_r$. The fraction of stars initially on loss-cone orbits is very small and they are rapidly eliminated. Subsequently, the close encounter event rate is set by the dynamical processes that refill the loss-cone.

The loss-cone formalism used for estimating the event rate (e.g.\cite{7}) usually assumes that the system is isolated and that the refilling process is 2-body relaxation. This typically leads to a low event rate, set by the long 2-body relaxation time.

Two-body relaxation, which is inherent to stellar systems, ensures a minimal loss-cone refilling rate. Other, more efficient but less general refilling mechanisms were also studied with the aim of explaining various open questions, or in the hope that they may lead to significantly higher event rates for close encounter processes (\cite{25}). However, most of these mechanisms require special circumstances to work, or are short-lived.

Here we explore another possibility, which is more likely to apply generally: accelerated relaxation and enhanced rates of close encounters driven by massive perturbers (MPs). Efficient relaxation by MPs were first suggested in this context by Zhao, Haehnelt & Rees (\cite{39}) as a mechanism for establishing the $M_\bullet/\sigma$ relation (\cite{10}) by fast accretion of stars and dark matter. Zhao et al. also noted the possibility of increased tidal disruption flares and accelerated MBH binary coalescence due to MPs. In this study we investigate in detail the dynamical implications of relaxation by MPs. We evaluate its effects on the different modes of close interactions with the MBH, in particular 3-body exchanges, which were not considered by Zhao et al. and apply our results to the Galactic Center (GC), where observations indicate that dynamical relaxation is very likely dominated by MPs.

## 2. Loss-cone refilling

In addition to stars, galaxies contain dense massive structures such as molecular clouds, open clusters and globular clusters with masses up to $10^4$–$10^7 M_\odot$. Such structures can perturb stellar orbits around the MBH much faster than 2-body stellar relaxation (hereafter “stellar relaxation”), provided they are numerous enough. The minimal impact parameter still consistent with a small
angle deflection in the MP-star scattering is \( b_{\text{min}} = \frac{GM_p}{v^2} \) (the capture radius), where \( v \) is of the order of the local velocity dispersion \( \sigma \). The total rate of scattering stars into the loss cone, \( \Gamma \), is obtained by integrating \( d\Gamma / dM_p db \) over all \( \text{MP masses and over all impact parameters between } b_{\text{min}} \text{ and } b_{\text{max}} \approx r \). The relaxation rate due to all MPs at all impact parameters is then

\[
\frac{1}{t_r} = \int_{b_{\text{min}}}^{b_{\text{max}}} dM_p \int dM_p \left( \frac{d^2 \Gamma}{dM_p db} \right) \sim \log \frac{b_{\text{max}}}{b_{\text{min}}} \int dM_p \left( \frac{dN_p}{dM_p} \right) M_p^2 ,
\]

where \( \log \Lambda = \log \frac{b_{\text{max}}}{b_{\text{min}}} \) is the Coulomb logarithm (here the dependence of \( \log \Lambda \) and \( v \) on \( M_p \) is assumed to be negligible). This formulation of the relaxation time is equivalent to its conventional definition \( \frac{1}{t_r} = \frac{v^2}{D(v^2)} \) as the time for a change of order unity in \( v^2 \) by diffusion in phase space due to scattering, \( t_r \sim v^2/D(v^2) \), where \( D(v^2) \) is the diffusion coefficient. If the stars and MPs have distinct mass scales with typical number densities \( N_* \) and \( N_p \) and rms masses \( \langle M_*^2 \rangle^{1/2} \) and \( \langle M_p^2 \rangle^{1/2} \) (\( \langle M^2 \rangle = \int M^2 (dN/dM) dM / N \)), then MPs dominate if the ratio of the 2nd moments of the MP and star mass distributions, \( \mu_2 = \frac{N_p \langle M_p^2 \rangle}{N_* \langle M_*^2 \rangle} \), satisfies \( \mu_2 > 1 \).

The central \( \sim 100 \) pc of the GC contain \( 10^8 - 10^9 \) solar masses in stars, and about \( 10^6 - 10^8 \) solar masses in MPs such as open clusters and GMCs of masses \( 10^7 - 10^7 M_\odot \). An order of magnitude estimate indicates that MPs in the GC can reduce the relaxation time by several orders of magnitude,

\[
\frac{t_{r,*}}{t_{r,\text{MP}}} = \mu_2 \sim \left[ \frac{(N_p M_p) M_p}{(N_* M_*) M_*} \right] = 10^3 \left[ \frac{(N_* M_*)}{N_p M_p} \right]^{-1} \left[ \frac{(M_p/M_*)}{10^5} \right] .
\]

This estimate is borne by more detailed calculations (Fig. 1 and table 1), using the formal definition \( t_r = v^2/D(v^2) \) with \( M_* \rho_* \rightarrow \int (dN_p/dM_p) M_p^2 dM_p \). In our calculations we follow the Fokker-Planck approach to the loss-cone problem, where we recalculate the diffusion coefficients obtained from stellar two body relaxation by taking into account the contribution from the much more massive MPs (for details see (32)).

Although some of the assumptions concerning the loss cone formalism are not necessarily valid in the case of MPs, the loss-cone formalism can be generalized to deal with MPs in an approximate manner with only few modifications (32). Thus to a good approximation our calculation follows the usual loss cone treatment (e.g. (24)) where the mass of the stars is replaced by that of the MPs, and the integration over the energies of deflected stars (mapped to their distance from the MBH) is done only for regions where MPs exist.

### 3. Massive perturbers in the Galactic Center

MPs can dominate relaxation only when they are massive enough to compensate for their small space densities. Here we consider only MPs with masses \( M_p \geq 10^3 M_\odot \). Such MPs could
be open or globular stellar clusters or molecular clouds of different masses, in particular giant molecular clouds (GMCs). Observations of the Galaxy reveal enough MPs to dominate relaxation in the central 100 pc. We adopt here a conservative approach, and include in our modeling only those MPs that are directly observed in the Galaxy. Based on the observations of such MPs we devise several possible models for the MPs in the GC, detailed in table I. As these observations (30; 11; 37; 16; 3) show that the MPs population is dominated by the GMCs, we consider only GMCs and gaseous clumps in our models. The observed MP species vary in their spatial distributions and mass functions, which are not smooth or regular. For our numeric calculations, we construct several simplified MP models (table I) that are broadly based on the observed properties of the MPs, and assume that the spatial distribution of MPs follows that of the stars (apart of the inner radius cutoff where no MPs are observed).

The three MP models in table I, Stars, GMC1 and GMC2, represent respectively the case of relaxation by stars only, by heavy GMCs and light GMCs.

Table 1: Massive perturber models

<table>
<thead>
<tr>
<th>Model</th>
<th>$r$ (pc)</th>
<th>$N_p$</th>
<th>$M_p$ ($M_\odot$)</th>
<th>$\beta$</th>
<th>$R_p$ (pc)</th>
<th>$\mu_2$ $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMC1</td>
<td>5–100</td>
<td>100</td>
<td>$10^4$–$10^7$</td>
<td>1.6</td>
<td>5</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td></td>
<td>1.5–5</td>
<td>30</td>
<td>$10^3$–$10^5$</td>
<td>0.9</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>GMC2</td>
<td>5–100</td>
<td>100</td>
<td>$10^3$–$10^6$</td>
<td>1.6</td>
<td>5</td>
<td>$3 \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>1.5–5</td>
<td>30</td>
<td>$10^2$–$10^4$</td>
<td>0.9</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Stars</td>
<td>5–100</td>
<td>$2 \times 10^8$</td>
<td>1</td>
<td>—</td>
<td>$\sim 0$</td>
<td>1</td>
</tr>
<tr>
<td>Stars</td>
<td>1.5–5</td>
<td>$6 \times 10^6$</td>
<td>1</td>
<td>—</td>
<td>$\sim 0$</td>
<td>1</td>
</tr>
</tbody>
</table>

$a N_p(r) \propto r^{-2}$ assumed.

$b \mu_2 \equiv N_p \langle M_p^2 \rangle / N_* \langle M_*^2 \rangle$, where $\langle M^2 \rangle = \int M^2 (dN/dM)dM/N$. 

Fig. 1.— Relaxation time as function of distance from the MBH, for stars (solid line) and for each of the 2 MP models separately, as listed in table I massive GMCs (dashed-dotted line) and intermediate GMCs (dashed line). The discontinuities are artifacts of the assumed sharp spatial cutoffs on the MP distributions. 2-body stellar processes dominate close to the MBH, where no MPs are observed to exist. However, at larger distances massive clumps (at $1.5 < r < 5$ pcs) and GMCs (at $5 < r < 100$) are much more important.
4. Massive perturber-driven interactions with a MBH

The maximal differential loss-cone refilling rate, which is also the close encounters event rate, $dΓ/dE$, is reached when relaxation is efficient enough to completely refill the loss cone during one orbit. Further decrease in the relaxation time does not affect the event rate at that energy. MPs can therefore increase the differential event rate over that predicted by stellar relaxation, only at high enough energies, $E > E_c$ (equivalently, small enough typical radii, $r < r_c$), the critical energy (radius), separating the full and empty loss cone regimes, where slow stellar relaxation fails to refill the empty loss-cone. The extent of the empty loss-cone region increases with the maximal periapse $q$, which in turn depends on the close encounter process of interest. For example, the tidal disruption of an object of mass $M$ and size $R$ occurs when $q < r_t$, the tidal disruption radius, $r_t \simeq R (M_*/M)^{1/3}$ . This approximate disruption criterion applies both for single stars ($M = M_*$, $R = R_*$) and for binaries, where $M$ is the combined mass of the binary components and $R$ is the binary’s semi-major axis, $a$. Stellar radii are usually much smaller than typical binary separations, but stellar masses are only $\sim 2$ times smaller than binary masses. Binaries are therefore disrupted on larger scales than single stars. In the GC this translates to an empty (stellar relaxation) loss-cone region extending out to $r_c^s \sim 3$ pc for single stars and out to $r_c^b > 100$ pc for binaries. In the GC $r_{MP} \lesssim r_c^s \ll r_c^b$ (where $r_{MP}$ is the smallest distance from the MBH where MPs are observed), and so MPs are expected to increase the binary disruption rate by orders of magnitude, but increase the single star disruption rate only by a small factor. Since for most Galactic MP types $r_{MP} > r_h$ (the radius of influence of the MBH), the disruption rate is dominated by stars near $r_{MP}$. For example, when the loss-cone is empty, $\sim 50\%$ of the total rate is due to MPs at $r < 2r_{MP}$; when the loss-cone is full, $\sim 75\%$ of the total rate is due to MPs at $r < 2r_{MP}$ (see details in (32)).

4.1. Interactions with single stars

Clusters, GMCs and gas clumps in the GC are abundant only beyond the central $r_{MP} \sim 1.5$ pc, whereas the empty loss-cone regime for tidal disruption of single stars extends only out to $r_c^s \sim 3$ pc. For inspiral processes such as GW emission, $r_c$ is $\sim 100$ times smaller still (19). The effect of such MPs on close encounter events involving single stars is thus suppressed (weaker tidal effects by MPs at $r > r_c^s$ are not considered here). This is contrary to the suggestion of Zhao et al. (39), who assumed that the effect of MPs fully extends to the empty loss-cone regime. We find that the enhancement of MPs over stellar relaxation to the single stars disruption rate is small, less than a factor of 3, and is due to stars scattered by gas clumps in the small empty-loss cone region between $r_{MP} \sim 1.5$ pc and $r_c^s \sim 3$ pc. A possible exception to this conclusion is the possible existence of IMBHs population, not modeled here.
4.2. Interactions with stellar binaries

The empty loss cone region for binary-MBH interactions extends out to $> 100$ pc because of their large tidal radius. On these large scales MPs are abundant enough to dominate the relaxation processes. Here we focus on 3-body exchange interactions $^{17,38}$, which lead to the disruption of the binary, the energetic ejection of one star, and the capture of the other star on a close orbit around the MBH. The event rate is highly dependent on the unknown binary fraction in these regions.

The binary fraction and typical binary semi-major axis depend on the binary mass, and on the rate at which binaries evaporate by encounters with other stars. This depends in turn on the stellar densities and velocities, and therefore on the distance from the MBH. We take these factors into account and estimate in detail the 3-body exchange rate for MP-driven relaxation. The rate is proportional to the binary fraction in the population, which is the product of the poorly-known binary IMF in the GC and the survival probability against binary evaporation.

The capture probability and the semi-major axis distribution of captured stars were estimated by simulations $^{18,38}$. Numeric experiments indicate that between 0.5–1.0 of the binaries that approach the MBH within the tidal radius $r_t(a)$ are disrupted. Here we adopt a disruption efficiency of 0.75. The harmonic mean semi-major axis for 3-body exchanges with equal mass binaries was found to be $^{18}$

$$\langle a_1 \rangle \simeq 0.56 \left( \frac{M_\bullet}{M_{\text{bin}}} \right)^{2/3} a \simeq 0.56 \left( \frac{M_\bullet}{M_{\text{bin}}} \right)^{1/3} r_t,$$

(3)

where $a$ is the semi-major axis of the infalling binary and $a_1$ that of the captured star (the MBH-star “binary”). Most values of $a_1$ fall within a factor 2 of the mean. This relation maps the semi-major axis distribution of the infalling binaries to that of the captured stars: the harder the binaries, the more tightly bound the captured stars. The periapse of the captured star is at $r_t$, and therefore its eccentricity is very high $^{18,27}$,

$$e = 1 - r_t / a_1 \simeq 1 - 1.8 (M_{\text{bin}}/M_\bullet)^{1/3} \gtrsim 0.98$$

for values typical of the GC.

We now consider the implications of 3-body exchange interactions of the MBH with old ($t_\star \gtrsim t_H$) binaries and massive young ($t_\star < 5 \times 10^7$ yr) binaries.

The properties of binaries in the inner GC are at present poorly determined. We use the period distribution of Solar neighborhood binaries for old low mass binaries $^{8}$ and young massive binaries $^{21}$. The total binary fraction of these binaries is estimated at $f_{\text{bin}} \sim 0.3$ for low mass $^{22}$ binaries, and $f_{\text{bin}} \sim 0.75$ for massive binaries $^{21}$. Adopting these values for the GC, the total binary disruption rate by the MBH can then be calculated by integrating $dN_{\text{bin}}/da$ over the binary $a$ distribution and over the power-law stellar density distribution of the GC from the minimal radius where such binaries exist (for young binaries we assume an inner cutoff at 1.5 pcs, where such young stars are not observed $^{31}$), up to 100 pc $^{13}$. Table (2) lists the capture rates for the
different perturber models, assuming a typical old equal-mass binary of $M_{\text{bin}} = 2 \, M_{\odot}$, or young equal-mass binary of $M_{\text{bin}} = 15 \, M_{\odot}$.

The old, low-mass binary disruption rate we derive for stellar relaxation alone is $\sim 5 \times 10^{-7} \, \text{yr}^{-1}$, $\sim 5$ times lower, but still in broad agreement with the result of Yu & Tremaine (38). Their rate is somewhat higher because they assumed a constant binary fraction and a constant semi-major axis for all binaries, even close to the MBH, where these assumptions no longer hold.

MPs increase the binary disruption and high-velocity star ejection rates by factors of $\sim 10^{1-3}$ and effectively accelerate stellar migration to the center. Thus we expect an increase in the number of stars captured close to the MBH and consequently a higher event rate of single star processes such as tidal disruption, tidal heating and GW emission from compact objects, in particular from compact objects on zero-eccentricity orbits (27). MPs may be implicated in the puzzling presence of a cluster of main sequence B-stars ($4 \lesssim M_* \lesssim 15 \, M_{\odot}$) in the inner $\sim 1''$ ($\sim 0.04$ pc) of the GC. This so-called “S-cluster” is spatially, kinematically and spectroscopically distinct from the young, more massive stars observed farther out, on the $\sim 0.05$–$0.5$ pc scale, which are thought to have formed from the gravitational fragmentation of one or two gas disks (31). There is however still no satisfactory explanation for the existence of the seemingly normal, young massive main sequence stars of the S-cluster, so close to a MBH (see review of proposed models by Alexander [1]; also a recent model by Levin [23], and in these proceedings).

Here we revisit an idea proposed by Gould & Quillen (15), that the S-stars were captured near the MBH by 3-body exchange interactions with infalling massive binaries. Originally, this exchange scenario lacked a plausible source for the massive binaries. We suggest that MP-driven 3-body exchanges can serve as a source for binaries, as they increase the rate of infall of young field binaries to the MBH. Such young field binaries should exist in the GC, taking into account the ongoing star formation in the central $\sim 100$ pc, which implies the presence of a large reservoir of massive stars there. Such stars are indeed observed in the central few $\times$ 10 pc both in dense

<table>
<thead>
<tr>
<th>Model</th>
<th>Disruption rate (yr$^{-1}$)</th>
<th>Young Stars$^a$</th>
<th>Young Stars$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r &lt; 0.04$ pc</td>
<td>$r &lt; 0.4$ pc</td>
<td>$&lt; 0.04$ pc</td>
</tr>
<tr>
<td>GMC1</td>
<td>$1 \times 10^{-4}$</td>
<td>$2.8 \times 10^{-4}$</td>
<td>33.5</td>
</tr>
<tr>
<td>GMC2</td>
<td>$2.1 \times 10^{-5}$</td>
<td>$6.4 \times 10^{-5}$</td>
<td>5</td>
</tr>
<tr>
<td>Stars</td>
<td>$3.4 \times 10^{-7}$</td>
<td>$5.3 \times 10^{-7}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Observed</td>
<td>?</td>
<td>?</td>
<td>10–35$^b$</td>
</tr>
</tbody>
</table>

$^a$Main sequence B stars with lifespan $t < 5 \times 10^7$ yr.

$^b$~10 stars with derived $a \lesssim 0.04$ pc. $\gtrsim 30$ stars are observed in the area.
clusters and in the field (11; 29). It is plausible that a high fraction of them are in binaries.

We assume star formation at a constant rate for 10 Gyr with a Miller-Scalo IMF (26), and use a stellar population synthesis code (35) with the Geneva stellar evolution tracks (34) to estimate that the present day number fraction of stars in the S-star mass range is $3.5 \times 10^{-4}$ (and less than 0.01 of that for $M_\star > 15 M_\odot$ stars). Note that if star formation in the GC is biased toward massive stars (36), this estimate should be revised upward. We adopt the observed Solar neighborhood distribution of the semi-major axis of massive binaries, which shows that massive binaries are thus typically harder than low-mass binaries (21), and will be tidally disrupted closer to the MBH and leave a more tightly bound captured star.

We represent the massive binaries by one with equal mass stars in the mid-range of the S-stars masses, with $M_{\text{bin}} = 15 M_\odot$ and $t_*(7.5 M_\odot) \simeq 5 \times 10^7$ yr, and integrate over the stellar distribution and the binary $a$ distribution as before, to obtain the rate of binary disruptions, $\Gamma$, the mean number of captured massive stars in steady state, $N_\star = \Gamma t_*$, and their semi-major axis distribution (Eq. 3). Table (2) compares the number of captured young stars in steady state, for the different MP models, on the $r < 0.04$ pc scale (the S-cluster) and $0.04 < r < 0.4$ pc scale (the stellar rings) with current observations (9; 31).

The number of captured massive stars falls rapidly beyond 0.04 pc (table 2) where the S-cluster is observed because wide massive binaries are rare. This capture model thus provides a natural explanation for the central concentration of the S-cluster (Fig 2). The absence of more massive stars in the S-cluster ($M_\star > 15 M_\odot$, spectral type O V) is a statistical reflection of their much smaller fraction in the binary population. Figure (2) and table (2) compare the cumulative semi-major axis distribution of captured B-stars, as predicted by the different MP models, with the total number of young stars observed in the inner 0.04 pc ($\sim 35$ stars (9; 14; 31). Of these, only $\sim 10$ have full orbital solutions (in particular $a$ and $e$) at present. The numbers predicted by the MP models are consistent with the observations, unlike the stellar relaxation model that falls short by two orders of magnitude.

The binary capture model predicts that captured stars have very high initial eccentricities. Most of the solved S-star orbits do have $e > 0.9$, but a couple have $e \sim 0.3–0.4$ (9). Normal stellar relaxation is too slow to explain the decrease in the eccentricity of these stars over their relatively short lifetimes. However, the much faster process of resonant relaxation (33) may be efficient enough to randomize the eccentricity of a fraction of the stars, and could thus possibly explain the much larger observed spread in eccentricities (20).

The companions of these captured stars are ejected from the GC at high velocities (17). Consequently, our model predicts the number of young hyper velocity stars ejected from the GC (such as observed (4; 12; 5)) to be comparable to that of observed B-stars in the S-cluster, in agreement
with current observation based estimations (4, 12, 5).

Fig. 2.— Cumulative number of young B-stars in the GC as predicted by the MP models and by stellar two-body relaxation (listed in table 2). The circles, squares and vertical bar represents the cumulative number of observed young stars inside 0.04 pc (9, 40). The dotted vertical line marks the approximate maximal distance in which captured B-stars are expected to be observed.

5. Summary

We presented here the results of a study of the effect of massive perturbers near the MBH in the GC. We have shown that current observations of MPs such as GMCs and clusters indicate that they dominate relaxation process in the GC, where they exist, and they are much more important than stellar two-body relaxation processes by stars which are usually considered. We have used the loss cone formalism in order to analyze the importance of this mechanism in generating low angular momentum stars and binaries to interact with the MBH. We have have computed the rates of these interactions, and showed that they can be highly important for the process of binaries disruption by the MBH and its consequent outcomes. The origin of the young massive B-stars at the central arcsecond of the GC can be explained by the capture of stars from binaries disrupted by the MBH after being scattered by MPs. Our calculations also show that some of the companions of these captured stars could be ejected at high velocities, thus explaining the observations of young massive high velocity stars observed recently. In addition MPs increase the number of compact stars captured close to the MBH, and thus increase the emission of zero-eccentricity GWs. We also suggest that such MPs may also help solve the last parsec problem of coalescing BMBHs. Although we focused on the GC of the milky-way, our results can be easily extended to MPs near other MBHs.

References

REFERENCES


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