Tension and Systematics in the Gold06 SnIa Dataset

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(Dated: December 22, 2006)

The Gold06 SnIa dataset recently released in astro-ph/0611572 consists of five distinct subsets defined by the group or instrument that discovered and analyzed the corresponding data. These subsets are: the SNLS subset (47 SnIa), the HST subset (30 SnIa), the HZSST subset (41 SnIa), the SCP subset (26 SnIa) and the Low Redshift (LR) subset (38 SnIa). These subsets sum up to the 182 SnIa of the Gold06 dataset. We use Monte-Carlo simulations to study the statistical consistency of each one of the above subsets with the full Gold06 dataset. In particular, we compare the best fit \(w(z)\) parameters \((w_0, w_1)\) obtained by subtracting each one of the above subsets from the Gold06 dataset (subset truncation), with the corresponding best fit parameters \((w_0', w_1')\) obtained by subtracting the same number of randomly selected SnIa from the same redshift range of the Gold06 dataset (random truncation). We find that the probability for \((w_0', w_1') = (w_0, w_1)\) is large for the Gold06 minus SCP (Gold06-SCP) truncation but is less than 5\% for the Gold06-SNLS, Gold06-HZSST and Gold06-HST truncations. This result implies that the Gold06 dataset is not statistically homogeneous. By comparing the values of the best fit \((w_0, w_1)\) for each subset truncation we find that the tension among subsets is such that the SNLS and HST subsets are statistically consistent with each other and ‘pull’ towards \(\Lambda\)CDM \((w_0 = -1, w_1 = 0)\) while the HZSST subset is statistically distinct and strongly ‘pulls’ towards a varying \(w(z)\) crossing the line \(w = -1\) from below \((w_0 < -1, w_1 > 0)\). We also isolate six SnIa that are mostly responsible for this behavior of the HZSST subset.

PACS numbers: 98.80.Es,98.65.Dx,98.62.Sb

I. INTRODUCTION

Current cosmological observations show strong evidence that we live in a spatially flat universe \cite{21} with low matter density \cite{2} that is currently undergoing accelerated cosmic expansion. The most direct indication for the current accelerating expansion comes from the accumulating type Ia supernovae (SnIa) data \cite{3, 4, 5, 6, 7, 8, 9} which provide a detailed form of the recent expansion history of the universe.

This accelerating expansion has been attributed to a dark energy component with negative pressure which can induce repulsive gravity and thus cause accelerated expansion (for recent reviews see \cite{16, 11, 12, 13, 14, 15, 16}). The simplest and most obvious candidate for this dark energy is the cosmological constant \(\Lambda\) \cite{17} with equation of state \(w = p/\rho = -1\). This model however raises theoretical problems related to the fine tuned value required for the cosmological constant. These difficulties have lead to a large variety of proposed models where the dark energy component evolves with time usually due to an evolving scalar field (quintessence) which may be minimally \cite{18} or non-minimally \cite{19} coupled to gravity. Alternatively, more general modified gravity theories \cite{20} have also been proposed based on \(f(R)\) theories \cite{21, 22, 23, 24}, braneworlds\cite{25, 26, 27, 28}, Gauss-Bonnet dark energy \cite{29} etc. The main prediction of the dynamical models is the evolution of the dark energy density parameter \(\Omega_X(z)\). Combining this prediction with the prior assumption for the matter density parameter \(\Omega_m\), the predicted expansion history \(H(z)\) is obtained as

\[
H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_X(z)]
\] (1.1)

The dark energy density parameter is usually expressed as

\[
\Omega_X(z) = \Omega_0 X e^{3 \int_0^z \frac{1}{X} (1+w(z'))} \] (1.2)

where \(w(z)\) is related to \(H(z)\) by \cite{30, 31, 32}

\[
w(z) = -\frac{3}{2} (1+z) \frac{d \ln H}{dz} - 1 \] (1.3)

If the dark energy can be described as an ideal fluid with conserved energy momentum tensor \(T^{\mu\nu} = diag(\rho, p, p, p)\) then the above parameter \(w(z)\) is identical with the equation of state parameter of dark energy

\[
w(z) = \frac{p(z)}{\rho(z)} \] (1.4)

Independently of its physical origin, the parameter \(w(z)\) is an observable derived from \(H(z)\) (with prior knowledge of \(\Omega_m\)) and is usually used to compare theoretical model predictions with observations.

The two most reliable and robust SnIa datasets existing at present are the Gold dataset \cite{9} (hereafter Gold06) and the Supernova Legacy Survey (SNLS) \cite{2} dataset. The Gold dataset compiled by Riess et al. is a set of 182 supernova data from various sources analyzed in a consistent and robust manner with reduced calibration errors arising from systematics. It contains 119 points...
from previously published data \cite{Gold04} (hereafter Gold04) plus 16 points with $0.46 < z < 1.39$ discovered recently by the Hubble Space Telescope (HST). It also incorporates 47 points ($0.25 < z < 1$) from the first year release of the SNLS dataset \cite{SNLS} out of a total of 73 distant SnIa. Some supernovae were excluded \cite{Gold04} due to highly uncertain color measurements, high extinction $A_V > 0.5$ and a redshift cut $cz < 7000$ km/s or $z < 0.0233$, to avoid the influence of a possible local “Hubble Bubble”, so as to define a high-confidence subsample. In addition, a single algorithm (MLCS2k2) was applied to estimate all the SnIa distances (including those originating from SNLS) thus attempting to minimize the non-uniformities of the dataset.

The total of 182 SnIa included in the Gold06 dataset can be grouped into five subsets according to the search teams/instruments that discovered them. These subsets are shown in Table I. A detailed table of all the data used in our analysis and their subset origin is shown in the Appendix. Notice that the early data of the Gold06 dataset were obtained mainly in the 90’s and consist of the High z Supernova Search Team (HZSST) subset, the Supernova Cosmology Project (SCP) subset and the Low Redshift (LR) subset.

The above observations provide the apparent magnitude $m(z)$ of the supernovae at peak brightness after implementing correction for galactic extinction, K-correction and light curve width-luminosity correction. The resulting apparent magnitude $m(z)$ is related to the luminosity distance $d_L(z)$ through

$$m_{th}(z) = M(M, H_0) + 5 \log_{10}(D_L(z))$$

(1.5)

where in a flat cosmological model

$$D_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z'; a_1, \ldots, a_n)}$$

(1.6)

is the Hubble free luminosity distance ($H_0d_L/c$), $a_1, \ldots, a_n$ are theoretical model parameters and $M$ is the magnitude zero point offset and depends on the absolute magnitude $M$ and on the present Hubble parameter $H_0$ as

$$M = M + 5 \log_{10}(cH_0^{-1}Mpc) + 25 =$$

$$= M - 5 \log_{10} h + 42.38$$

(1.7)

The parameter $M$ is the absolute magnitude which is assumed to be constant after the above mentioned corrections have been implemented in $m(z)$.

The data points of the Gold06 dataset are given after the corrections have been implemented, in terms of the distance modulus

$$\mu_{obs}(z_i) \equiv m_{obs}(z_i) - M$$

(1.8)

The theoretical model parameters are determined by minimizing the quantity

$$\chi^2(a_1, \ldots, a_n) = \sum_{i=1}^N \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma_{\mu i}^2 + \sigma_{v i}^2}$$

(1.9)

where $\sigma_{\mu i}$ and $\sigma_{v i}$ are the errors due to flux uncertainties and peculiar velocity dispersion respectively. These errors are assumed to be gaussian and uncorrelated. The theoretical distance modulus is defined as

$$\mu_{th}(z_i) \equiv m_{th}(z_i) - M = 5 \log_{10}(D_L(z)) + \mu_0$$

(1.10)

where

$$\mu_0 = 42.38 - 5 \log_{10} h$$

(1.11)

and $\mu_{th}(z_i)$ also depends on the parameters $a_1, \ldots, a_n$ used in the parametrization of $H(z)$ in equation (1.6).

The parametrization used in our analysis is the CPL parametrization\cite{CPL1, CPL2}:

$$w(z) = w_0 + \frac{w_1}{1 + z}$$

(1.12)

$$H^2(z) = H_0^2[\Omega_m(1 + z)^3 + (1 - \Omega_m)(1 + z)^3(1+w_0+w_1)e^{3w_1/(1+(1+z)-1)}]$$

(1.13)

with a prior of the matter density parameter $\Omega_m = 0.28$ (as in Ref. \cite{SNLS}), assuming flatness, according to the methods described in detail in Ref.\cite{SNLS, SNLS2}.

The previous version of the Gold sample \cite{Gold04} (Gold04) had been shown to be in mild ($2\sigma$) tension with the SNLS dataset \cite{SNLS, SNLS}. While the Gold04 mildly favored an evolving dark energy equation of state parameter $w(z)$ (crossing the phantom divide line $w=-1$) over the cosmological constant ($\Lambda$CDM) at almost 2$\sigma$ level \cite{Gold2, Gold3, Gold4, Gold5, Gold6, GOLD7, GOLD8, GOLD9, GOLD10}, the SNLS data had shown no such trend and provided \cite{SNLS} a best fit $w(z)$ very close to $w = -1$ (ACDM). The trend towards phantom divide crossing can not be explained in the context of minimally coupled quintessence and could be viewed as an indication for more exotic models \cite{Exotic1, Exotic2, Exotic3, Exotic4, Exotic5, Exotic6, Exotic7}. This mild tension could have been attributed to systematic errors due eg to the different algorithm used in the analysis of the two datasets. The new version of the Gold sample however, (Gold06) involves an improved uniform analysis and incorporates a large part of the SNLS sample. Thus there could have been an anticipation that the mild tension with SNLS would be ameliorated or even disappear. As shown in Fig. 1 however, this anticipation has not been fulfilled (see also \cite{Gold2, Gold3}).

<table>
<thead>
<tr>
<th>Subsets</th>
<th>Total</th>
<th>Redshift Range</th>
<th>Years of discovery</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNLS</td>
<td>47</td>
<td>$0.25 \leq z \leq 0.96$</td>
<td>2003-2004</td>
<td>\cite{SNLS}</td>
</tr>
<tr>
<td>HST</td>
<td>30</td>
<td>$0.46 \leq z \leq 1.76$</td>
<td>1997-2005</td>
<td>\cite{SNLS}</td>
</tr>
<tr>
<td>HZSST</td>
<td>41</td>
<td>$0.28 \leq z \leq 1.20$</td>
<td>1995-2001</td>
<td>\cite{SNLS}</td>
</tr>
<tr>
<td>SCP</td>
<td>26</td>
<td>$0.17 \leq z \leq 0.86$</td>
<td>1995-2000</td>
<td>\cite{SNLS}</td>
</tr>
<tr>
<td>LR</td>
<td>38</td>
<td>$0.024 \leq z \leq 0.12$</td>
<td>1990-2000</td>
<td>\cite{SNLS}</td>
</tr>
</tbody>
</table>

**Table I**: The subsets of the Gold06 dataset (see also \cite{SNLS}).
The mild (almost 2σ) tension between the Gold04 and the SNLS samples (Figs. 1a and 1b) has not decreased by using the Gold06 sample (Fig. 1c)! The investigation of the origin of this tension and the statistical uniformity of the Gold06 dataset consist the main focus of the present paper.

II. TENSION IN THE GOLD06 DATASET

The 182 SnIa included in the Gold06 dataset originate mainly from the search teams/instruments shown in Table I. The low redshift subset (LR) is a mixture of various early SnIa by different groups and instruments but we consider it as a single subset because otherwise we would have to increase the number of subsets beyond a reasonable number.

In order to investigate the statistical uniformity of the Gold06 dataset and also the origin of the tension with the SNLS, we have decomposed the Gold06 dataset into the subsamples of Table I and constructed new datasets by subtracting each one (or two) of the subsets from the full Gold06 dataset. We thus obtained the following six subset truncations:

1. \[182_{\text{G06}} - 47_{\text{SNLS}} - 30_{\text{HST}}\]
2. \[182_{\text{G06}} - 47_{\text{SNLS}}\]
3. \[182_{\text{G06}} - 30_{\text{HST}}\]
4. \[182_{\text{G06}} - 26_{\text{SCP}}\]
5. \[182_{\text{G06}} - 41_{\text{HZSST}}\]
6. \[182_{\text{G06}} - 41_{\text{HZSST}} - 26_{\text{SCP}}\]

We did not consider the subset \[182_{\text{G06}} - 38_{\text{LR}}\] with low redshift truncation because the LR subset is not uniform and also because subtracting it cannot be associated with a corresponding random truncation in the same low redshift range (the range \(z < 0.124\) is spanned completely by the LR subset). We then addressed the following two questions:

• How do the best fit \((w_0, w_1)\) values for each of the six truncations compare with the corresponding best fit value of the full Gold06 dataset?

• How do the best fit \((w_0, w_1)\) values for each of the six truncations compare with the corresponding best-fit value of a random truncation of the full Gold06 dataset made in the same redshift range as that of the subtracted subset?

The answer to the first question is provided in Fig. 2 where we show the best fit values \((w_0, w_1)\) for each one of the above six truncations. Notice that the two multiple truncations: \[182_{\text{G06}} - 41_{\text{HZSST}} - 26_{\text{SCP}}\] (point 1) and \[182_{\text{G06}} - 47_{\text{SNLS}} - 30_{\text{HST}}\] (point 6) correspond to more extreme best fit values of \((w_0, w_1)\). The best fit \((w_0, w_1)\) of the Gold06 dataset along with its 1σ and 2σ contours is also shown in Fig. 2 (point 0).

The following comments can be made on the basis of Fig. 2:

• The truncation \[182_{\text{G06}} - 26_{\text{SCP}}\] leaves the best fit \((w_0, w_1)\) of the Gold06 dataset practically unchanged

• No single subset truncation is able to shift the best fit \((w_0, w_1)\) values beyond the 1σ contours of the Gold06 dataset.
truncations can be used to obtain the 1σ range for the expected values of the best fit \((w_0^r, w_1^r)\) of the randomly truncated Gold06 dataset.

If the best fit values \((w_0, w_1)\) of the subset truncation is within the 1σ range of the best fit values \((w_0^r, w_1^r)\) of the random truncation then the considered subset truncation is a typical truncation representative of the Gold06 dataset and statistically consistent with it. If on the other hand \((w_0, w_1)\) differs by 2σ or more from the mean best fit values \((w_0^r, w_1^r)\) of the random truncation then the considered subset truncation is not a typical truncation and is systematically different from the full dataset. We have implemented the above comparison for the six subset truncations referred above and the results are shown in Table II and in Fig. 3.

The following comments can be made on the basis of Table II and Fig. 3:

- The SCP is a typical, statistically consistent subset of the Gold06 dataset because its truncation does not significantly alter the statistical properties of the Gold06 dataset. In particular the best fit \((w_0, w_1)\) value of the 182G06 – 26SCP truncation differs only by 0.2σ from the corresponding mean random truncation best fit \((\bar{w}_0^r, \bar{w}_1^r)\) which involves random subtraction of the same number of SNIa from the same redshift range as the SCP subset.

- The other five subsets considered in Fig. 3 are not typical subsets of the Gold06 dataset. The best fit \((w_0, w_1)\) values of the truncations considered in Fig. 3 differ by more than 2σ from the mean best fit values \((\bar{w}_0^r, \bar{w}_1^r)\) of the corresponding random truncations.

- An extreme case is the truncation 182G06 – 47SNLS – 30HST whose best fit values are 3.7σ away from the corresponding mean best fit values of a random truncation! This implies that the combination of the 38LR + 41HZSST + 26SCP which is left over from the truncation 182G06 – 47SNLS – 30HST...
strongly favors an evolving $w(z)$ and is statistically inconsistent with the Gold06 dataset. This result is consistent with Fig. 2 which also shows that best fit $(w_0, w_1)$ of the truncation $182_{\text{G06}} - 47_{\text{SNLS}} - 30_{\text{HST}}$ is about $3\sigma$ away from the Gold06 best fit!

- The $182_{\text{G06}} - 47_{\text{SNLS}}$ and $182_{\text{G06}} - 30_{\text{HST}}$ are statistically very similar to each other (with a trend towards $\Lambda$CDM) even though they are both significantly different (more that $2\sigma$) from the corresponding random truncations of Gold06 (see also Fig. 2).

- Both Figs 2 and 3 indicate that the trend towards $\Lambda$CDM increases for more recent ($\text{HST}$ and $\text{SNLS}$) data while earlier data ($\text{HZSST}$ and $\text{SCP}$) seem to favor and evolving $w(z)$.

The above results can also be verified by considering the ‘pure’ Gold06 dataset which does not include the 47 SnIa of SNLS. This dataset (Gold06p) consists of 135 SnIa and is essentially a filtered version of the Gold04 dataset with the addition of the 16 SnIa with $0.46 < z < 1.39$ discovered recently by the HST. The best fit parameter values for the Gold06p dataset are somewhat shifted in the direction of varying $w(z)$ compared to the full Gold06 (compare Figs. 2 and 4) as expected since SNLS favors $\Lambda$CDM. As shown in Fig. 4 and Table III, the effect of each subset truncation in this case is more prominent due to the smaller number of points in the Gold06p dataset.

For example, the $135_{\text{G06p}} - 41_{\text{HZSST}} - 26_{\text{SCP}}$ truncation shifts the best fit parameter values of the Gold06p by about $3\sigma$ in the direction of $\Lambda$CDM (and beyond it) while the shift with respect to the random truncations of Gold06p is $3.7\sigma$ (Fig. 5). The corresponding shifts with respect to the Gold06 dataset were about $1\sigma$ and $2.6\sigma$ respectively (Figs. 2 and 3).
FIG. 5: Comparison of the best fit parameters to the subsample truncations shown in Table III with corresponding random truncations of the Gold06p dataset. In all truncation cases (except of the SCP truncation) the best fit parameter values are shifted (in different directions) by more than $\sigma$ from the mean random truncation values. The best fit parameter shift of the $135_{\text{Gold06p}} - 41_{\text{HZSST}} - 26_{\text{SCP}}$ is $3.7\sigma$ compared to the corresponding random truncation. The point corresponding to $\Lambda$CDM ($w_0 = -1, w_1 = 0$) is also shown.

### III. DISCUSSION-CONCLUSION

The fact that more recent SNIa data (HST and SNLS) seem to favor $\Lambda$CDM significantly more than earlier data (HZSST) makes it possible that earlier data may be more prone to systematic errors. It is therefore interesting to identify a small subset of SNIa from the HZSST data that is mostly responsible for the trend of HZSST towards an evolving $w(z)$. We have isolated the group of SNIa in the HZSST subset whose distance modulus differs by more than $1.8\sigma$ from the $\Lambda$CDM predictions ($\Omega_m = 0.28$). The group which consists of just six SNIa is also significantly responsible for the trend of the HZSST subset towards an evolving $w(z)$. These SNIa are: (SN99Q2, SN00ee, SN00ec, SN99S, SN01fo, SN99fv). The shifted best fit parameter values ($w_0, w_1$) due to these six SNIa data truncation are shown in Fig. 6a superposed on a Monte-Carlo simulation of corresponding random 6 point truncations to the HZSST subset. We anticipate that the possible systematic errors that lead to the distinct behavior of the HZSST subset are maximal for these six SNIa and it may be easier to identify them and correct them in this set of six SNIa. Alternatively these 6 SNIa could be discarded from the Gold06 dataset as outliers in an effort to improve its statistical uniformity and bring it to line with the more recent data.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>$w_0$</th>
<th>$w_1$</th>
<th>$w_0^C$ (MC)</th>
<th>$w_1^C$</th>
<th>$w_{w} - \omega_{w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$135 - 30_{\text{HST}}$</td>
<td>$-2.21$</td>
<td>$7.53$</td>
<td>$-1.63\pm0.17$</td>
<td>$3.98\pm0.97$</td>
<td>$-3.6\sigma$</td>
</tr>
<tr>
<td>$135 - 26_{\text{SCP}}$</td>
<td>$-1.75$</td>
<td>$4.52$</td>
<td>$-1.63\pm0.17$</td>
<td>$3.91\pm0.75$</td>
<td>$-0.8\sigma$</td>
</tr>
<tr>
<td>$135 - 41_{\text{HZSST}}$</td>
<td>$-1.20$</td>
<td>$1.90$</td>
<td>$-1.60\pm0.21$</td>
<td>$3.76\pm0.95$</td>
<td>$+1.9\sigma$</td>
</tr>
<tr>
<td>$135 - 41_{\text{HZSST}} - 26_{\text{SCP}}$</td>
<td>$-0.42$</td>
<td>$-1.83$</td>
<td>$-1.67\pm0.37$</td>
<td>$3.95\pm1.55$</td>
<td>$+3.7\sigma$</td>
</tr>
</tbody>
</table>

TABLE III: The four subset truncations of Fig 5.
FIG. 6: a. The best fit parameter values for the Gold06p dataset for random 6 point truncations from the HZSST subset. The parameter shift is maximized at 3.9σ when the following six points are truncated: (SN99Q2, SN00ee, SN00ec, SN99S, SN01fo, SN99fv) (red dot). These are also the points whose distance modulus differs by more than 1.8σ from the ΛCDM predictions. b. The best fit distance modulus (dashed line) relative to ΛCDM and the data of the Gold06 dataset in the redshift range of the HZSST subset. The six points of the HZSST subset which differ from ΛCDM by more than 1.8σ are colored in red. They are also the most favorable points for an evolving $w(z)$.

A visual display of the six datapoints (points in red) compared to other datapoints is shown in Fig. 6b where we show the distance modulus relative to ΛCDM ($\Omega_m = 0.28$) of the Gold06 data in the redshift range of the HZSST subset. In the same plot we show (thick dashed line) the distance modulus corresponding to the best fit values ($w_0 = -1.62$, $w_1 = 3.95$) obtained from the Gold06p data (dashed line) indicating that all of the six red datapoints strongly favor the best fit $w(z)$ over ΛCDM.

In conclusion we have demonstrated that despite the careful filtering and the improved calibration, the Gold06 dataset is plagued with statistical inhomogeneities which are possibly due to systematic errors. Given the fact that the more recent data (SNLS and HST) are statistically consistent with each other and homogeneous, it is highly probable that the possible source of systematic errors lies within the earlier data and in particular in the HZSST subset.

**Numerical Analysis:** The mathematical files and the datafile used in the numerical analysis of this work may be found at [http://leandros.physics.uoi.gr/gold06/gold06.htm](http://leandros.physics.uoi.gr/gold06/gold06.htm).

**Acknowledgements**

This work was supported by the European Research and Training Network MRTNP-CT-2006 035863-1 (UniverseNet). SN acknowledges support from the Greek State Scholarships Foundation (I.K.Y.).

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TABLE IV: The Gold06 dataset with its subsets. The six outliers of the HZSST subset are denoted by a ∗.

<table>
<thead>
<tr>
<th>SN</th>
<th>$z$</th>
<th>$\mu_0$</th>
<th>$\sigma_{\mu_0}$</th>
<th>Subsample</th>
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<tbody>
<tr>
<td>SN03D1au</td>
<td>0.504</td>
<td>42.61</td>
<td>0.17</td>
<td>SNLS</td>
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<tr>
<td>SN03D1aw</td>
<td>0.582</td>
<td>43.07</td>
<td>0.17</td>
<td>SNLS</td>
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<tr>
<td>SN03D1ax</td>
<td>0.496</td>
<td>42.36</td>
<td>0.17</td>
<td>SNLS</td>
</tr>
<tr>
<td>SN03D1cm</td>
<td>0.870</td>
<td>44.28</td>
<td>0.34</td>
<td>SNLS</td>
</tr>
<tr>
<td>SN03D1co</td>
<td>0.679</td>
<td>43.58</td>
<td>0.19</td>
<td>SNLS</td>
</tr>
<tr>
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<td>41.13</td>
<td>0.17</td>
<td>SNLS</td>
</tr>
<tr>
<td>SN03D1fl</td>
<td>0.688</td>
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<td>0.17</td>
<td>SNLS</td>
</tr>
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</tr>
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<td>0.17</td>
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</tr>
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<td>0.17</td>
<td>SNLS</td>
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<td>SN03D4at</td>
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<td>43.32</td>
<td>0.18</td>
<td>SNLS</td>
</tr>
<tr>
<td>SN03D4cx</td>
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<td>43.69</td>
<td>0.32</td>
<td>SNLS</td>
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<tr>
<td>SN03D4eg</td>
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<td>43.21</td>
<td>0.19</td>
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</tr>
<tr>
<td>SN03D4fl</td>
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<td>SNLS</td>
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<tr>
<td>SN03D4fp</td>
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<td>41.96</td>
<td>0.17</td>
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</tr>
<tr>
<td>SN03D4fr</td>
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