Generalized parton distributions of few body systems

S. Scopetta

Dipartimento di Fisica, Università degli Studi di Perugia, via A. Pascoli, 06100 Perugia, Italy and INFN, sezione di Perugia

The relevance of measuring Generalized Parton Distributions (GPDs) for few nucleon systems is illustrated. An approach which permits to calculate the GPDs of hadrons made of composite constituents by proper convolutions is described. The application of the method to the nucleon target, assumed to be made of composite constituents is reviewed. Calculations of GPDs for few nucleon systems are summarized, with special emphasis to the $^3\text{He}$ target.

In the last few years [1], Generalized Parton Distributions (GPDs) have become one of the most relevant issues in Hadronic Physics. GPDs enter the non trivial part of exclusive lepton Deep Inelastic Scattering (DIS) off hadrons and can be measured, e.g., in Deeply Virtual Compton Scattering (DVCS), i.e. the process $eH \rightarrow e'H'\gamma$ when $Q^2 \gg m_H^2$ (here and in the following, $Q^2$ is the momentum transfer between the leptons $e$ and $e'$, and $\Delta^2$ the one between the hadrons $H$ and $H'$) so that experimental DVCS programs are taking place. As observed in [2], the knowledge of GPDs of nuclei would permit the study of their short light-like distance structure, and thus the interplay of nucleon and parton degrees of freedom. In DIS off a nucleus with four-momentum $P_A$ and $A$ nucleons of mass $M$, this information can be accessed in the region where $Ax_{Bj} \simeq Q^2/(2M\nu) > 1$, being $x_{Bj} = Q^2/(2P_A \cdot q)$ and $\nu$ the energy transfer in the laboratory system, but measurements are difficult, because of vanishing cross-sections. As explained in Ref. [2], the same physics can be accessed in DVCS at lower values of $x_{Bj}$ [3]. In this talk, a method is reviewed for calculating the GPDs of hadrons made of composite constituents, in an Impulse Approximation (IA) framework. In this scheme, GPDs are given by convolutions between the light cone non-diagonal momentum distribution of the hadron and the GPD of the constituent. Results are presented for the nucleon and for the $^3\text{He}$ nucleus. The simplest GPD is the unpolarized one, $H_q(x, \xi, \Delta^2)$. Usually one works in a system of coordinates where the photon 4-momentum, $q^\mu = (q_0, \vec{q})$, and $\vec{P} = (P + P')/2$ are collinear along $z$, $\xi$ is the so called “skewness”, defined by the relation $\xi = -n \cdot \Delta/2 = -\Delta^+/(2P^+ = x_{Bj}/(2 - x_{Bj}) + O(\Delta^2/Q^2)$, where $n$ is a light-like 4-vector with $n \cdot \vec{P} = 1$. One should notice that $\xi$ can be fixed experimentally. The constraints of $H_q(x, \xi, \Delta^2)$ are: i) the “forward” limit, $\Delta^2 = \xi = 0$, where one recovers the usual parton distribution; ii) the integration over $x$, giving the contribution of the quark of flavor $q$ to the Dirac form factor; iii) the polynomiality property, involving higher moments of GPDs. In Ref. [4], an IA expression for $H_q$ of a given hadron target $A$ has been obtained. Assuming that the interacting parton belongs to a bound constituent $N$
Figure 1. Left panel: the valence quark parton distribution in a relativistic model for a quark-antiquark system (dashed) and its non relativistic (NR) limit (full); once a structure is introduced for the constituents, the relativistic and NR models give the long dashed and dot-dashed results, respectively. Right panel: The GPD \( H^{u,NS}(x, \xi, \Delta^2) \) for the flavor \( u \), evolved from \( \mu_0^2 = 0.34 \text{ GeV}^2 \) to \( Q^2 = 10 \text{ GeV}^2 \), for \( \Delta^2 = -0.1 \text{ GeV}^2 \) and \( \xi = 0.1 \).

with momentum \( p \) and removal energy \( E \), for small values of \( \xi^2 \) and \( \Delta^2 \ll Q^2, M^2 \), it reads:

\[
H^A_q(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} P^A_N(\vec{p}, \vec{p}+\vec{\Delta}, E) \frac{\xi'}{\xi} H^N_q(x', \xi', \Delta^2).
\]  

(1)

In the above equation, the kinetic energies of the residual system and of the recoiling target have been neglected, \( P^A_N(\vec{p}, \vec{p}+\vec{\Delta}, E) \) is the one-body off-diagonal spectral function for the constituent \( N \) in the target \( A \), the quantity \( H^N_q(x', \xi', \Delta^2) \) is the GPD of the bound constituent up to terms of order \( O(\xi^2) \), and \( \xi' = -\Delta^+ / 2\vec{p}^+ , \ x' = (\xi'/\xi)x \). In Ref. [4], it is discussed that Eq. (1) fulfills the constraints i) – iii) listed above. This formalism has been applied in Ref [4] to the nucleon target. The spectral function of the composite constituent quarks has been approximated by a momentum distribution, calculated within the Isgur and Karl model [5], convoluted with the GPDS of the constituent quarks themselves. The latter are modeled by using the structure functions of the constituent quark, obtained generalizing to the GPDs case the approach of [6] which is, in turn, built following Ref [7], the double distribution representation of GPDs [8], and a recently proposed constituent quark form factor [9]. In ref. [10], an analysis has been done to show that, in calculating parton distributions from quark models, as it is done here, the role played by the structure of the constituent quarks is different from that arising from relativistic effects. In particular, the structure parameterized following Ref. [7], improves the description of the low-x behavior, while relativistic effects governs the high-x tail of
Figure 2. The GPD $H_3^{3q}$, for $\Delta^2 = -0.15$ GeV$^2$, as a function of $x_3 = 3x$, for the allowed values of $\xi_3 = 3\xi$. Results for the flavors $u$ and $d$ are shown in the left and right panels, respectively.

The data. A typical example of this trend is shown in the left panel of Fig.1. Details can be found in Ref. [10]. Results for GPDs have been discussed in Ref. [4] for the helicity-independent and helicity independent GPDs. The model has been built to be valid at the so-called hadronic scale, $\mu_0^2 = 0.34$ GeV$^2$, and in the unpolarized case also the NLO QCD evolution of the results up to experimental scales has been discussed. In the right panel of Fig.1, typical results are shown for $H_q(x, \xi, \Delta^2)$. This approach permits to access the so-called ERBL region, difficult to study within quark models. To this aim, another approach has been recently proposed, adding a meson cloud to a light-front quark model scenario [11].

Let us discuss now the GPDs for nuclear few-body systems. The deuteron target has been studied carefully [2, 3]. In the coherent, no-break-up channel, at low values of $\Delta^2$, the differential cross section for exclusive photon production off the deuteron target has been found to be comparable to that obtained for the proton target, demonstrating the possibility of its measurement. The study of GPDs for $^3$He is very interesting, since for $^3$He realistic studies are possible, so that conventional effects can be distinguished from the exotic ones. Besides, $^3$He is an effective polarized free neutron [12] and it will be a natural target to study the helicity-dependent GPDs of the free neutron. In what follows, the results will be reviewed of an IA calculation of $H_3^{3q}$, Eq. (1) for the quark of flavor $q$ of $^3$He [13]. Use has been made of a realistic non-diagonal spectral function, so that Fermi motion and binding effects are rigorously estimated. The scheme is valid for $\Delta^2 \ll Q^2, M^2$ and it permits to calculate GPDs in the kinematical range relevant to the coherent channel of DVCS off $^3$He. $H_3^{3q}$ has been evaluated in the nuclear Breit Frame. The non-diagonal spectral function appearing in Eq. (1) has been calculated along the lines of Ref. [14], by
The other ingredient in Eq. (1), i.e. the nucleon GPD $H_N^{q'}$, has been modelled in agreement with the Double Distribution representation [8]. In this model, whose details are found in Ref. [13], the $\Delta^2$-dependence of $H_N^{q'}$ is given by $F_q(\Delta^2)$, i.e. the contribution of the quark of flavor $q$ to the nucleon form factor. Typical numerical results are shown in Fig. 2. Anyway, the main result of this investigation is not the size and shape of the obtained $H_N^{q'}$ for $^3$He, but the nature of nuclear effects on it. This permits to test the accuracy of prescriptions proposed to estimate nuclear GPDs [3]. In [13] nuclear effects are thoroughly discussed. Some general trends of them can be summarized as follows: i) nuclear effects, for $x_3 \leq 0.7$, are as large as 15% at most; ii) Fermi motion and binding have their main effect for $x_3 \leq 0.3$, at variance with what happens in the forward limit; iii) nuclear effects increase with increasing $\xi$ and $\Delta^2$, for $x_3 \leq 0.3$; iv) nuclear effects for the $d$ flavor are larger than for the $u$ flavor. In general, it is found that the realistic calculation yields a rather different result with respect to a simple parameterizations of nuclear GPDs, as some of the ones proposed in Ref. [3]. In Ref. [15], it is shown that nuclear effects are found to depend also on the choice of the NN potential, at variance with what happens in the forward case. The study of nuclear GPDs turns out therefore to be very fruitful, being able to detect relevant details of the nuclear structure at short light-cone distances. The obtained $^3$He GPDs have now to be used to estimate cross-sections. Calculations for the helicity dependent channels are in progress.

REFERENCES