A quantum field theory of simplicial geometry and
the emergence of spacetime

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Abstract. We present the case for a fundamentally discrete quantum spacetime and for Group
Field Theories as a candidate consistent description of it, briefly reviewing the key properties
of the GFT formalism. We then argue that the outstanding problem of the emergence of a
continuum spacetime and of General Relativity from fundamentally discrete quantum structures
should be tackled from a condensed matter perspective and using purely QFT methods, adapted
to the GFT context. We outline the picture of continuum spacetime as a condensed phase of a
GFT and a research programme aimed at realizing this picture in concrete terms.

1. Introduction: quantum gravity, discreteness and the problem of the continuum
What is the problem of quantum gravity? The most straightforward and naive answer is that it
is to construct a quantum field theory of the gravitational field, obtained by quantizing somehow
the corresponding classical field theory. However, since the classical theory is General Relativity,
the above answer is unsatisfactory. The key insight of GR is that the gravitational field is to be
understood as spacetime geometry, and to quantize it means understanding what it means to
have a ‘quantum spacetime geometry’. Even more, given the strict relation between spacetime
geometry and topology, it has long been suggested that a quantization of geometry should involve
a dynamical spacetime topology as well. Therefore, a theory of quantum gravity aims to be not
just a theory of a certain physical interaction, but the codification of a new understanding of
what spacetime is at the most fundamental (quantum and microscopic) level [1].

1.1. Spacetime discreteness, the ‘atoms of space’ and the problem of the continuum
One fascinating possibility is that, at high energies and small distances, spacetime loses its
continuum appearance and is instead best described in terms of discrete structures. This
possibility is a basic assumption in some approaches to quantum gravity, e.g. causal set theory
[2], as a convenient computational tool in others, e.g. simplicial quantum gravity [3][4], and it
is the natural (preliminary) outcome of standard quantization techniques applied to GR, e.g. in
Loop Quantum Gravity [5], being also suggested by several results in string theory.

We refer to the literature for all the arguments leading to the hypothesis of spacetime
discreteness (e.g. [5]). Among them, we can mention the many results establishing the laws of
black hole thermodynamics [6]. A thermodynamics is usually the macroscopic encoding of
an underlying microscopic statistical mechanics for the system, i.e. black holes, governing the
dynamics of its microscopic degrees of freedom. The finiteness of black hole entropy further
suggests the discreteness of at least the degrees of freedom constituting the horizon of the black hole (to which the entropy is associated). But a black hole is just a particular region of spacetime, and it is therefore the statistical mechanics of spacetime itself that we are forced to unravel, and express in terms of some unknown microscopic discrete constituents. Another body of arguments suggests a special role for the Planck length \( l_p = \sqrt{\hbar G/c^3} \) in any future theory of quantum gravity [7]. In absence of such a complete theory, of course, such arguments remain just motivations, but certainly the appearance of the Planck length as either a minimal length or as a minimal resolution for geometric measurements would prove the usual description of spacetime as a continuum inadequate at the more fundamental quantum level.

Now, to what extent do existing approaches support a discrete picture of spacetime? What description do they give of the hypothetical building blocks, the ‘atoms of space’?

Kinematical states of the gravitational field in Loop Quantum Gravity [8] are given by spin networks, graphs whose links are labeled by representations of the symmetry group chosen, and whose vertices are labeled by intertwiners of the same group. Thus spacetime as a continuum disappears at the quantum level and is replaced by purely combinatorial and algebraic degrees of freedom. In turn, any spin networks can be thought of as built from the composition of their individual vertices (better, of open spin networks with a single vertex each) at the combinatorial level, and of the corresponding intertwiners at the algebraic level. So one could say that, as far as LQG is concerned, the ‘atoms of space’ are group intertwiners or spin network vertices. Let us consider now simplicial quantum gravity approaches, by which we mean both quantum Regge calculus [1] and dynamical triangulations [3]. While the way these two approaches encode the geometric degrees of freedom of spacetime is different, in both cases spacetime is represented by a simplicial complex, at the quantum level, and a continuum D-dimensional spacetime emerges from the dynamics of a (infinite) collection of (D-1)-simplices, which can be interpreted as the ‘atoms of space’. In the most studied spin foam models [9], strictly related to GFTs, the atoms of space are spin networks, as in LQG, but the graph on which these spin networks are based is dual to a simplicial complex; thus these atoms can be represented, in spin foam models, and GFTs, both as group intertwiners and as fundamental simplices.

In all the above mentioned approaches, just as in any current discrete approach to quantum gravity, the outstanding problem is that of the continuum: given a fundamental description of spacetime as made out of discrete quantum building blocks, and a well-defined quantum dynamics for them, can we show that this dynamics leads to a continuum description of spacetime, and is effectively described by General Relativity (or some modification of it) in some (low energy/large scale) approximation? How does the continuum emerge from a fundamentally discrete picture? How does the dynamics of General Relativity emerge from a microscopic dynamical theory that does not refer to the continuum at all, and likely neither does refer to geometry in the first place? This entails a kinematical question, that of the approximation of the fundamentally discrete structures with continuum ones, but also as a dynamical process, in which the evolution of the fundamentally discrete system leads, under certain conditions and to some approximation, to a configuration where the continuum description is possible. The same problem is faced, although of course it manifests itself and it is tackled in a different way, in all discrete approaches. Each of them has developed techniques and ideas for tackling this problem: from weave states and ‘statistical geometry’ techniques in LQG [8, 10], to refinement and lattice renormalization group techniques in simplicial quantum gravity [3], to the large body of methods of causal set theory [2]. In spite of this intense activity, and also of the many promising results (e.g. [9]), it is fair to say that the problem of the continuum remains wide open.

1.2. An emergent spacetime? The condensed matter analogy

Concerning the problem of the continuum in quantum gravity, we feel that the results obtained in condensed matter analogs of gravitational phenomena [11, 12] are very inspiring, and suggest
a perspective that is particularly suited for group field theories. Let us summarize some of these results. We will be of course very sketchy, and also consider only one example of a condensed matter system that is of interest for gravitational physics: $^3He$. We refer to the vast literature on the subject, and in particular to [11], for a more extensive, detailed and competent review.

$^3He$ is made of fermionic atoms. The ‘fundamental’ theory at work for this system, its own ‘theory of everything’[12], is therefore an interacting relativistic field theory of fermions. Such description is however useless for all practical purposes, and the system is perfectly described in the non-relativistic approximation. At temperatures well above the superfluid transition temperature $T_c$, the system is a gas; as the temperature drops, but remaining above $T_c$, the atoms condense to a liquid phase. It can be described by the action:

$$S(\psi) = \int \, dt d^3x \left[ \bar{\psi}_{\alpha} \left( i \partial_t - \frac{p^2}{2m} + \mu \right) \psi_{\alpha} \right] + S_{int}$$

where $\psi$ is the fundamental field operator for the $^3He$ atoms with mass $m$, $\mu$ is the chemical potential and $S_{int}$ is the quartic term describing the pair interaction of the atoms. It is a system with a Fermi surface, described by a very large number of degrees of freedom. As the temperature drops further, however, the number of degrees of freedom is drastically reduced and the system can be described solely in terms of non-interacting fermionic quasi-particles above the Fermi surface. This effective theory is valid for temperatures well below the Fermi temperature $\Theta = \frac{\hbar^2}{m a^2}$ ($a$ is the average inter-particle distance in the liquid), which plays the role of the ‘Planck energy’, and can be dealt with in detail using the BCS methods [11]. At this level we can use an approximate hydrodynamic description of the liquid, expressing its dynamics in terms of collective liquid variables, e.g. the velocity field $v_s(x)$, and quasi-particles degrees of freedom, i.e. collective propagating excitations of the liquid, while the liquid itself is well-approximated as a continuum. The BCS treatment shows that the system belongs to the universality class of Fermi systems with a Fermi point (in the so-called ‘A-phase’), and the quasi-particles possess, in the vicinity of the Fermi point, the effective ‘relativistic’ dispersion relation:

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0$$

in which one has introduced an effective metric and an effective electromagnetic field, as seen by the propagating quasi-particles. We are concerned here only with the effective metric, given by the collective variables of the liquid-continuum: the superfluid velocity $v_s^i$ and the $l$-field, which measures the vorticity of $^3He - A$, or, in geometric terms, the anisotropy of space, as:

$$g_{ij} = \frac{1}{c_\parallel^2} l^i l^j + \frac{1}{c_\perp^2} (\delta^{ij} - l^i l^j) \quad g_{00} = -(1 - g_{ij} v_s^i v_s^j) \quad g_{0i} = -g_{ij} v_s^j \quad \sqrt{-g} = \frac{1}{c_\parallel c_\perp},$$

where the parameters $c_\parallel$ and $c_\perp$ arise from the microscopic physics [11], and play the role of the velocity of light in the direction parallel and orthogonal to $l$ respectively. The effective action for the collective variables is composed of various terms, some of them depending on the superfluid velocity field $v_s$, others depending on the $l$-field instead. In general, the terms depending on $v_s$ do not have an interpretation in terms of a gravitational theory and do not relate easily to General Relativity; at the same time they tend to dominate over those, depending on the $l$-field, that do have such interpretation. However, these terms are suppressed in the limit of heavy atoms, since $v_s \sim 1/m$, i.e. the limit of inert vacuum. In this limit, together with other terms which have an electromagnetic interpretation, we are left with an effective action proportional to: $\int \, d^3x \left( l \cdot (\nabla \times l) \right)^2$, which in turn is nothing but $\frac{\hbar c_\parallel}{4 c_\perp} \int \sqrt{-g} R(g)$, i.e. the Einstein action, for the effective metric $g_{\mu\nu}(l)$ in which the $v_s$-dependent terms have been dropped, and the effective Newton constant can be expressed in terms of the microscopic parameters of the superfluid.
Let us summarize. We have a discrete system whose building blocks are fermionic atoms and a microscopic field theory describing its dynamics. At low temperature, the system undergoes a phase transition and condenses to a liquid phase. In this phase most of the microscopic details are irrelevant (not all of them, of course; e.g. if the system was a made of atoms living in 6 space dimensions, say, the collective velocity of the fluid would be 6-dimensional, or in $^4$He vorticity is absent and a different class of metrics emerge, etc), and one can adopt an effective hydrodynamic description in terms of a continuum, and of collective variables describing the fluid, together with low energy excitations (quasi-particles) propagating in it. The microscopic theory provides the values of the effective ‘fundamental constants’of the macroscopic theory. The collective variables of the liquid behave like an effective metric field (and other interaction fields as well), and their dynamics admits a description, at least in some limit, in terms of the Einstein action. The theory is thus approximately generally covariant (internal symmetries emerge as well). This behaviour is only approximate (preferred directions can be identified) and only valid at very low temperature; moreover, the class of metrics that emerge is large but does not include all the configuration space of General Relativity. These limitations are a consequence, ultimately, of the non-relativistic nature of the fundamental atomic system considered, and of other details of it. These limitations notwithstanding, we have a concrete example of how: 1) the continuum can be understood as a convenient, if not necessary, approximation of a fundamentally discrete system; 2) spacetime and of geometry can emerge from a theory that is not about spacetime nor geometry nor gravity; 3) General Relativity itself can emerge as an effective description of the (hydro-)dynamics of the collective continuum variables of a microscopically discrete system.

How these results should change our views about spacetime and gravity is of course a matter of debate [11,12]. It seems to us that these results fit nicely with the point of view outlined in the beginning, which sees a continuum spacetime as an approximation of some yet to be discovered ‘atoms of space’, described by a theory that is not expressed in terms of a pre-existing spacetime to start with, and from which General Relativity emerge in a low energy or macroscopic limit. The question then is not of principles, but very practical: can we make a more direct use of the insights provided by condensed matter systems? can we identify a context in which these ideas and techniques can be directly applied? We will argue below that Group Field Theories can represent such a context, and outline a research programme aimed at implementing the tools and ideas of condensed matter gravity analogs in quantum gravity, solving in the process the outstanding difficulties of existing discrete quantum gravity approaches.

2. Group Field Theories

We now give a very brief sketch of Group Field Theories, referring to the literature for more details [14,15]. GFTs can be seen, on the one hand, as a generalization of matrix models for 2d quantum gravity to higher dimensions, and on the other hand as a complete definition of spin foam models in that they provide a natural prescription for the sum over spin foam 2-complexes that is necessary to capture in full the dynamics of these models; as such they share ideas and mathematical structures with both simplicial approaches and Loop Quantum Gravity. GFTs can also be understood as a local and discrete realization of the idea of a 3rd quantization of gravity, including a sum over topologies alongside a covariant sum over geometries for given topology. For GFTs aimed at describing D-dimensional quantum gravity, the field is a $\mathbb{C}$-valued function of D group elements $\phi(g_1,...,g_D)$, for a generic group $G$ being either the $SO(D-1,1)$ Lorentz group (or $SO(D)$ for Riemannian gravity), or some extension of it. It can be interpreted as a second quantized (D-1)-simplex, and each argument corresponds to one of its boundary (D-2)-faces. The closure of the D (D-2)-faces to form a (D-1)-simplex is encoded in the invariance under diagonal action of the group $G$: $\phi(g_1,...,g_D) = \phi(g_1g_1,...,g_Dg_D)$. The mode expansion gives:

$$\phi(g_i) = \sum_{J_i,A,k_i} \phi^{J_iA}_{k_i} \prod_i D^{J_i}_{k_i}(g_i) C^{J_i...J_D}_{A}$$,
with the $J$’s labeling representations of $G$, the $k$’s vector indices in the representation spaces, and the $C$’s being intertwiners of the group $G$, labeled by an extra parameter $\Lambda$. Group variables represent configuration space, while the representation parameters label the corresponding momentum space. Geometrically, the group variables represent parallel transport of a connection along elementary paths dual to the (D-2)-faces, while the representations $J$ can be put in correspondence with the volumes of the same (D-2)-faces, the details of this correspondence depending on the specific model. A simplicial space built out of $N$ such (D-1)-simplices is then described by the tensor product of $N$ field operators, with suitable contractions implementing the fact that some of the (D-2)-faces are identified. States of the theory are then labeled, in momentum space, by spin networks of the group $G$. Spacetime, represented by a D-dimensional simplicial complex, emerges in the perturbative expansion of the GFT partition function, as a particular interaction process among (D-1)-simplices, i.e. as a GFT Feynman diagram. The action is chosen, with this goal in mind, to be of the form:

$$S = \frac{1}{2} \int dg_i d\tilde{g}_i \phi(g_i) K(g_i \tilde{g}_i^{-1}) \phi(\tilde{g}_i) + \frac{\lambda}{(D + 1)!} \int dg_{ij} \phi(g_{ij})...\phi(g_{D+1j}) V(g_{ij} g_{ji}^{-1}),$$

where the choice of kinetic and interaction functions $K$ and $V$ define the specific model. The interaction term describes the interaction of D+1 (D-1)-simplices to form a D-simplex by gluing them along their (D-2)-faces (arguments of the fields). The nature of this interaction is specified by the choice of function $V$. The kinetic term involves two fields each representing a given (D-1)-simplex seen from one of the two D-simplices (interaction vertices) sharing it, so that the choice of kinetic functions $K$ specifies how the geometric degrees of freedom corresponding to their D (D-2)-faces are propagated from one vertex of interaction (fundamental spacetime event) to another. A GFT is an almost ordinary field theory, with a fixed background metric structure and the usual splitting between kinetic (quadratic) and interaction (higher order) term in the action, allowing for a straightforward perturbative expansion. However, the action is also non-local in that the arguments of the D+1 fields in the interaction term are not all simultaneously identified, but only pairwise. Most of the work up to now has focused on the perturbative aspects of quantum GFTs, using the expansion in Feynman diagrams of the partition function:

$$Z = \int D\phi e^{-S[\phi]} = \sum_{\Gamma} \frac{\Lambda^N}{\text{sym} [\Gamma]} Z(\Gamma),$$

where $N$ is the number of interaction vertices in the Feynman graph $\Gamma$, $\text{sym}[\Gamma]$ is a symmetry factor for the graph and $Z(\Gamma)$ the corresponding Feynman amplitude. Each edge of the Feynman graph is made of $D$ strands, one for each argument of the field, and each one is then re-routed at the interaction vertex, following the pairing of field arguments in the vertex operator. Each strand goes through several vertices, coming back where it started, for closed Feynman graphs, and therefore identifies a 2-cell. Each Feynman graph $\Gamma$ is then a collection of 2-cells (faces), edges and vertices, i.e. a 2-complex, that, because of the chosen combinatorics for the arguments of the field in the action, is topologically dual to a D-dimensional simplicial complex. Clearly, the resulting complexes/triangulations can have arbitrary topology, each corresponding to a particular scattering process of the fundamental building blocks of space, i.e. (D-1)-simplices. In momentum space, each Feynman graph is given by a spin foam (a 2-complex with faces labeled by representation variables), and each Feynman amplitude (a complex function of the representation labels) by a spin foam model: $Z(\Gamma) = \sum_{\{J_f\}} A(\{J_f\})$. The representation variables have a geometric interpretation (edge lengths, areas, etc) and so each of these Feynman amplitudes defines a sum-over-histories for discrete quantum gravity on the specific triangulation dual to the Feynman graph. At the same time, this sum over geometries is generated within a sum over simplicial topologies corresponding to the perturbative sum over Feynman diagrams.
The transition amplitude between certain boundary data represented by two spin networks, of arbitrary combinatorial complexity, can be expressed as the expectation value of the field operators having the same combinatorial structure of the two spin networks. Moreover, the restriction of the GFT perturbative expansion to tree level, involving then only classical GFT information and generating simplicial spacetimes of trivial topology only, for given boundary spin networks, can be considered [15] as the GFT definition of the canonical inner product, implementing the action of the Hamiltonian constraint operator on spin network states.

Most of the model building has been based on the description of classical gravity as a topological BF theory in 3d, or as a constrained BF theory in higher dimensions, and leads to very simple GFT models, with kinetic and vertex terms given just by a product of delta functions over the group $G$ or over suitable homogeneous subspaces of it:

$$
\mathcal{K}(g_i, \tilde{g}_i) = \int_G dg \prod_i \delta(g_i \tilde{g}_i^{-1} g), \quad \mathcal{V}(g_{ij}, g_{ji}) = \prod_i \int_G dg_i \prod_{i<j} \delta(g_i g_{ij} g_{ji}^{-1} g_j^{-1}),
$$

where the integrals impose the $G$-invariance. These models have thus a very simple structure, and can be motivated in more than one way as candidate descriptions of quantum gravity. At the same time, they are rather peculiar field theories, because, on top of their non-locality, they lack the usual derivative operators in the kinetic term.

Much more is known about the general structure of GFTs, and about the various specific GFT models that have been constructed up to now. For all this, we refer again to the literature.

GFTs, just as matrix models in the 2d case, provide thus a picture of a discrete spacetime as emergent as a Feynman graph from a theory that is not about spacetime at all. While remarkable, this is of course not the solution to the problem of the continuum, as outlined above. For this, a different perspective is needed, and we will outline it in the next section.

Before doing so, however, we want to mention another potentially important feature of the GFT formalism. This is the possibility to identify, within the GFT approach, many of the ingredients and structures that are present in other discrete approaches to quantum gravity. Boundary states are spin networks, as in LQG, but at the same time they have a dual description as simplicial spaces, as in simplicial quantum gravity. Their dynamics is expressed as a covariant sum over geometries as in spin foam models, or a discrete gravity path integral, as in quantum Regge calculus, but involves also a sum over inequivalent triangulations, as in the dynamical triangulations approach. The histories summed over are GFT Feynman diagrams, which are directed graphs or pre-orders from a mathematical point of view, interpreted as collection of fundamental spacetime events linked by fundamental causal relations, thus sharing some structures and ideas with the causal set approach to quantum gravity. More links can be found, as well as more work is needed to clarify or strengthen such links, as discussed, for example in [14, 16], but we believe that GFT can be useful in providing bridges between these various approaches and for understanding them from a single, although un-conventional, perspective. A key step toward this goal would be to construct a GFT that has quantum amplitudes having the form of the exponential of a classical action, say the Regge action for simplicial gravity. Work on this is in progress, and a candidate model [16] is based on the following kinetic term:

$$
K = \prod_{i=1}^D \int_G dg_i \int \mathbb{R} ds_i \left\{ \phi^* (g_i, s_i) \left[ \prod_i (-i \partial s_i + \nabla_i) \right] \phi (g_i, s_i) \right\},
$$

where $\nabla$ is the Laplacian on the group manifold $G$, and the $s_i$ are additional real variables. On top of representing, possibly, a unified framework for discrete quantum gravity approaches, this (class of) model(s) is also closer to conventional field theories (even if still non local) for the presence of derivatives in the action, and this makes the analysis of its quantum (e.g. canonical) structure easier, as we will discuss briefly in the following.
3. The emergence of a continuum spacetime from Quantum Gravity: a different perspective and a research programme

3.1. The problem of the continuum from a GFT perspective

Because the GFT formalism incorporates many ingredients that are present in other approaches to quantum gravity, techniques that have been developed to tackle the problem of the continuum within them can be applied in GFT models as well. For example, one can construct spin network weave states that approximate a given semi-classical geometry at the kinematical level, as done in LQG, insert them as boundary states (observables) in an appropriate GFT model, use then the latter to define (using its tree level perturbative expansion) the physical inner product among the corresponding physical states, and finally compute observable quantities that could be compared with the predictions of continuum GR (or to experiments) to test the model. Alternatively, one can use a GFT model with Feynman amplitudes given by the exponential of a classical gravity action, truncate the sum over geometric degrees of freedom in the perturbative expansion of the GFT to reduce it to a purely combinatorial sum over triangulations, possible further reduced to involve only trivial topology (again, the tree level restriction is probably one way to achieve this), and then apply the methods of the dynamical triangulations approach: refinement and renormalization of the free parameters of the GFT model. Finally, one can see the GFT just as a definition of a particular spin foam model, to which one can apply the methods for background independent coarse graining developed in [17]; this last option is particularly revealing, because such methods have been originally developed as a convenient and elegant encoding of perturbative renormalization of ordinary QFTs, and indeed, from the perspective of GFTs, what one is doing when applying those methods to spin foam models is exactly performing perturbative renormalization moves to the corresponding GFTs.

All this is very good, but it also suggests that there is much more in the GFT formalism than what can be seen from the perspective of the other approaches. In particular, the structures that play such a prominent role in these approaches all appear at the perturbative level in GFTs, and there is much more in a quantum field theory that its perturbative expansion and a lot of physics is either not captured at all or non very conveniently described by it. Now let us instead take the GFT formalism seriously. It seems to us that this means one thing: if quantum spacetime is described (to some approximation and in some limited context) by a specific class of GFTs, and if the emergence of a continuum spacetime is a result of some non-perturbative physics acting on a large number of its fundamental quantum discrete constituents, then to work at the GFT perturbative level (in whatever restriction corresponding to one of these other approaches), is not the most convenient thing to do. We should instead change perspective.

We propose to look at GFTs as fundamental quantum field theories for the elementary quantum constituents of space, the fundamental ‘atoms of space’ mentioned in the introduction. These can be seen, in GFTs, as simplices or as intertwiners, as discussed above, and the theory describes their interaction in purely field theoretic terms to form a discrete spacetime at the perturbative level. Just as atoms of matter are described by quantum field theories, these atoms of space would be described by group field theories. Just as atoms of matter (or elementary particles), when their number is limited, can be very well described by perturbative QFT, in terms of Feynman diagrams and retain their discrete appearance, the interactions of a few ‘atoms of space’ is well-dealt with perturbatively in terms of spin foam models, or of others discrete approaches, like simplicial quantum gravity or causal sets, and gives rise to a discrete picture of a quantum spacetime. At the same time, just as when the number of atoms of matter is very large and their temperature is low, they condense to a liquid (or some other condensed) phase and the field theory picture in terms of elementary quanta gives way to an effective hydrodynamic continuum description of the system, the emergence of a continuum
space(time) should be understood as the condensation of a large number of interacting quantum atoms of space to a new condensed phase, in which an hydrodynamic effective description is more appropriate, but that can be deduced (at least in part) from the underlying microscopic field theory. In other words we are advocating a condensed matter picture for quantum gravity, and we are suggesting that GFTs are the appropriate context to realize this picture in detail, by taking them seriously as microscopic field theories for the elementary quantum constituents of space, i.e. the quantum gravity analog of the field theory for \(^3\)He. We will now outline briefly a possible research programme, whose goal is exactly to realize the condensed matter picture we are advocating here and to solve the problem of the emergence of a continuous classical spacetime and of General Relativity as the effective description of its geometry.

3.2. The emergence of spacetime: a research programme

Suppose now that we believe the proposed interpretation of Group Field Theories, that we believe that the fundamental quanta of the theory are indeed elementary atoms of space, and that the continuum is to emerge from their interactions when their number is large, after a phase transition from a ‘gas’phase to a ‘liquid’phase, and that General Relativity is to appear as the effective description of their collective dynamics in this condensed phase. How should we proceed, starting from the GFT action and partition function, once we understand this action and partition function as the quantum gravity analog of \(^3\)He, to be dealt with using condensed matter physics tools and ideas? The answer is obviously statistical field theory, i.e. we should develop and study a statistical group field theory.

There are several techniques that are used in condensed matter physics to deal with systems made of a large number of constituents. One of them is the renormalization group, and in fact one way to study the phase structure of GFTs would certainly be in terms of exact renormalization group transformations. No such study of GFT renormalization has been done until now, neither at the non-perturbative level nor in perturbation expansion, and it would certainly be of great value. This can be done using Lagrangian setting, already available for GFTs. Here, however, we would like to expand a bit more on an alternative strategy: the development and use of a Hamiltonian statistical group field theory. The development of such a picture requires two main steps. 1) *The Hamiltonian formulation of GFTs with the definition of a clear Fock space structure* of the space of quantum states; this has been developed partially in [18] for the generalised GFT models of [16], mentioned above, but that analysis should be completed and then extended to other GFT models. An interesting preliminary result of [18] is that the basic ‘atoms of space’are fermions, which is important because, fermionic condensed matter systems show a richer set of emergent phenomena in the low temperature regime. It has to be checked whether this is confirmed for other GFT models. 2) One needs to *identify the GFT notion of temperature*. Usually this is related to a Wick rotation (euclideanisation) of a time variable. In the generalised models of [16], mentioned above, there is a multiplicity of time variables \(s_i\), a consequence of the non-local nature of GFTs, that is likely to be present in any other model. A Wick rotation of these variables is possible, but it is not obvious that this is what one should look for, and in any case their multiplicity requires a non-trivial adaptation of the traditional techniques. This time multiplicity is also the main difficulty faced in the development of an Hamiltonian formulation of GFTs. Also, if the notion of temperature comes out of some of the variables of the GFT, as it is expected, this would correspond from the quantum gravity point of view to using some of the geometric degrees of freedom as physical temperature. Once an Hamiltonian formulation and a notion of temperature is obtained, the stage is set for the statistical treatment of GFTs. The task will then be to analyze, with the newly developed tools (RG methods and Hamiltonian methods), the phase structure of various GFT models. The goal is to *prove the existence of a liquid or generically condensed phase*, in which only the collective behaviour of the atoms of space is relevant. This will be by definition the phase of the theory where a continuum...
approximation is appropriate, or in other words, where a continuum spacetime emerges. The search for such a phase will be done by studying the properties of the GFT system: i) **varying the number of atoms**; ii) **varying the temperature of the system** (one expects condensation in the low temperature regime); iii) **varying type and strength of the interaction** (which is also related, from the quantum gravity point of view, to the strength of spatial topology changing processes; iv) **in the GFT analogue of both the relativistic and non-relativistic regimes** (from a purely formal point of view, the generalised GFT models mentioned above have a non-relativistic Schroedinger-like kinetic term in the time variables $s_i$); v) **varying of course the type and symmetries of the fields and the group manifold** on which they are defined (thus the specific GFT model considered).

Once the condensed/liquid phase has been found, and thus the issue of the emergence of a continuum spacetime solved, the final goal is to **show the emergence of General Relativity as the hydrodynamic description of the collective variables in this phase**. Here one could build up directly on the insights gained in condensed matter analogues of gravitational phenomena [11].

The first thing to do is to identify and study directly the collective or hydrodynamic variables for the GFT system in the condensed phase. Given the microscopic GFT variables, one of them is likely to be a connection field, which is also the right set of variables for a gauge theory formulation of continuum gravity, like those from which GFT have originated in the first place. The argument for believing so is to look at what type of collective variables emerge in the case of $^3$He, for example: there one has the steps: $\psi(v_i) \rightarrow |v^1_i, v^2_i, ..., v^N_i\rangle \rightarrow v_i(x)$, where the first step is the Fock or multi-particle description of the system following from the microscopic field theory with field $\psi(v_i)$ one starts from, and the last step is the passage to an hydrodynamic continuous velocity field in the liquid phase. The analogues of these steps in the GFT case would be something like: $\phi(g_1, ..., g_D) \rightarrow \phi(g_\mu) \rightarrow |g^1_\mu, g^2_\mu, ..., g^N_\mu\rangle \rightarrow g_\mu(x) \rightarrow A_{\mu}^{IJ}(x)$, where $\mu = 1...D$ and the last step is the correspondence between elementary (infinitesimal) parallel transports and a connection field. Of course the exact nature and number of the collective variables is model-dependent. Having done so, the second step is to **derive the energy functional (hamiltonian) or the Lagrangian** governing the effective dynamics of these collective variables, and compare it with that of General Relativity. The second way to proceed is based on the **analysis of quasi-particles**, i.e. the small excitations above the ground state. One has first to **determine the energy spectrum** of these quasi-particles, which characterizes the *universality class* of the condensed matter system at hand, and can thus give immediate evidence of the possibility of the emergence of gravity in the effective description. One has then to **derive the equations of motion and the effective Lagrangian for quasi-particle dynamics**, in the vicinity of the Fermi points or of the Fermi surface, according to the universality class to which the GFT belongs (if it is a fermionic system); this will serve also to check that the collective variables identified above are indeed seen by the quasi-particle degrees of freedom as an effective metric field. Finally, one can use for example Sakharov method of *induced gravity* [11] and derive the effective action for the collective variables by integrating out the fluctuations of the quasi-particles modes, and once more compare it with that of General Relativity.

4. Conclusions

We have outlined above one possible strategy to solve the problem of the continuum in Quantum Gravity using Group Field Theory methods. Other strategies can be followed and different methods from those proposed above can of course turn out to be more suited to the task and more effective. Only future work can tell. The only message that we think it is useful to put forward, even if only as a suggestion, at the present stage, is the following: we may have already found the right (class of) microscopic theory (theories) of the fundamental constituents of a quantum spacetime. We have in fact at our disposal a quantum field theory of simplicial geometry, describing the interaction of elementary building blocks of space, and with a discrete spacetime emerging from this interaction; a large number of approaches converged to this class
of models, thus supporting and motivating them with a variety of partial results and arguments; also, as we have discussed, GFTs incorporate many of the key structures of other approaches to discrete quantum gravity, and can thus also be seen as a framework in which to realize and complete the insights of these other approaches. The analysis of the perturbative structure of these models has already proven to be fruitful and interesting, but the probably much richer non-perturbative structure of them is basically un-explored territory. In particular, we have argued in the present contribution that GFTs may be the right framework in which to realize the idea of spacetime as a condensate and of General Relativity as an emergent effective theory for the collective behaviour of the atoms of space in this phase, i.e. as a geometro-hydrodynamics [12], solving in this way the outstanding problem of the continuum approximation that all current discrete quantum gravity approaches still face. We do not know at present if GFTs can really be all this; once more, only much more future work can tell. For now, we only have to take them seriously and start exploring.