Infrared-suppressed gluon propagator in 4d Yang-Mills theory in a Landau-like gauge

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The infrared behavior of the gluon propagator is directly related to confinement in QCD. Indeed, the Gribov-Zwanziger scenario of confinement predicts an infrared vanishing (transverse) gluon propagator in Landau-like gauges, implying violation of reflection positivity and gluon confinement. Finite-volume effects make it very difficult to observe (in the minimal Landau gauge) an infrared suppressed gluon propagator in lattice simulations of the four-dimensional case. Here we report results for the $SU(2)$ gluon propagator in a gauge that interpolates between the minimal Landau gauge (for gauge parameter $\lambda$ equal to 1) and the (complete) minimal Coulomb gauge (corresponding to the limit $\lambda \to 0$). We find that — sufficiently close to the (complete) minimal Coulomb gauge — the spatially-transverse gluon propagator, considered as a function of the spatial momentum $|\vec{p}|$, is clearly infrared suppressed. This result is in agreement with the Gribov-Zwanziger scenario and with previous numerical results in the (complete) minimal Coulomb gauge. We also discuss the nature of the limit $\lambda \to 0$ and its relation to the standard Coulomb gauge ($\lambda = 0$). Our findings are corroborated by similar results in the three-dimensional case, where the infrared suppression is observed for all considered values of $\lambda$.

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I. INTRODUCTION

The infrared behavior of the gluon propagator is linked to the confinement of gluons [1]. In particular, the confinement scenario of Gribov and Zwaniger [2, 3, 4, 5] predicts a (transverse) gluon propagator vanishing at zero (Euclidean) momentum in Landau gauge and in Landau-like gauges (or $\lambda$-gauges). The latter class refers to gauges interpolating between the Landau and the Coulomb gauge [6], with a gauge condition (in d-dimensional space) given by

$$\lambda \partial_0 A_0^a + \partial_1 A_1^a + \ldots + \partial_{d-1} A_{d-1}^a = 0 ,$$ (1)

where the gauge parameter $\lambda$ is between 1 and 0. Let us recall that an infrared (IR) null gluon propagator has far-reaching consequences. Indeed, such a particle cannot have a positive semi-definite spectral function [2] or, as a consequence, a Källen-Lehmann representation. This is regarded as a manifestation of confinement [7, 8], when considering Euclidean correlation functions.1

The question of whether the Landau-gauge gluon propagator is indeed null at zero momentum is a longstanding one. Various continuum methods, based on functional approaches, yield a vanishing gluon propagator [9, 10, 12]. This result is rather tightly constrained [10], i.e. it seems to be the only possible solution satisfying both Dyson-Schwinger equations and functional renormalization-group equations. At the same time, lattice calculations in four dimensions have obtained an IR-suppressed Landau-gauge gluon propagator $D(p)$ only when using strongly-asymmetric lattices [11] or a coupling constant in the strong-coupling regime [12]. For lattice couplings in the scaling region and using symmetric lattices, one finds for the Landau-gauge gluon propagator an increase slower than $1/p^2$ as one approaches the IR region [13], with a finite value for $p = 0$ [14, 15]. The fact that a propagator decreasing at small momenta is not observed in the 4d case, even for volumes of almost $(10 \text{ fm})^4$ [15], is probably due to very strong finite-size effects [14, 15, 16, 17]. This assumption is supported by numerical results in the 3d case, where much larger lattice sides are accessible. In this case, there is substantial evidence for an IR-suppressed gluon propagator in Landau gauge [18, 19, 20], in agreement with continuum calculations [4, 21]. However, also in this case, a reliable extrapolation of $D(0)$ to the infinite-volume limit is still lacking [20].

Let us recall that lattice Landau calculations [16, 22] have also obtained direct evidence for the non-positivity of the gluon spectral function, both in the three- and in the four-dimensional cases. On the other hand, the existence of negative-norm states is also expected due to the Oehme-Zimmermann superconvergence relation [5], i.e. verifying violation of reflection positivity for the gluon field is a necessary (not a sufficient) condition for the validity of the Gribov-Zwanziger scenario.

The Gribov-Zwanziger confinement scenario applies also to Coulomb gauge [2, 23, 24]. In this case it is important to observe that the standard Coulomb-gauge-fixing
condition
\[ \partial_t A^a_\mu + \ldots + \partial_{d-1} A^a_{d-1} = 0 \] (2)
is not a complete one, due to the residual gauge degrees of freedom \( g(t) \). On the other hand, a possible complete Coulomb gauge condition \( \mathbf{1} \) can be obtained using the class of gauges defined in \( \mathbf{1} \). Indeed, the parameter \( \lambda \) interpolates between the Landau (\( \lambda = 1 \)) and a complete Coulomb gauge, corresponding to the limit \( 2 \lambda \rightarrow 0 \). Therefore, the complete Coulomb gauge condition is, by definition, a smooth limiting case of the interpolating gauge \( \mathbf{1} \) while, of course, this is not the case for the standard Coulomb condition (\( \lambda = 0 \)). Let us recall that the gauge condition \( \mathbf{1} \) above can be obtained by minimizing the (lattice) functional\(^3\)
\[ \mathcal{E}[g] = - \text{Tr} \sum_x \left\{ \lambda U_0(x) + \sum_{i=1}^{d-1} U_i(x) \right\} \] , (3)
where \( U_\mu(x) \) indicates a lattice link variable in the \( \mu \) direction. Then, the limiting case \( \lambda \rightarrow 0 \) corresponds to minimizing the following two functionals \( \mathbf{23} \)
\[ \mathcal{E}_{\text{hor}}[g(x)] = - \text{Tr} \sum_x \sum_{i=1}^{d-1} U_i(x) \] (4)
\[ \mathcal{E}_{\text{ver}}[g(t)] = - \text{Tr} \sum_x U_0(x) \] . (5)
The minimization of the first functional is equivalent to a Landau gauge condition fixed on each time slice, using \( g(x) \) gauge transformations, i.e. it corresponds to the standard (incomplete) Coulomb gauge. The minimization of the second functional, considering only \( g(t) \) gauge transformations, provides additional constraints, necessary to eliminate the residual gauge degrees of freedom \( g(t) \). Note that we can also write
\[ \mathcal{E}_{\text{ver}}[g(t)] = - \text{Tr} \sum_t Q_0(t) \] , (6)
with
\[ Q_0(t) = \sum_{\bar{x}} U_0(t, \bar{x}) \] . (7)
Then, the minimization of \( \mathcal{E}_{\text{ver}}[g(t)] \) is like a one-dimensional Landau gauge fixing. Of course, instantaneous quantities, i.e. defined on each time slice, are not affected by the residual gauge condition obtained by minimizing the functional \( \mathcal{E}_{\text{ver}}[g(t)] \).

Numerical studies in minimal Coulomb gauge have shown [for the \( SU(2) \) case in \( 4d \)] that the instantaneous transverse gluon propagator \( D^{tr}(\vec{p}) \) is indeed suppressed in the IR limit \( \mathbf{23, 26, 27} \). Also, in the infinite-volume limit, it has been found \( \mathbf{23, 26} \) that \( D^{tr}(\vec{p}) \) is well described by a Gribov-like propagator with a pair of purely imaginary poles \( m^2 = \pm iy \). These results are in agreement with the Gribov-Zwanziger confinement scenario \( \mathbf{2, 23} \). The fact that, for a given lattice size \( L \) (in fm), one sees an IR-suppressed transverse gluon propagator in \( 4d \)-Coulomb gauge and in \( 3d \)-Landau gauge but not in \( 4d \)-Landau gauge may be related to a quantitatively different IR suppression in the two cases. Indeed, functional methods \( \mathbf{4, 21, 28} \) predict a stronger suppression in \( 4d \)-Coulomb gauge and in \( 3d \)-Landau gauge than in \( 4d \)-Landau gauge, i.e. the so-called IR gluon exponent \( \alpha_P \) should be larger for the \( 4d \)-Coulomb and the \( 3d \)-Landau cases.

One should note that, for any non-zero value of \( \lambda \), the gauge condition \( \mathbf{1} \) is essentially a deformed Landau gauge, i.e. the infrared exponents of the propagators do not depend on \( \lambda \) \( \mathbf{29} \). In particular, calculations using Dyson-Schwinger equations suggest \( \mathbf{29} \) that, at momenta sufficiently small compared to a separation momenta \( p_s \), the transverse gluon propagator behaves as in Landau gauge, i.e. all Lorentz and color components of the gluon propagator vanish at zero four-momentum. However, the limit \( \lambda \rightarrow 0 \) can also be thought of as sending to zero the momentum \( p_s \), which separates Coulomb-like from Landau-gauge-like behavior. Indeed, for any finite value \( \lambda \neq 0 \) and given the Landau gauge condition \( \partial_\mu A^a_\mu = 0 \), we can obtain the gauge condition
\[ \partial_0 A^a_0 + \frac{1}{\lambda} \left[ \partial_1 A^a_1 + \ldots + \partial_{d-1} A^a_{d-1} \right] = 0 \] (8)
by using the rescaling\(^4\) \( x_i \rightarrow x_i/\lambda \) for \( i \neq 0 \). This implies, in momentum space, the rescaling \( p_i \rightarrow \lambda p_i \). Thus, if we consider only spatial momenta, we find\(^5\) that \( p_s \) is rescaled to \( \lambda p_s \) and goes to zero when \( \lambda \rightarrow 0 \). As a consequence, one should expect that, for very small values of \( \lambda \), all correlation functions would show a Coulomb-like behavior for momenta \( p > p_s \), with \( p_s \) very small. In particular, the correlation function that corresponds to the transverse (instantaneous) gluon propagator in Coulomb gauge should become more and more infrared suppressed as the parameter \( \lambda \) approaches zero. Verifying this expectation is the aim of this work.

II. NUMERICAL RESULTS

Following Eqs. (8) and (9) in Ref. \( \mathbf{23} \), we can consider on the lattice the three-dimensionally transverse gluonic

\( ^2 \) Clearly, since the gauge fixing \( \mathbf{1} \) is complete for any \( \lambda \neq 0 \), it is also a complete one when considering the limit \( \lambda \rightarrow 0 \).

\( ^3 \) A similar functional can be defined in the continuum.

\( ^4 \) In Ref. \( \mathbf{23} \), a similar rescaling was used to show that Dyson-Schwinger equations for \( \lambda \)-gauges are equivalent to the Landau case for all \( \lambda \neq 0 \).

\( ^5 \) Of course, this simple explanation is correct only at tree-level and can be modified by the renormalization of \( \lambda \).
correlation function
\[ D(t, p) = \langle A^a_i(t, p) A^a_j(t, -p) \rangle / \left( N_c^2 - 1 \right) / d V. \] (10)

Here, \( N_c \) is the number of colors, \( V \) is the d-dimensional lattice volume, Lorentz indices \( i, j \) are summed only over the \( d-1 \) spatial directions and \( A^a_i(p) \) is the d-dimensional Fourier transform of the gluon field. Let us recall that, by considering only spatial momenta (i.e. \( p_0 = 0 \)), the function \( D(t, |\vec{p}|) \), defined by the expression above, is predicted to be infrared suppressed — and vanishing at zero momentum — for all non-zero values of \( \lambda \) in three and in four dimensions [24, 30]. Also, when \( \lambda \) is null, the above definition yields the instantaneous part of the three-dimensional transverse gluon propagator, i.e. [23]

\[ D_{\text{inst}}(|\vec{p}|) = \sum_i \left( \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \right) < A^0_i(t, \vec{p}) A^0_j(t, -\vec{p}) > / \left( N_c^2 - 1 \right) / d V, \] (11)

where now the Fourier transform of the gluon field is evaluated for each time slice. Indeed, when \( \lambda \) is null, the gauge transformations \( g(t) \) are independent of the gauge transformation \( g(\vec{x}) \) and the two sets of transformations commute. Then, the terms in Eqs. (10) that depend explicitly on \( g(t) \) are averaged to zero and one is left with the expression above. Note that this explanation is valid whether the residual gauge freedom \( g(t) \) is fixed or not.

Here we evaluate numerically \( D(t, |\vec{p}|) \) as a function of \( |\vec{p}| \) for \( SU(2) \) Yang-Mills theory in three and in four dimensions for several values of the parameter \( \lambda \). Details of the simulations can be found in [20, 31]. Let us note that for \( \lambda \neq 1 \) the numerical gauge fixing is very similar to the usual Landau gauge fixing [19, 30]. On the other hand, when \( \lambda \) goes to zero one sees [32] that more iterations are needed in order to satisfy a given numerical accuracy for the gauge fixing, in agreement with a recent analytic study [33]. Finally, we did not consider here possible systematic effects related to the breaking of rotational symmetry and to the existence of Gribov copies. The former type of effects is not expected to play a significant role in the infrared region, considered here. As for the latter type of effects, in the infinite-volume limit, averages taken over configurations belonging to the so-called Gribov region \( \Omega \) should coincide with averages obtained by restricting the functional integral to the so-called fundamental modular region \( \Gamma \), whose interior is free of Gribov copies.

Our results are reported in Figure 1. In three dimensions, a well-defined maximum (and thus an infrared suppression) is visible for all values of \( \lambda \). This includes also Landau gauge (\( \lambda = 1 \)), confirming earlier results [18, 19, 20]. As can be seen from the plot, the maximum value attained by the propagator seems to move to larger momenta with decreasing \( \lambda \), going from about 400 MeV in the Landau case [20] to about 600 MeV for the Coulomb case. At the same time, as \( \lambda \) decreases, the...
maximum becomes more visible.

In four dimensions, on the other hand, no discernible peak is visible in Landau gauge. Decreasing $\lambda$, however, leads to a suppression of the propagator in the infrared region. In particular, at the smallest value of $\lambda$ considered, i.e. $\lambda = 1/100$, a maximum is seen also in four dimensions. This maximum is as visible as the one in three dimensions in Landau gauge. Thus for small $\lambda$, just as for Coulomb gauge [23], one sees a maximum of the transverse gluon propagator already for small volumes [30].

III. CONCLUSIONS

We have presented the first direct observation of an infrared-suppressed gluonic correlation function on a symmetric 4d lattice in a Landau-like gauge, with gauge parameter $\lambda$ as in Eq. (1). (Landau gauge is obtained for $\lambda = 1$, while Coulomb gauge corresponds to $\lambda = 0$.) The suppression is seen for sufficiently small $\lambda$. Judging from the results shown for the 3d case, it is conceivable that a similar suppression might be observed for any $\lambda$ if a large enough lattice side is considered. Furthermore, since the limit $\lambda \to 1$ is smooth [24, 22], we expect to see an infrared suppression for sufficiently large lattices also in Landau gauge. Our results provide additional support to the Gribov-Zwanziger scenario of confinement, establishing an infrared suppression of gluonic correlation functions in four dimensions beyond Coulomb gauge.

As said in the Introduction, the infrared exponent $\alpha_D$ is predicted to show a discontinuity in the limit $\lambda \to 0$. This is not seen from our data. However, a reliable check of this prediction can only be obtained if one has control over the infinite-volume and the continuum limits, which we have not yet achieved. A more extensive study would be important in order to get a better understanding about the gauge dependence of correlation functions of confined objects.

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