Might EPR particles communicate through a wormhole?*

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Abstract

We consider the two-particle wave function of an EPR system given by a two dimensional relativistic scalar field model. The Bohm-de Broglie interpretation is applied and the quantum potential is viewed as modifying the Minkowski geometry. In such a way a black hole metric appear in one case and a particular metric with singularities appear in other case, opening the possibility, following Holland, of interpret the EPR correlations as originated by a wormhole effective geometry, through which physical signals can propagate.

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I. INTRODUCTION

There is an increasing interest on the application of the Bohm - de Broglie (BdB) interpretation of quantum mechanics to several areas as quantum cosmology, quantum gravity and quantum field theory, see for example [1][2][3][4][5]. In this work we develop a causal approach to the Einstein- Podolsky-Rosen (EPR) problem i.e. a two-particle correlated system. We attack the problem from the point of view of quantum field theory considering the two-particle function for a scalar field. In the BdB approach it is possible to interpret the quantum effects as modifying the geometry in such a way that the scalar particles see an effective geometry. As a first example we show that a two dimensional EPR model, in a particular quantum state and under a non-tachyonic approximating condition, can exhibit an effective metric that is analog to a two dimensional black hole (BH). In a second example, for a two-dimensional static EPR model we are able to show that quantum effects produces an effective geometry with singularities in the metric, a key ingredient of a bridge construction or wormhole. Following a suggestion by Holland [6] this open the possibility of interpret the EPR correlations as driven by an effective wormhole, through which physical signals can propagate. This letter is organized as follows: in the next section we recall the basic features of a relativistic scalar field and we write the two-particle wave’s equation. Then we applies the BdB interpretation to it and, from the generalized Hamilton-Jacobi equation, we visualize the quantum potential as generating an effective metric. Having done that we next study two dimensional EPR models and we show how the effective metric appears, being a BH metric in a first example and a particular metric with singularities in a second one. Last section is for conclusions.

II. SCALAR FIELD THEORY AND ITS BDB INTERPRETATION

The Schrödinger functional equation for a quantum relativistic scalar field is given by

\[ i\hbar \frac{\partial \Psi(\phi, t)}{\partial t} = \int d^3 x \left\{ \frac{1}{2} \left[ -\hbar^2 \frac{\partial^2}{\partial \phi^2} + \left( \nabla \phi \right)^2 \right] + U(\phi) \right\} \Psi(\phi, t), \tag{1} \]

where \( \Psi(\phi, t) \) is a functional with respect to \( \phi(x) \) and a function with respect to \( t \). A normalized solution \( \Psi(\phi, t) \) can be expanded as
\[ \Psi[\phi, t] = \sum_{n=0}^{\infty} \int d^3k_1...d^3k_n c_n(\vec{K}^n, t) \Psi_{n, \vec{K}^n}[\phi] \] (2)

where \( \vec{K}^n \equiv \{k_1, ..., k_n\} \) and \( k_j \) is the momenta of particle \( j \), being the functionals \( \Psi_{n, \vec{K}^n}[\phi] \) a complete orthonormal basis.

For free fields the \( n \)-particle wave function is given by (see for example [7][8])

\[ \psi_n(\vec{x}^{(n)}, t) = \langle 0 | \hat{\phi}(t, x_1)...\hat{\phi}(t, x_n) | \Psi \rangle, \] (3)

where \( \vec{x}^{(n)} \equiv \{x_1, ..., x_n\} \).

The wave function (3) satisfies

\[ \sum_{j=0}^{n} [(\partial^\mu \partial_\mu)_j + m^2] \psi_n(\vec{x}^{(n)}, t) = 0. \] (4)

For the two-particle wave function we have

\[ \sum_{j=1}^{2} [(\partial^\mu \partial_\mu)_j + m^2] \psi_2(\vec{x}^{(2)}, t) = 0 \] (5)

which is

\[ [(\partial^\mu \partial_\mu)_1 + m^2] \psi_2(x_1, x_2, t) + [(\partial^\mu \partial_\mu)_2 + m^2] \psi_2(x_1, x_2, t) = 0. \] (6)

In order to apply the BdB interpretation we substitute \( \psi_2 = R \exp(iS/\hbar) \) in eq. (6) obtaining two equations, one of them for the real part and the other for the imaginary part. The first equation reads

\[ \partial^\mu \partial_\mu S - m^2 \hbar^2 - \hbar^2 \frac{(\partial^\mu \partial_\mu)_1 R}{R} + \partial^\mu \partial_\mu S - m^2 \hbar^2 - \hbar^2 \frac{(\partial^\mu \partial_\mu)_2 R}{R} = 0 \] (7)

that can be written as

\[ \eta^{\mu_1\nu_1} \partial_{\mu_1} S \partial_{\nu_1} S + \eta^{\mu_2\nu_2} \partial_{\mu_2} S \partial_{\nu_2} S = 2M^2 \] (8)

where

\[ M^2 \equiv m^2 \hbar^2 (1 - \frac{Q}{2m^2 \hbar^2}) \] (9)

with
\[ Q \equiv Q_1 + Q_2 \tag{10} \]

being

\[ Q_1 = -\hbar^2 \frac{(\partial^\mu \partial_\mu) R}{R} \tag{11} \]

and

\[ Q_2 = -\hbar^2 \frac{(\partial^\mu \partial_\mu) R}{R}. \tag{12} \]

The equation that comes from the imaginary part is

\[ \eta^\mu_1 \nu_1 \partial_\mu_1 (R^2 \partial_\nu_1 S) + \eta^\mu_2 \nu_2 \partial_\mu_2 (R^2 \partial_\nu_2 S) = 0 \tag{13} \]

which is a continuity equation.

Equation (8) is the Hamilton-Jacobi equation for a 2-particle system of mass \( 2M \). The term \( Q \) is the quantum potential whose effect can be interpreted as a modification of the system’s mass with respect to its classical value \( 2m \). We see that \( M^2 \) is not positive-definite, feature that is associated to the existence of tachyonic solutions. To overcome this problem one can, for example, choose initial conditions in such a way that a positive \( M^2 \) value is obtained for an initial time. Because continuity equation this will be true for any time.

Now, following an idea proposed by L. De Broglie \[9\] and fruitfully applied to gravity in \[3\] and \[4\], we rewrite the Hamilton-Jacobi equation (8) as

\[ \frac{\eta^\mu_1 \nu_1}{(1 - \frac{Q}{2mh^2})} \partial_\mu_1 S \partial_\nu_1 S + \frac{\eta^\mu_2 \nu_2}{(1 - \frac{Q}{2mh^2})} \partial_\mu_2 S \partial_\nu_2 S = 2m^2 \hbar^2. \tag{14} \]

Here \( \eta^{\mu \nu} \) is the Minkowski metric and we can interpret the quantum effects as realizing a conformal transformation of the metric in such a way that the effective metric is

\[ g_{\mu \nu} = (1 - \frac{Q}{2m^2 \hbar^2}) \eta_{\mu \nu} \tag{15} \]

and eq. (14) can be written as

\[ D_{\mu_1} S D^\mu_1 S + D_{\mu_2} S D^\mu_2 S = 2m^2 \hbar^2 \tag{16} \]

where \( D_\mu \) stands for a covariant differentiation with respect to the metric \( g_{\mu \nu} \) and it was used that \( \partial_\mu S = D_\mu S \) because \( S \) is a scalar.
Then, as was already shown by Shojai et al. in [3], the quantum potential modifies the background geometry giving a curved space-time with metric given by eq. (15). In some sense, according Shojai, space-time geometry have a dual aspect: sometimes looks as (semi-classical) gravity and sometimes looks as quantum effects.

III. TWO DIMENSIONAL MODELS

Two dimensional models have been studied for a long time in order to address subjects as gravitational collapse, black holes and quantum effects. We analyse two dimensional models because some aspects in a low dimensional model have the same behavior as the more realistic four-dimensional models. In this section we are going to show two examples in two dimensions of a two-particle EPR system that exhibit an effective metric as in eq. (15). Because of the singularities of this effective metric it resembles a two dimensional BH type solution, as presented in [4] [10], and this is the key that could allow us to connect the EPR correlations with an effective wormhole geometry. Before present our examples we briefly recall the basic features of that solution.

The two dimensional BH presented in [10] consist of a point particle situated at the origin, with density \( \rho = \frac{M}{2\pi G_N} \delta(x) \), where \( M \) is the mass of the particle and \( G_N \) is the Newton gravitational constant. A symmetric solution of the field equation of this problem without cosmological constant (see [10] section 3) is given by the metric:

\[
ds^2 = -(2M|x| - C)dt^2 + \frac{dx^2}{2M|x| - C}
\]  

(17)

where \( C \) is a constant. The sign of the quantity \( \alpha \equiv 2M|x| - C \) determines the type of region: timelike regions are for positive \( \alpha \) and spacelike regions are for negative \( \alpha \). The points at which \( \alpha(x) = 0 \) are coordinate singularities and locate the event horizons of the space, which for the present case are at:

\[
|x| = \frac{C}{2M}.
\]  

(18)

The horizons only exist if \( C \) and \( M \) are of the same sign. For example for \( C \) and \( M \) positives there are two horizons, at \( x_h = \pm \frac{C}{2M} \), with the source located in a spacelike

\[\text{1 Furthermore low dimensional models appears naturally in effective string theories.}\]
region surrounded by two timelike ones. In particular if $C = 0$ and $M > 0$ there is only one horizon at $x = 0$ surrounded by a timelike region. The metric (17) can be cast in “conformal coordinates” $(t, y)$ (see \[10\] section 3). For example in the case $M, C$ positives, the transformation $x = \frac{C + e^{2My}}{2M}$, for $x \in (\frac{C}{2M}, \infty)$, transform the metric in:

$$ds^2 = e^{2My}(-dt^2 + dy^2).$$  \hspace{1cm} (19)

In the following part of this section we consider two dimensional models for the EPR problem.

First example: non-tachyonic EPR model. For the two-particle system, we obtained a conformal transformation of the metric where the conformal factor is associated with the quantum potential, eq. (15). Here we consider the non tachyonic case i.e. we need to impose the positivity of $\mathcal{M}^2$, and one way to do that is approximating eq. (9) by an exponential, assuming that

$$\mathcal{M}^2 = m^2\hbar^2 \exp(-\frac{Q}{2m^2\hbar^2}).$$  \hspace{1cm} (20)

which differs from Eq (9) at order $\hbar^4$.

Then, with this assumption, we have for the effective metric, eq. (15):

$$g_{\mu\nu} = \exp(-\frac{Q}{2m^2\hbar^2})\eta_{\mu\nu}.$$  \hspace{1cm} (21)

Now we assume that our two dimensional two-particle system satisfies an EPR condition, i.e., their positions $x_1$ and $x_2$ are correlated in such a way that $x_1 + x_2 = constant$ [11]. Then the dependence of the quantum potential $Q$ on coordinates $x_1$ and $x_2$ can be cast as a function of only one coordinate, say $x_1$, and defining $z \equiv x_1$ we can write, with a little abuse of notation,

$$Q = Q(x_1, x_2) = Q(z)$$  \hspace{1cm} (22)

being the line element

$$ds^2 = \exp(-\frac{Q(z)}{2m^2\hbar^2})(-dt^2 + dz^2).$$  \hspace{1cm} (23)
Defining a coordinate transformation from \(z\) to \(y\) by mean \(2My = -\frac{Q(z)}{2m^2\hbar^2}\) and assuming a particular quantum state such that \(\frac{dQ}{dz} = 4Mm^2\hbar^2\) with \(M\) a constant, the line element \(ds^2\) in \((t, y)\) ”conformal coordinates” is

\[
ds^2 = e^{2My}(-dt^2 + dy^2).
\] (24)

Now we can make a coordinate transformation from \(y\) to \(x\) by mean

\[
2Mx = C + e^{2My}
\] (25)

where \(C\) is a constant and \(x \in (\frac{C}{2M}, \infty)\). The line element \(ds^2\) in terms of the \((t, x)\) coordinates is given now by

\[
ds^2 = -(2M|x| - C)dt^2 + \frac{dx^2}{2M|x| - C}
\] (26)

where we made a symmetrical extension for the others values of \(x\) other than \((\frac{C}{2M}, \infty)\) (see [10]). Then we arrived to the same metric defining a two dimensional BH type solution, eq. [17] and, in some sense, we can consider it as an analog model for the particular EPR problem analysed. The coordinate singularities are located at \(x = \pm \frac{C}{2M}\), at the poles of the quantum potential \(Q(z)\).

Second example: a static model. In the last part of this section we shall show a very simple example where singularities in the transformed metric appears. We consider again the two-particle wave function of a scalar field in two dimensions. Following the approach of Alves in [4] we shall see that, for the static case, it is possible to obtain a solution as a metric of the curved space-time (the effective metric) which comes from Eqs. (8) and (13). For the present case these equations are:

\[
\eta^{11}\partial_{x_1}S\partial_{x_1}S + \eta^{12}\partial_{x_1}S\partial_{x_2}S = 2m^2\hbar^2(1 - \frac{Q}{2m\hbar^2})
\] (27)

\[
\partial_{x_1}(R^2\partial_{x_1}S) + \partial_{x_2}(R^2\partial_{x_2}S) = 0
\] (28)

Now we consider that our two-particle system satisfies the EPR condition \(p_1 = -p_2\) which in the BdB interpretation, using the Bohm guidance equation \(p = \partial_xS\), can be written as

\[
\partial_{x_1}S = -\partial_{x_2}S.
\] (29)
Using this condition in eq. (28) we have

$$\partial_{x_1}(R^2 \partial_{x_1} S) = \partial_{x_2}(R^2 \partial_{x_1} S)$$

(30)

and this equation has the solution

$$R^2 \frac{\partial S}{\partial x_1} = G(x_1 + x_2)$$

(31)

where $G$ is an arbitrary (well behaved) function of $x_1 + x_2$.

Substituting eq. (31) in eq. (27) we have

$$2m^2 \hbar^2 (1 - \frac{Q}{2m \hbar^2}) = 2\left(\frac{G}{R^2}\right)^2$$

(32)

and taking into account the expressions (10), (11) and (12) for the quantum potential, the last equation reads

$$8G^2 + (\partial_{x_1}(R^2))^2 - 2R^2 \partial_{x_1}^2 R^2 + (\partial_{x_2}(R^2))^2 - 2R^2 \partial_{x_2}^2 R^2 - 8m^2 R^4 = 0.$$  

(33)

A solution of this nonlinear equation is

$$R^4 = \frac{1}{2m^2}(C_1 \sin(m(x_1 + x_2) + C_2))$$

(34)

provided an adequate function $G(x_1 + x_2)$, which can be obtained from (33) by substituting the solution.

In order to interpret the effect of the quantum potential we can re-write eq. (27) using (32) obtaining

$$\eta^{11} \partial_{x_1} S \partial_{x_1} S + \eta^{11} \partial_{x_2} S \partial_{x_2} S = 2\left(\frac{G}{R^2}\right)^2$$

(35)

or

$$m^2 \frac{\eta^{11}}{(G/R^2)^2} \partial_{x_1} S \partial_{x_1} S + m^2 \frac{\eta^{11}}{(G/R^2)^2} \partial_{x_2} S \partial_{x_2} S = 2m^2$$

(36)

that we write as

$$g^{11} \partial_{x_1} S \partial_{x_1} S + g^{11} \partial_{x_2} S \partial_{x_2} S = 2m^2$$

(37)
and then we see that the quantum potential was “‘absorbed’” in the new metric \( g_{11} \) which is:

\[
g_{11} = \frac{1}{g^{11}} = \frac{\eta_{11}}{m^2} \left( \frac{G}{R^2} \right)^2 = \frac{-C_1^2}{16m^2} + \frac{3C_2^2}{16m^2} \sin^2(m(x_1 + x_2) + C_2) \frac{\sin(m(x_1 + x_2) + C_2)}{2m^2}. \tag{38}
\]

We can see that this metric is singular at the zeroes of the denominator in (38). According to the model reviewed at the beginning of this section, this is characteristic of a two dimensional BH solution (see [4] and [10]). Then our two-particle system ”see” an effective metric with singularities, a fundamental component of a wormhole [12], through which physical signals can propagate.

IV. CONCLUSION

We studied the two-particle state of a scalar field under the EPR condition, for the two dimensional case in two situations, a non-tachyonic case and a static one. We found that the quantum potential can be interpreted as realizing a conformal transformation of the Minkowski metric to an effective metric. In the first situation this effective metric is analog to a BH metric and in the second one this metric present singularities, a key ingredient of a bridge construction or wormhole. This open the possibility, following a suggestion by Holland [6], of interpret the EPR correlations of the entangled particles as driven by an effective wormhole. Obviously a more realistic (i.e. four dimensional) and more sophisticated model (i.e. including the spin of the particles) must be studied.

It is interesting to note that a wormhole coming from a (Euclidean) conformally flat metric with singularities was shown by Hawking [13]. Consider the metric:

\[
ds^2 = \Omega^2 dx^2 \tag{39}
\]

with

\[
\Omega^2 = 1 + \frac{b^2}{(x - x_0)^2}. \tag{40}
\]

This looks like a metric with a singularity at \( x_0 \). However, the divergence of the conformal factor can be though as the space opening out to another asymptotically flat region connected with the first one by mean a wormhole of size \( 2b \).
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