Scalar Dark Matter Effects in Higgs and Top Quark Decays
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Abstract:
We study possible observational effects of scalar dark matter, the darkon $D$, in Higgs $h$ and top quark $t$ decay processes, $h \to DD$ and $t \to cDD$ in the minimal Standard Model (SM) and its two Higgs doublet model (THDM) extension supplemented with a SM singlet darkon scalar field $D$. We find that the darkon $D$ can have a mass in the range of sub-GeV to several tens of GeV, interesting for LHC and ILC colliders, to produce the required dark matter relic density. In the SM with a darkon, $t \to cDD$ only occurs at loop level giving a very small rate, while the rate for Higgs decay $h \to DD$ can be large. In THDM III with a darkon, where tree level flavor changing neutral current (FCNC) interaction exists, a sizable rate for $t \to cDD$ is also possible.

I. INTRODUCTION

Understanding the nature of dark matter is one of the most challenging problems in particle physics and cosmology. Although dark matter contributes about 20\% to the energy density of our universe\cite{1}, the identity of the basic constituents of the dark matter is still not known. One of the popular candidates for dark matter is the Weakly Interacting Massive Particle (WIMP). Detection of WIMP candidate is extremely important in understanding the nature of dark matter and also the fundamental particle physics models. The traditional way is to measure the dark matter flux at earth detectors. It is interesting to see whether WIMP can be produced and detected at collider experiments directly. Among the many possible WIMPs, the lightest supersymmetric particle is the most studied one. But no direct experimental evidence has been obtained for supersymmetry so that other possibilities of WIMP which explain dark matter relic density in our universe should be studied and searched for.

The simplest model which has a candidate of WIMP is the Standard Model (SM) with a singlet SM real scalar field $D$ (SM+D). We will call the field $D$ as darkon. The darkon field as dark matter was first considered by Silveira and Zee\cite{2}, and further studied later by several others groups\cite{3, 4, 5, 6, 7, 8}. In this work we concentrate on the darkon observable effects on Higgs $h$ and top quark $t$ decays. We find that a darkon of mass in the range of sub GeV to tens of GeV can play the role of dark matter with very constrained parameters and also have interesting collider physics signatures. We will also extend the studies to two Higgs doublet models (THDM). The LHC to be in operation soon and the planned ILC offer excellent possibilities to study darkon signatures through $h \to DD$ and $t \to cDD$.

II. DARK MATTER CONSTRAINTS ON DARKON

The darkon field $D$ must interact weakly with the standard matter field sector to play the role of dark matter. The simplest way of introducing the darkon $D$ is to make it a SM real singlet which can only be created and annihilated in pairs, the SM+D model. If the interaction of $D$ is required to be renormalizable, it can only couple to the Higgs doublet field $H$. Beside the kinetic energy term $-(1/2)\partial_\mu D^\mu D$, the general form of other terms are given by\cite{6, 7}

$$-\mathcal{L}_D = \frac{\lambda_D}{4} D^4 + \frac{m_0^2}{2} D^2 + \lambda D^2 H^\dagger H .$$ (1)

Note that the above Lagrangian is invariant under a $D \to -D \, Z_2$ symmetry. The parameters in the potential should be chosen such that the $D$ field will not develop vacuum expectation value (vev) and the $Z_2$ symmetry is not broken, after $SU(2)_L \times U(1)$ spontaneously breaks down to $U(1)_{em}$, to make sure that darkons can only be produced or annihilated in pairs. The relic density of $D$ is then decided, to the leading order, by annihilation of a pair of $DD$ into SM particles through Higgs exchange\cite{3, 4, 8}, $DD \to h \to X$ where $X$ indicates SM particles.

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Eliminating the pseudo-goldstone boson “eaten” by $W$ and $Z$, we have the physical Higgs $h$ coupling to $D$ as

$$- L_D = \frac{\lambda_D}{4} D^4 + \frac{1}{2}(m_0^2 + \lambda v^2) D^2 + \frac{1}{2} \lambda h^2 D^2 + \lambda v h D^2,$$

where $v = 246$ GeV is the vev of $H$. The $D$ field has a mass $m_D^2 = m_0^2 + \lambda v^2$. The last term $\lambda v h D^2$ plays an important role in determining the relic density of the dark matter.

The annihilation of a $DD$ pair into SM particles is through s-channel $h$ exchange. To have some idea how this works, let us consider $DD \rightarrow h \rightarrow f \bar{f}$. We parameterize Higgs-fermion and Higgs-darkon interactions as

$$- L_V = a_{ij} \bar{f}_i f_j h + bhD^2,$$

where $R(L) = (1$ $\pm \gamma_5)/2$. In the SM, $a_{ij} = m_i \delta_{ij}/v$ and $b = \lambda v$.

The total averaging annihilation rate of a pair $DD$ to fermion pairs is then given by

$$\langle v, \sigma \rangle = \frac{16\pi^2}{32\pi m_D^2} \frac{1}{(4m_D^2 - m_H^2)^2 + \Gamma_H^2 m_H^2} \sum_f N_f^2 |a_{ff}|^2 (4m_D^2 - 4m_f^2)^3/2.$$

where $N_f^2$ is the number of colors of the $f$-fermion. For a quark $N_f^2 = 3$ and a lepton $N_f^2 = 1$. $f$ sums over the fermions with $m_f < m_D$. In the above $v_r$ is the average relative velocity of the two $D$ particles. We have used the fact that for cold dark matter $D$, the velocity is small, therefore to a good approximation the average relative speed of the two $D$ is $v_r = 2p_{D,\text{cm}}/m_D$ and $s = (p_f + p_f)^2$ is equal to $4m_D^2$.

If there are other decay channels, the sum should also include these final states. The above can be re-written and generalized to $h$

$$\langle v, \sigma \rangle = \frac{8\pi^2}{32\pi m_D^2} \frac{1}{(4m_D^2 - m_H^2)^2 + \Gamma_H^2 m_H^2} \sum_f \frac{\Gamma(h \rightarrow X)}{2m_D},$$

where $\Gamma(h \rightarrow X') = \sum_i \Gamma(h \rightarrow X_i)$ with $h$ being a “virtual” Higgs having the same couplings to other states as the Higgs $h$, but with a mass of $2m_D$. $X_i$ indicate any possible decay modes of $h$. For a given model $\Gamma(h \rightarrow X')$ is obtained by calculating the $h$ width and then set the mass equal to $2m_D$.

To produce the right relic density for dark matter $\Omega_D$, the annihilate rate needs to satisfy the following

$$\langle v, \sigma \rangle \approx 1.07 \times 10^9 x_f \sqrt{x_f m_D GeV(\Omega_D h^2)}, \quad x_f \approx 0.038 m_D m_D \langle v, \sigma \rangle,$$

where $m_D = 1.22 \times 10^{19}$ GeV, $x_f = m_D/T_f$ with $T_f$ being the freezing temperature, and $g_*$ is the relativistic degrees of freedom with mass less than $T_f$. Note that the ‘$h$’ in $\Omega_D h^2$ is the normalized Hubble constant, not the Higgs field.

For given values of $m_D$ and $\Omega_D h^2$, $x_f$ and $g_*$ can be determined and therefore also $\langle v, \sigma \rangle$. Then one can determine the parameter $b$. In Fig. 1 we show the allowed range for the parameter $b/v = \lambda$ as a function of the darkon mass $m_D$ for several values of Higgs mass $m_h$ with $\Omega_D h^2$ set in the range 0.095 $\sim$ 0.112 determined from cosmological observations. We see that the darkon mass can be as low as a GeV. Since we are interested in producing the darkons and study their properties at colliders, we will limit ourselves to study darkon with a mass less than 100 GeV. We note that when the darkon mass decreases, $\lambda$ becomes larger. For small enough $m_D$ $\lambda$ can be close to one which may upset applicability of perturbative calculation. We will only show region of parameters with $\lambda < 1$.

III. EFFECTS OF DARKON ON HIGGS AND TOP DECAYS

At LHC and ILC, a large number of Higgs and top quark particles may be produced if kinematically accessible\cite{10, 11}. The various production cross sections of Higgs at LHC and ILC are typically a few pb level\cite{11}. Assuming the integrated luminosities at LHC and ILC to be $200 fb^{-1}$, a large number of Higgs can be copiously produced and its properties studied in details. The main effect of the darkon field on the Higgs properties is to add an invisible decay mode $h \rightarrow DD$ to the Higgs particle. Due to this additional mode, the Higgs width will be broader affecting determinations of the Higgs mass, and also decay properties in processes such as $pp \rightarrow Xh \rightarrow XX'$ and $e^+e^- \rightarrow Z^* \rightarrow Zh \rightarrow XX'$. Here $XX'$ indicates the final states used to study $h$ properties.

Top quark properties will be studied in details at LHC and ILC. Assuming an integrated luminosity of $200 fb^{-1}$, the sensitivity for $B(t \rightarrow ch)$ can reach $3 \times 10^{-5}$ at LHC\cite{10, 12} and $4.5 \times 10^{-5}$ at ILC\cite{10, 12}. Branching ratio for $t \rightarrow cDD$ at that level will therefore significantly affect the results and should be accounted. The decay mode
$t \to cDD$ is interesting for several reasons. Since $D$ is neutral if it is produced without tagging it is not possible to identify its production. Through top decay, one can in some way to tag it if one considers pair production of top. The signal will be a charm jet plus missing energy. There may be other processes producing similar signal, such as $t \to c\bar{\nu}$. This decay mode in the SM is, however, very small. Another reason for studying this flavor changing neutral current (FCNC) process is that the top quark is heavy, the mass of $D$ up to about 80 GeV can be studied compared with other quark decays, such as $b$ quark decay where mass of $D$ below 2.5 GeV may be studied.

In our previous discussions on dark matter density we have seen that the coupling $\lambda = b/v$ in a wide range of darkon mass is not much smaller than 1, it is clear that the introduction of darkon will affect processes mediated by Higgs exchange and Higgs decay itself. In Figs. 2 and 3 we show the decay width $\Gamma(h \to DD)$ and the branching ratio $B(h \to DD)$ as a function of $m_D$ for several values of $m_h$. We see that the invisible decay $h \to DD$ dominates over the Higgs decay width if $m_D$ is significantly below the $h \to DD$ threshold. However such invisible domination becomes weaker when $h \to VV$ modes become kinematically allowed. This will affect the bounds set on the Higgs mass for low mass $m_D$.

For Higgs mediated $t \to cDD$, one needs to know the couplings $a_{ij}$ defined in eq.(3). The decay amplitude is

$$M(f_i \to f_j DD) = \frac{2b}{s - m_h^2 + i\Gamma_h m_h} \tilde{f}_j (a_{ji} R + a_{*ji} L) f_i.$$

(7)

In the SM+D, flavor changing coupling of Higgs to fermions are generated at loop level and therefore small. Using the expression in Ref. [15] for the SM Higgs couplings to fermions, we find that the branching ratio $B(t \to ch \to cDD)$ to be less than $10^{-13}$ for the Higgs $h$ mass in the hundred GeV range. This is too small to be observed by future experiments at either LHC or ILC.

To have detectable effects for $t \to cDD$, there must be new physics beyond the SM+D model where tree level FCNC interaction mediated by Higgs exists. To this end we take the two Higgs doublet model as an example for discussion in the following.
As far as dark matter relic density is concerned, one can treat concreteness, in our numerical analysis we will neglect contributions from the quark masses, respectively. In models I and II the parameters used, the width of Higgs decay to SM particle for small D mass is larger.

Depending on how the two Higgs doublets \( H^T_{1,2} = \left( h^+_{1,2}, (v_{1,2} + h_{1,2} + i a_{1,2})/\sqrt{2} \right) \) couple to quarks and leptons, there are different models with darkon field added (THDM+D). We will come back to this later. We first discuss how darkon \( D \) couples to the two Higgs doublets. In analog to \( \text{(1)} \), we write down the most general renormalizable interaction of \( D \) with the Higgs doublets \( H_{1,2} \) in the Higgs potential. We have

\[
- \mathcal{L}_D = \frac{\lambda_D}{4} D^4 + \frac{m^2_D}{2} D^2 + D^2 (\lambda_1 H^\dagger_1 H_1 + \lambda_2 H^\dagger_2 H_2 + \lambda_3 (H^\dagger_1 H_2 + H^\dagger_2 H_1)).
\]  

We have again imposed the \( Z_2 \) symmetry previously discussed. For the same reason we need to keep it unbroken. We have also assumed CP conservation in the above Lagrangian. Eliminating the pseudo-goldstone boson “eaten” by \( W \) and \( Z \), the two Higgs doublets have physical components \( H^T = (\sin \beta h^+, (v_1 + \cos \alpha H - \sin \alpha h - i \sin \beta A)/\sqrt{2}) \), and \( H^T_2 = (\cos \beta h^+, (v_2 + \sin \alpha H + \cos \alpha h + i \cos \beta A)/\sqrt{2}) \), where \( \tan \beta = v_2/v_1 \). \( \alpha \) is the mixing angle for the scalar Higgs fields \( h \) and \( H \) with \( h \) playing a similar role as the SM Higgs. \( A \) is a physical pseudoscalar field.

Using the above information, we obtain the mass of \( D \) and the \( hD^2 \) interaction after \( H_{1,2} \) develop vevs,

\[
m_D^2 = m_0^2 + v^2 (\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta + 2 \lambda_3 \cos \beta \sin \beta),
\]

\[
-L_{hD^2} = [-\lambda_1 \cos \beta \sin \alpha + \lambda_2 \sin \beta \cos \alpha + \lambda_3 \cos (\beta + \alpha)] v h D^2 = \lambda_h v h D^2.
\]  

The mass parameter \( m_D^2 \) and the effective coupling \( \lambda_h \) are free parameters in this model. The couplings of \( H \) and \( A \) to \( D \) are: \(-L_{HD^2} = (\lambda_1 \cos \beta \cos \alpha + \lambda_2 \sin \beta \sin \alpha + \lambda_3 \sin (\beta + \alpha)) v h D^2 = \lambda_H v h D^2 \), and \(-L_{AD^2} = 0 \). If both the two neutral scalar particles \( h \) and \( H \) contribute, the analysis will be complicated. For concreteness, in our numerical analysis we will neglect contributions from the \( H \) by requiring small \( \lambda_H \). In this case as far as dark matter relic density is concerned, one can treat \( \lambda_h \) as an effective coupling \( \lambda_h = b/v \) defined before. The constraint on \( \lambda_h \) can be obtained in a similar way as that for SM+D. The results for illustrating parameters chosen are shown in Fig.1. We note that \( \lambda_h \) in this case needs not to be as large as that in the SM+D model because, for the parameters used, the width of Higgs decay to SM particle for small \( D \) mass is larger.

The couplings of the Higgs fields to quarks distinguish different models \( \text{(10)} \). In the literature, these different models are called THDM I, THDM II and THDM III which are defined by only one Higgs gives masses to both up and down quark masses, \( H_1 \) gives down quark and \( H_2 \) gives up quark masses, and both \( H_1 \) and \( H_2 \) give up quark and down quark masses, respectively. In models I and II \( t \rightarrow c D \overline{D} \) are generated at one loop level hence the decay rate are too small to be detected at LHC and ILC \[ \text{II,} \text{(10)} \], although can be substantially larger than that predicted by SM. THDM III offers a possibility to have a large detectable rate. We therefore will only show some details for THDM III. We will refer this model as THDM III+D. The couplings of \( h \) to fermions are given by \( \text{(10)} \).

\[
L_{III} = -\bar{Q}L \lambda_{qL}^2 \bar{H}_1 U_R - \bar{Q}L \lambda_{qL}^2 \bar{H}_2 U_R - \bar{Q}L \lambda_{qL}^2 H_1 D_R - \bar{Q}L \lambda_{qL}^2 H_2 D_R
-\bar{L}L \lambda_{qL}^2 \bar{H}_1 E_R - \bar{L}L \lambda_{qL}^2 \bar{H}_2 E_R + h.c.,
\]  

where \( \bar{H}_i = i \tau_2 H^*_{1i} \).
We obtain the $h$ coupling to fermions as

$$L_{III} = -\bar{U}_L M^{u} U_R \frac{\cos \alpha}{v \sin \beta} h + \bar{U}_L M^{u} U_R \frac{\cos (\alpha - \beta)}{v \sin \beta} h + \bar{D}_L M^d D_R \frac{\sin \alpha}{v \cos \beta} h + \bar{E}_L \tilde{M}^E \tilde{E}_R \frac{\sin \alpha}{v \cos \beta} h - \bar{E}_L \tilde{M}^E \tilde{E}_R \frac{\cos (\alpha - \beta)}{v \cos \beta} h + h.c.,$$

(11)

where $M^{u,d,l} = (\lambda_1^{u,d,l} v_1 + \lambda_2^{u,d,l} v_2)/\sqrt{2}$ are the diagonalized masses of the up-quarks, down-quarks and charged leptons. The off diagonal entries $\tilde{M}^u = \lambda_1^u v/\sqrt{2}$ and $\tilde{M}^d = \lambda_2^d v/\sqrt{2}$ are not fixed. We have chosen a parametrization for $\tilde{M}$ in the limit that when they are set to zero, the Yukawa couplings reduce to the minimal SUSY ones. In our later discussions, we will follow Ref. [17] to parameterize the off diagonal entries to have the geometric mean form

$$\tilde{M}_{ij}^{u,d,l} = \rho_{ij}^{u,d,l} \sqrt{m_i m_j}$$

with $\rho_{ij} \simeq 1$ for concreteness, and $\rho_{ii}$ to be negligibly small, for illustration.

The couplings of $h$ to $W$, $Z$ will also be changed to

$$L_{hhW} = \frac{2m_W^2}{v} \sin(\beta - \alpha) h W^2, \quad L_{hZZ} = \frac{m_Z^2}{v} \sin(\beta - \alpha) h Z^2,$$

which affect the Higgs decay width.

Using the above information, we can obtain the total $h$ decay width without and with the $h \rightarrow DD$ mode. The resulting $\Gamma(h \rightarrow DD)$ and $B(h \rightarrow DD)$ are shown in Figs. 2 and 3. We see that $\Gamma(h \rightarrow DD)$ and $B(h \rightarrow DD)$ in THDM III+D model can be smaller than those in the SM+D because $\Gamma(h \rightarrow X')$ in Eq.(5) is bigger in THDM III+D when $m_D$ is small.

A large difference of THDM III+D compared with SM+D can show up in $t \rightarrow cDD$ decays. The results are shown in Fig. 4. For the above parametrization of $h$ coupling to fermions, we find that the branching ratio $B(t \rightarrow cDD)$ can be as large as $10^{-3}$ if $h$ mass is below the $h \rightarrow VV$ threshold which can be investigated at LHC and ILC [12].

![Graph](image_url)

**FIG. 4:** The branching ratios of $t \rightarrow cDD$ in SM+D (left) and THDM III+D (right) as a function of $m_D$.

V. CONCLUSIONS

We have studied the effects of scalar dark matter, the darkon $D$, in Higgs $h$ and top quark $t$ decay processes, $h \rightarrow DD$ and $t \rightarrow cDD$ in the SM+D and THDM III+D. Requiring renormalizable interaction for these models, the darkon field can only couple to the Higgs field in the Lagrangian. We find that the darkon $D$ can have a mass in the range of sub-GeV to several tens of GeV, interesting for LHC and ILC colliders, to produce the required dark matter relic density with restricted darkon and Higgs coupling.

In both SM+D and THDM III+D models, the darkon field can have significant effects on the Higgs decay width through the invisible Higgs decay mode $h \rightarrow DD$. In the SM+D, if the darkon mass is significantly lower than the $h \rightarrow DD$ threshold, this invisible decay width dominates the Higgs decay making determination of Higgs mass in this region more difficult than that in the SM. In the THDM III+D, the invisible decay width can be made not so dominating.

In the SM+D model, $t \rightarrow cDD$ only occurs at loop level giving a very small rate. In THDM III+D, where tree level FCNC interaction exists, a sizable rate of order $10^{-3}$ for $t \rightarrow cDD$ is possible for $m_h$ below $2m_W$. Experiments at LHC and ILC will be able to provide important information about darkon field interaction.
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