Resonantly Enhanced Axion-Photon Regeneration

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Photon-regeneration experiments which search for the axion, or axionlike particles, may be resonantly enhanced by employing matched Fabry-Perot optical cavities encompassing both the axion production and conversion magnetic field regions. Compared to a simple photon-regeneration experiment, which uses the laser in a single-pass geometry, this technique can result in a gain in rate of order \( F^2 \), where \( F \) is the finesse of the cavities. This gain could feasibly be \( 10^{10-12} \), corresponding to an improvement in sensitivity in the axion-photon coupling \( g_{\alpha \gamma \gamma} \) of order \( F^{1/2} \). This factor may be \( 10^{2.5-3} \).

In this Letter, we point out that matched Fabry-Perot cavities incorporated into the production and detection magnets can improve the sensitivity in the axion-photon coupling \( g_{\alpha \gamma \gamma} \) by the square root of the cavities’ finesse \( F^{1/2} \).

Recent discussions of the axion’s properties and potential as a dark matter candidate [1] have led to renewed interest in the detection of axions. Theoretical models of axions can have couplings to electromagnetic fields such that a laser incident on an axion-producing device produces a polarization ellipticity and a non-equipartitioned dichroism of the vacuum [4], which may be interpreted as a measurement of the square root of the axion-photon coupling [2].

The action density for the dynamics of photons and axions is

\[
\mathcal{L} = \frac{1}{2} (\epsilon E^2 - B^2) + \frac{1}{2} (\partial_\mu a^\mu)^2 - \frac{1}{2} (\nabla a)^2 - \frac{1}{2} m_a^2 a^2 - ga\tilde{E} \cdot \tilde{B},
\]

(1)

where \( \tilde{E} \), \( \tilde{B} \), and \( a \) are, respectively, the electric, magnetic, and axion fields. The electromagnetic fields are given in terms of scalar and vector potentials, \( \tilde{E} = -\nabla \Phi - \partial_t \tilde{A} \)

\( \tilde{B} = \nabla \times \tilde{A} \), as usual. Henceforth, the coupling \( g = g_{\alpha \gamma \gamma} \) is, for simplicity, written without subscripts. The dielectric function \( \epsilon \) is assumed constant in both space and time. In the presence of a large static magnetic field \( \tilde{B}_0(x) \), the equations of motion are

\[
e\nabla \cdot \tilde{E} = g \tilde{B}_0 \cdot \nabla a, \quad \nabla \times \tilde{B} - e\partial_\tau \tilde{E} = -g\tilde{B}_0 \partial_\tau a, \quad \partial_\tau^2 a - \nabla^2 a + m_a^2 a = -g\tilde{E} \cdot \tilde{B}_0.
\]

(2)

\( \tilde{B}_0 \) now represents the magnetic field minus \( \tilde{B}_0 \), and terms of order \( gE \) and \( gB \) are neglected. Equations (2) describe the conversion of axions to photons and vice versa.

Using these equations, it can be shown [5,8,9] that the axion-to-photon conversion probability in a region of length \( L \), permeated by a constant magnetic field \( B_0 \) transverse to the direction of propagation, is given by

\[
p = \frac{1}{4\beta_a \sqrt{\epsilon}} (gB_0 L)^2 \frac{2}{qL} \sin \frac{qL}{2} (qL)^2
\]

(3)

with \( \beta_a \) the axion speed and \( q = k_x - k_y \) the momentum transfer. In terms of the energy \( \omega_a \), which is the same for the

The axion remains the most attractive solution to the strong-\( CP \) problem and is one of two leading dark-matter candidates [1]. Recently, it has been realized that the axion represents a fundamental underlying feature of string theories; there could be several or even a great number of axions or axionlike particles within any particular string theory [2]. From the experimental viewpoint, there are now several photon-regeneration experiments in various stages of preparation [3], motivated in part by the report of the PVLAS Collaboration of a nonzero magnetically induced dichroism of the vacuum [4], which may be interpreted as the production of a light boson with a two-photon coupling [5]. Although the particle interpretation of the PVLAS experiment is, in principle, excluded by the much more stringent limit set by the CAST solar axion search [6], theories have been proposed to reconcile them. In any case, it is important to check this result by a purely laboratory experiment, particularly if one could ultimately improve such a measurement to reach previously unexplored regions of axion mass \( m_a \) and its coupling to two photons \( g_{\alpha \gamma \gamma} \). Recent discussions of the axion’s properties and experimental searches for the axion are found in Ref. [7].

The simplest and most unambiguous purely laboratory experiment to look for light scalars or pseudoscalars is photon regeneration (“shining light through walls”) [8]. A laser beam traverses a dipole magnet, wherein a small fraction of the photons are converted into axions with the same energy. An optical barrier blocks the primary laser beam, whereas the axion component of the beam travels through the wall unimpeded and enters a second dipole magnet, where it is reconverted to photons with the same probability (we will assume magnets of identical length \( L \) and field strength \( B_0 \) without loss of generality). However, because the photon-regeneration rate goes as \( g_{\alpha \gamma \gamma}^4 \), the sensitivity of the experiment to small values of \( g_{\alpha \gamma \gamma} \) is poor in its basic form, improved appreciably only by increasing \( B_0 \) or \( L \).
axion and the photon, $k_a = \sqrt{\omega^2 - m_a^2}$, $\beta_a = k_a/\omega$, and $k_\gamma = \sqrt{\epsilon_0 \omega}$. The photon-to-axion conversion probability in this same region is also equal to $p$. Everything else being the same, the conversion probability is largest when $q = 0$. For $m_a \ll \omega$, and propagation in a vacuum (henceforth we set $\epsilon = 1$),

$$q = -\frac{m_a^2}{2\omega}. \quad (4)$$

Figure 1(a) shows the axion-photon-regeneration experiment as usually conceived. If $P_0$ is the power of the laser, the power of the axion beam traversing the wall is $pP_0$, where $p$ is the conversion probability in the magnet on the left-hand side of Fig. 1(a). Let $p'$ be the conversion probability in the magnet on the right-hand side. The power in regenerated photons is $P = p'pP_0$.

Figure 1(b) shows two improvements that may be made to the experiment. The first, which is not new, is to build up the power on the photon-to-axion conversion side of the experiment using a Fabry-Perot cavity, as illustrated. Photons in the production cavity will then convert to axions with probability $p$ for each pass through the cavity. The standing wave in the production cavity is the sum of left-moving and right-moving components of equal amplitude. If the reflectivity of the cavity mirrors is given by

$$R = 1 - \eta, \quad (5)$$

and the power of the laser is $P_0$, the power of the right-moving wave in the production cavity is $\frac{1}{\eta}P_0$. Therefore, the axion power through the wall in the setup of Fig. 1(b) is $\frac{1}{\eta}PP_0$. Assuming the lasers in Figs. 1(a) and 1(b) have the same power, the axion flux is increased by the factor $\frac{1}{\eta}$.

As we have said, increasing the axion production rate (and thus the photon-regeneration rate) by building up the optical power in the first magnet is not a new idea. In fact, the only photon-regeneration experiment performed and published to date, Ref. [10], utilized an “optical delay line,” i.e., an incoherent cavity encompassing the production magnet, causing the laser beam to traverse the magnet 200 times before exiting. With relatively modest magnets (4.4 m, 3.7 T each), a limit of $g < 6.7 \times 10^{-7}$ GeV$^{-1}$ was set. There is substantial gain from building up the laser power in the axion production magnet; however, it is immaterial whether one “recycles” the photons incoherently, as in an optical delay line, or coherently, as in a Fabry-Perot (FP) cavity. In contrast, the coherent case alone can provide a large additional gain in sensitivity for photon regeneration. Thus, the second improvement is to also install a Fabry-Perot cavity on the regeneration side of the experiment, making a symmetric arrangement, as illustrated in Fig. 1(b). When the second FP cavity is locked to the first, the probability of axion-to-photon conversion in the second FP cavity is $\frac{1}{\eta} p'' = \frac{2}{\pi} f'' p'$, where $f''$ is the finesse of the cavity, and $p'$ is the axion-to-photon conversion probability in the absence of the cavity. The calculation which yields this result is outlined in the next paragraph.

The cavity modes are described by

$$\tilde{A}_n = A_n(t)\hat{y} \sin\left(\frac{n\pi}{L}z\right). \quad (6)$$

where $\hat{z}$ is in the direction of light propagation and $\hat{y}$ is a transverse direction. The dependence of the mode function on the transverse coordinates ($x$ and $y$) is neglected here but will be discussed later. Using Eqs. (2), one can show that, in the presence of an axion beam traveling through the cavity in the $z$ direction

$$a(z, t) = \mathcal{A} \sin(k_a z - \omega t), \quad (7)$$

the coefficients $A_n(t)$ satisfy

$$\left(\frac{d^2}{dt^2} + \gamma \frac{d}{dt} + \omega_n^2\right) A_n(t) = C \sin\left(\omega t - \frac{qL}{2}\right), \quad (8)$$

where $\omega_n = \frac{n\pi}{L}$ and

$$C = g\omega B_0 \mathcal{A} \frac{2}{Lq} \sin\left(\frac{qL}{2}\right). \quad (9)$$

As before, $q = k_a - k_n = \sqrt{\omega^2 - m_a^2 - \frac{L^2}{q^2}}$ is the momentum transfer. When the production cavity and the regeneration cavity are tuned to the same frequency, $\omega_n = \omega$ for some $n$. Then

FIG. 1 (color online). (a) Simple photon regeneration. (b) Resonant photon regeneration, employing matched Fabry-Perot cavities. The overall envelope schematically shown by the thin dashed lines indicates the important condition that the axion wave, and thus the Fabry-Perot mode, in the conversion magnet must follow that of the hypothetically unimpeded photon wave from the Fabry-Perot mode in the production magnet. Between the laser and the cavity is the injection optics (IO), which manages mode matching of the laser to the cavity, imposes rf sidebands for reflection locking of the laser to the cavity, and provides isolation for the laser. The photon detectors are also preceded by matching and beam-steering optics. Not shown at all is the electro-optical system required to lock the two cavities together in frequency.
\[A_\eta = \frac{\gamma}{\omega} C \sin \left( \frac{n\pi}{L} \right) \sin \left( \omega t - \frac{qL}{2} - \frac{\pi}{2} \right).\]  

(10) up to transients. The energy stored in the cavity is 
\[E = \frac{1}{2} SL^2 \omega^2,\]
where \(S\) is the cross-sectional area of the cavity mode. The power emitted by the cavity is 
\[P = \gamma E,\]
assuming that there are no losses other than by transmission through the mirrors. The power of the axion beam is 
\[P_a = \frac{1}{2} S A_\eta \omega k_a,\]
assuming it has the same cross-sectional area \(S\) as the cavity mode. The axion-to-photon conversion probability in the FP cavity is therefore
\[p_{FP} = \frac{P}{P_a} = \frac{1}{2} \frac{g^2 B_0^2}{\beta_a \omega} Q L \left( \frac{2}{qL} \sin \left( \frac{qL}{2} \right) \right)^2.\]  

(11)

where \(Q = g^2/\gamma\) is the quality factor of the cavity. In terms of the conversion probability \(p'\) in the same region (length \(L\) and magnetic field \(B_0\)) without the cavity, we have 
\[p_{FP} = \frac{2}{\gamma} p' \frac{pP_0}{P_0}.\]  

(12)

Half of the power \(P\) is right-moving and half is left-moving. To detect all of the regenerated photons, detectors are installed on both sides of the Fabry-Perot cavity, as illustrated. The combined improvements yield an increase by a factor \(2/\eta\eta'\) in signal power. With present technology, this factor may be as large as \(10^2\).

In general, the loss of power from the cavity will have other contributions \(\eta' = \eta'_\text{trans} + \eta'_\text{abs} + \eta'_\text{scatt}\), where the latter two terms represent absorption and scattering (including diffraction) losses. If transmission is not entirely dominant, then Eq. (12) should be modified by a multiplicative factor \(f = \eta'_\text{trans}/\eta'\). There is no such factor corresponding to the production side, as the transmitted power is irrelevant, and the laser power can always be brought up to the limit established by optical damage.

There are practical limitations to the optics which establish the achievable sensitivity of a resonant photon-regeneration experiment. Here we present a plausible experimental realization, utilizing a 10 W continuous wave Nd:YAG (\(\lambda = 1.064 \mu m\)) laser, similar to the LIGO laser [11], and a total of eight LHC dipole magnets (9.5 T, 14.3 m, 50 mm \(\varnothing\) cold bore), which are well suited to the experiment.

With the magnets set end to end, four for the production leg and four for the regeneration leg, the overall length of each cavity will exceed 60 m. We use 66 m in this estimate, giving a cavity free spectral range (FSR) of 2.3 MHz. We will refer to this as a \(4+4\) configuration of LHC dipole magnets. The magnet diameter determines the maximum size of the TEM\(_{00}\) Gaussian mode of the cavities. Furthermore, the spatiotemporal profiles of the axion and photon modes are identical; i.e., the axion mode follows that of a hypothetically unobstructed photon beam. Thus, the cavities should be symmetric, with the beam waist at the optical barrier, such that the two end mirrors would support the optical mode if the barrier and inboard mirrors were removed [Fig. 1(b)]. There is no constraint that the length of the two cavities be exactly equal nor that their relative separation equal a multiple number of wavelength. For the case of 1.064 \(\mu m\) light, and for 130 m between end mirrors, the beam is everywhere smaller than 36 mm and is not clipped by the sagitta of the LHC dipoles [12].

Another limit is that the intracavity power density at the mirrors be below the damage threshold, which for the best multilayer dielectric mirrors approaches 1 GW/cm\(^2\). Putting the waist of the mode at the barrier implies that the limitation on power density will be first encountered for the slightly convex inboard mirrors. The total circulating power in the production magnet is \(\sim 1\) MW.

The best Fabry-Perot resonators have achieved a finesse of a few million; here, we take for the finesse \(\mathcal{F} \sim 3.1 \times 10^5\), an order of magnitude lower, which is still ambitious but feasible. For the linewidth implied by this finesse, 7.3 Hz, the vibration tolerance for the cavity mirrors is of order \(10^{-3}\) nm, well within the experience of LIGO detectors [11]. Also for this finesse, we can design the cavity to ensure that transmission dominates loss (scattering and diffraction), and this has been assumed.

Intrinsic to resonantly enhanced photon regeneration is the requirement that the production and regeneration cavities remained locked in frequency together, within their bandwidth \(\Delta \nu = 2\nu/Q\), where the quality factor \(Q = \mathcal{F}/\nu\) of the cavity, and \(n\) is its mode index. The 10 W laser is reflection locked to one of the cavity modes of the production cavity using the Pound-Drever-Hall reflection locking scheme [13]. Then the challenge is to implement the locking of the regeneration cavity in a way that does not introduce any spurious photons into the detectors. The scheme we envision is to use a low power Nd:YAG laser, offset locked by (integer) \(\times\) FSR of the production cavity (say, 50 MHz) from the main laser and use the same Pound-Drever-Hall reflection locking method to control the length of the regeneration cavity. For additional rejection, the locking could be done in the orthogonal polarization state, i.e., perpendicular to the dipole magnetic field \(B_0\).

There are several possible schemes for detection of the weak signal from the regeneration cavity. The simplest is to focus the cavity output on a cooled InGaAs charge-coupled device (CCD), with pixels of the order of 15 \(\mu m\). Modern CCDs developed for astronomy have very low dark current rates, of order 1–2 e\(^-\) min\(^{-1}\) pixel\(^{-1}\). However, the best scheme seems to be heterodyne detection, mixing the generation cavity output with the locking laser at an rf photodiode and detecting both in-phase and quadrature signals at the difference frequency between the laser in the generating cavity and the locking beam, especially as...
While resonant photon regeneration marks a significant improvement over the simple experiment, in fact the sensitivity in $g$ still gains (or loses) only as $\mathcal{F}^{1/2}$. Thus, in the example above, the experiment would still reach a limit of $g = 5.7 \times 10^{-11}$ GeV$^{-1}$ even if the Fabry-Perot achieved only a finesse of $\mathcal{F} \approx 30\,000$. As it will likely be easier to attain higher values of $\mathcal{F}$ for shorter baselines, we further note that, as $g \propto (BL)^{-1}$, a limit of $g = 7.2 \times 10^{-11}$ GeV$^{-1}$, i.e., still better than the CAST and the horizontal branch star limits, should still be achievable even with only one LHC dipole in each leg.

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[3] All of the photon-regeneration experiments either currently operating, in preparation, or proposed are found in A. Ringwald, hep-ph/0612127.
[12] P. Pugnat (private communication). The LHC dipoles were built with a 9-mm sagitta to accommodate the orbit of a 2804-m mean radius; the cold mass could be flexed to take the sagitta out if necessary.