U-Spin Tests of the Standard Model and New Physics

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Within the standard model, a relation involving branching ratios and direct CP asymmetries holds for the six B-decay pairs that are related by U-spin. The violation of this relation indicates new physics (NP). In this paper, we assume that the NP affects only the ΔS = 1 decays, and show that, for most decay pairs, the NP operators are the same as those appearing in B → πK decays. The fit to the latest B → πK data shows that only one NP operator is sizeable. As a consequence, only one decay pair – B^0_d → K^0\pi^0 and B^0_s → K^0\pi^0 – is expected to exhibit measurable U-spin violation.

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At present, several different experiments are focusing on the measurement of CP violation in various B decays. The principal hope is to find a discrepancy with the predictions of the standard model (SM). This would indicate the presence of physics beyond the SM, which is one of the main aims of these experiments.

U-spin relations between different B decays provide several tests of the SM. This is discussed in detail in Ref. [1]. U-spin is the symmetry that places quarks on an equal footing, and is often given as transposing d and s quarks: d → s. Assuming a perfect U-spin symmetry, the effective Hamiltonian describing a ΔS = 0 transition (b → d) is equal to that of the corresponding ΔS = 1 transition (b → s) with d → s (the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are changed appropriately). Using the CKM unitarity relation [2], Im(V^\ast_{ub}V_{ud}V_{cb}V^\ast_{cd}) = -Im(V^\ast_{ub}V_{ud}V_{cb}V^\ast_{cd}), this implies that there exists a U-spin relation between the CP-violating rate differences of the ΔS = 0 and ΔS = 1 decays [1]:

\[ |A(B → f)|^2 - |A(\bar{B} → \bar{f})|^2 = -[(A(UB → Uf))^2 - (A(U\bar{B} → U\bar{f}))^2] , \]

in which U is the U-spin operator that transposes d and s quarks. This expression can be written as

\[ \frac{-A_{CP}^{dir}(\text{decay} \ #1)}{A_{CP}^{dir}(\text{decay} \ #2)} = BR(\text{decay} \ #2), \]

where A_{CP}^{dir} and BR refer to the direct CP asymmetry and branching ratio, respectively, and where decays #1,2 are the ΔS = 0 and ΔS = 1 decays, in either order, related by U-spin.

The pairs of B → PP decays (P is a pseudoscalar meson) which are related by U-spin are

1. B^0_d → K^+\pi^- and B^0_s → \pi^+K^-, 
2. B^0_s → K^+K^- and B^0_d → \pi^+\pi^-, 
3. B^0_d → K^0\pi^0 and B^0_s → K^0\pi^0, 
4. B^+ → K^0\pi^+ and B^+ → \bar{K}^0K^+, 
5. B^0_s → K^0\bar{K}^0 and B^0_d → \bar{K}^0K^0, 
6. B^0_s → \pi^+\pi^- and B^0_d → K^+K^-.

In all cases, the first decay is ΔS = 1; the second is ΔS = 0. Note that the decays described in pair #6 can only come about if the two quarks in the initial state interact with each other (annihilation). We therefore expect the branching ratios for these decays to be considerably smaller than those of the other decays. Throughout this paper, we refer to a given pair of decays by its number in the list above.

If Eq. 2 is not satisfied for any of these pairs of decays, this indicates the presence of physics beyond the SM.

Technically, this is not quite true, as U-spin is only an approximate symmetry. U-spin breaking effects are typically given by ratios of form factors and decay constants. These effects can total O(30%), but leaving aside the (known) form-factor and decay-constant ratios, the unknown factors are generally much smaller. In any case, U-spin breaking does not lead to a gross violation of Eq. 2. Thus, it is more correct to say that “if Eq. 2 is greatly violated for any of the pairs of decays – such as the direct CP asymmetries having the same sign – this indicates the presence of physics beyond the SM.”

This new physics (NP) can take the form of new contributions to the ΔS = 0 and/or the ΔS = 1 decays. Now, there have been many measurements of quantities in B decays. To date, there have been several hints of discrepancies with the SM. However, none of them lie in the ΔS = 0 sector; they all point to NP in b → s transitions. For example, the CP asymmetry in b → s\bar{q}q modes (q = u, d, s) is found to differ from that in b → c\bar{c}s decays by 2.6σ (they are expected to be approximately equal in the SM) [3, 4]. One also sees a discrepancy with the SM in triple-product asymmetries in B → ϕK^* [5, 6], and in the polarization measurements of B → ϕK^* [7, 8, 9] and B → ρK^* [10, 11]. Finally, some B → ϕK measurements disagree with SM expectations [12], although
it has been argued that the so-called $B \to \pi K$ puzzle [13] has been somewhat reduced [14, 13]. Although none of the discrepancies are statistically significant, together they give an interesting hint of NP. In this paper we follow this indication and assume that the NP appears only in $b \to \bar{s}$ decays ($\Delta S = 1$) but does not affect $\bar{b} \to \bar{d}$ decays ($\Delta S = 0$).

There are a great many NP operators which can contribute to $\Delta S = 1$ decays. However, it was recently shown in Ref. [14] that this number can be reduced considerably. Briefly, the argument is as follows. Following the experimental hints, we assume that NP contributes significantly to those decays which have large $b \to \bar{s}$ penguin amplitudes, and take the NP operators to be roughly the same size as the SM $b \to \bar{s}$ penguin operators, so the new effects are sizeable. Each NP matrix element can have its own weak and strong phase. Now, all strong phases arise from rescattering. In the SM, this comes mainly from the $b \to \bar{s}c\bar{c}$ tree diagram. The NP strong phases must come from rescattering of the NP operators. However, the tree diagram is quite a bit larger than the $b \to \bar{s}$ penguin diagram (the expected size of the NP operator). As a consequence, the generated NP strong phases are correspondingly smaller than those of the SM. That is, the NP strong phases are negligible compared to the SM strong phases.

The neglect of all NP strong phases considerably simplifies the situation. At the quark level, each NP contribution to the decay $B \to f$ takes the form $(f \mid O^{ij}_{N,F} \mid B)$, where $O_{ij}^{N,F} \sim s_{ij} b_{ij} q_{ij} \bar{q}_{ij} (q = u, d, s, c)$, in which the $s_{ij}$ represent Lorentz structures, and colour indices are suppressed. If one neglects all NP strong phases, one can now combine all NP matrix elements into a single NP amplitude, with a single weak phase:

$$\sum (f \mid O^{ij}_{N,F} \mid B) = A_Q e^{i\Phi_Q}. \quad (3)$$

Thus, all NP effects can be parametrized in terms of a small number of NP quantities. For $\Delta S = 1$ decays, there are two classes of NP operators, differing in their colour structure: $s_{ia} b_{ia} q_{ia} \bar{q}_{ia}$ and $s_{ia} b_{ia} q_{ia} \bar{q}_{ia}$. The first class of NP operators contributes with no colour suppression to final states containing $q\bar{q}$ mesons. The second type of operator can also contribute via Fierz transformations, but there is a suppression factor of $1/N_c$, as well as additional operators involving colour octet currents.) Similarly, for final states with $s\bar{q}$ mesons, the roles of the two classes of operators are reversed. As in Ref. [17], we denote by $A^{i\bar{q}e^{i\Phi_Q}}$ and $A^{i\bar{q}e^{i\Phi_Q}}$ the sum of NP operators which contribute to final states involving $q\bar{q}$ and $s\bar{q}$ mesons, respectively (the primes indicate a ($\Delta S = 1$) $b \to s$ transition). Here, $\Phi_Q$ and $\Phi_{q\bar{q}}$ are the NP weak phases; the strong phases are zero. We stress that, despite the “colour-suppressed” index $C$, the operators $A^{i\bar{q}e^{i\Phi_Q}}$ are not necessarily smaller than the $A^{i\bar{q}e^{i\Phi_Q}}$. It is now possible to easily compute the effect of the NP operators on the $\Delta S = 1$ decays mentioned in this paper. However, before doing so, we return to Ref. [17] and consider $B \to \pi K$ decays. In Ref. [18], the relative sizes of the SM $B \to \pi K$ diagrams were roughly estimated as

$$1 : \mid |P| \mid, \quad \mid |P_{EW}| \mid, \quad \mid |P_{SM}| \mid, \quad P_{SM}, \quad P_{EW}, \quad (4)$$

where $\lambda \sim 0.2$. These estimates are expected to hold approximately in the SM. If one ignores the small $\mid \lambda^2 \mid$ diagrams, but includes the NP operators, the $B \to \pi K$ amplitudes $(i, j$ are electric charges) can be written

$$A^{+0} = -P_{\pi e} + A^{i\bar{q}e^{i\Phi_Q}}, \quad (5)$$

$$\sqrt{2} A^{+0} = P_{\pi e}^{i\bar{q}} - T_{\pi e}^{i\bar{q}} - P_{\pi e}^{i\bar{q}} + A^{\text{comb}, e^{i\Phi_Q}} - A^{i\bar{q}e^{i\Phi_Q}},$$

$$A^{--} = P_{\pi e}^{i\bar{q}} - T_{\pi e}^{i\bar{q}} - A^{i\bar{q}e^{i\Phi_Q}},$$

$$\sqrt{2} A^{00} = -P_{\pi e}^{i\bar{q}} - P_{\pi e}^{i\bar{q}} + A^{\text{comb}, e^{i\Phi_Q}} + A^{i\bar{q}e^{i\Phi_Q}},$$

where $A^{\text{comb}, e^{i\Phi_Q}} \equiv A^{i\bar{q}e^{i\Phi_Q}}$, $A^{i\bar{q}e^{i\Phi_Q}}$. It is not possible to distinguish the two component amplitudes in $B \to \pi K$ decays. $\gamma$ is the SM weak phase.

The NP operators mentioned above correspond to the decay $B \to \pi K$. But the other $\Delta S = 1$ decays in the U-spin list include $B_{s}^{0} \to K^{+}K^{-}$, $B_{s}^{0} \to K^{0}\bar{K}^{0}$, and $B_{b}^{0} \to \pi^{+}\pi^{-}$. One might think that the NP operators affecting these decays bear no relation to those in $B \to \pi K$. In fact, this is not true: the other $\Delta S = 1$ decays are the same as $B \to \pi K$ in the limit of flavour SU(3) (which treats $u, d$ and $s$ quarks identically). The point is that the matrix elements differ only in the quarks involved, which affects their hadronization. If all quarks are identical [flavour SU(3)], then the hadronization is the same (the NP affects this hadronization only at the level of $m_b/M_{NP}$, which is tiny). Thus, the NP operators for all $\Delta S = 1$ decays are equal, up to SU(3)-breaking effects. We will therefore denote all NP operators as $A^{i\bar{q}e^{i\Phi_Q}}$, $A^{i\bar{q}e^{i\Phi_Q}}$, $A^{i\bar{q}e^{i\Phi_Q}}$, and $A^{\text{comb}, e^{i\Phi_Q}}$. We can now compute the contribution of NP operators to all $\Delta S = 1$ decays in the U-spin list. $B_{s}^{0} \to K^{+}K^{-}$ (pair #1) and $B_{s}^{0} \to K^{0}\bar{K}^{0}$ (pair #2) receive a NP contribution of the form $A^{i\bar{q}e^{i\Phi_Q}}$; $B_{s}^{0} \to K^{0}\pi^{0}$ (pair #3) receives $A^{i\bar{q}e^{i\Phi_Q}} + A^{i\bar{q}e^{i\Phi_Q}} + A^{i\bar{q}e^{i\Phi_Q}}$. $B_{b}^{0} \to K^{0}\pi^{0}$ (pair #4) and $B_{s}^{0} \to K^{0}\bar{K}^{0}$ (pair #5) receive $A^{i\bar{q}e^{i\Phi_Q}}$, depending on the expected size of the NP operators, not all decays will be equally affected by the NP. In particular, if a given NP operator is expected to be small, Eq. 2 will be satisfied for this pair of decays. Conversely, if Eq. 2 is violated for a particular decay pair, this points to the presence of a specific NP operator.

Note that the NP contribution to $B_{s}^{0} \to \pi^{+}\pi^{-}$ (pair #6) is not an operator that enters $B \to \pi K$ decays. As such, it will not be constrained by such decays. However,
as mentioned above, the processes in this pair are both annihilation-type decays, and their branching ratios are considerably suppressed compared to those of the other B decays. For this reason, we ignore this pair from here on.

<table>
<thead>
<tr>
<th>Mode</th>
<th>BR[10^-6]</th>
<th>$A_{CP}$</th>
<th>$S_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to \pi^+ K^0$</td>
<td>23.1 ± 1.0</td>
<td>0.009 ± 0.025</td>
<td></td>
</tr>
<tr>
<td>$B^+ \to \pi^0 K^+$</td>
<td>12.8 ± 0.6</td>
<td>0.047 ± 0.026</td>
<td></td>
</tr>
<tr>
<td>$B^0 \to \pi^- K^+$</td>
<td>19.7 ± 0.6</td>
<td>−0.093 ± 0.015</td>
<td></td>
</tr>
<tr>
<td>$B^0_d \to \pi^0 K^0$</td>
<td>10.0 ± 0.6</td>
<td>−0.12 ± 0.11</td>
<td>0.33 ± 0.21</td>
</tr>
</tbody>
</table>

TABLE I: Branching ratios, direct CP asymmetries $A_{CP}$, and mixing-induced CP asymmetry $S_{CP}$ (if applicable) for the four $B \to \pi K$ decay modes. The data is taken from Refs. [8] and [9].

In Ref. [20], a fit was done to the 2006 $B \to \pi K$ data, shown in Table 1. We summarize the results here. If one defines the ratios

$$R = \frac{\tau_{B^+}BR[B^0 \to \pi^- K^+] + BR[B^0 \to \pi^+ K^-]}{\tau_{B^0}BR[B^0_d \to \pi^0 K^0] + BR[B_d \to \pi^0 K^0]} ,$$
$$R_a = \frac{1}{2} BR[B^0 \to \pi^- K^+] + BR[B^0 \to \pi^+ K^-] ,$$
$$R_c = \frac{2}{BR[B^0 \to \pi^0 K^0] + BR[B^0 \to \pi^0 K^0]} ,$$

one can show that the present values of $R$, $R_a$, and $R_c$ agree with the SM [21]. This has led some authors to posit that there is no longer a $B \to \pi K$ puzzle [21, 22]. Unfortunately, this analysis is incomplete.

In Eq. (6), if one ignores the NP contributions, the amplitudes $\sqrt{2A(B^+ \to \pi^+ K^+)}$ and $A(B^0 \to \pi^- K^+)$ are equal, up to a factor of $P_{EW}$ [Eq. (13)]. Since $|P_{EW}| < |P_{C^I}|$, we therefore expect $A_{CP}(B^+ \to \pi^+ K^+)$ to be approximately equal to $A_{CP}(B^0 \to \pi^- K^+)$ in magnitude (multiplicative factors cancel between the numerator and denominator of asymmetries). However, as can be seen in Table 1, these asymmetries are very different. Thus, the present $B \to \pi K$ data cannot be explained by the SM. It is only by considering the CP-violating asymmetries that one realizes this.

The only way to account for the data within the SM is to include a large $C'$. Indeed, if one includes $C'$, a good fit was obtained [24], but $|C'|/T'| = 1.6 ± 0.3$ is required. Since this value is much larger than the naive estimates of Eq. (13), this shows that the $B \to \pi K$ puzzle is still present; the small error indicates that the puzzle is much worse in 2006 than in 2004 [23].

The three NP operators were then included in the fit in Ref. [24], one at a time. It was found that the fit remained poor if $A^{c,u}_{CP} e^{i\Phi_u^C}$ or $A^{c,d}_{CP} e^{i\Phi_d^C}$ was added. That is, large values of these NP operators may be allowed, but there is no experimental evidence that these are needed. Below we therefore assume that $A^{c,u}_{CP} e^{i\Phi_u^C}$ and $A^{c,d}_{CP} e^{i\Phi_d^C}$ are small. It was also found that the discrepancy could be removed if the NP operator $A^{c,c}_{CP} e^{i\Phi_c^C}$ was implied. In summary, the present $B \to \pi K$ puzzle suggests that the NP operators $A^{c,u}_{CP} e^{i\Phi_u^C}$ and $A^{c,d}_{CP} e^{i\Phi_d^C}$ are small, but that $A^{c,c}_{CP} e^{i\Phi_c^C}$ is big. This is what was found previously [24].

In this case, only the decay $B^0_s \to K^0\pi^0$ (pair #3) can be significantly affected by the NP. We therefore conclude that the present $B \to \pi K$ data predicts that, of the six U-spin pairs, one expects a measurable discrepancy with the SM only for pair #3: $B^0_s \to K^0\pi^0$ and $B^0 \to K^0\pi^0$.

In general, measurements have not yet been made to test the U-spin relation between pairs of $B$ decay. The one exception is pair #1. Recently, the branching ratio and direct CP asymmetry of $B^0_s \to \pi^+ K^-$ have been measured by the CDF experiment [24]: $BR(B^0_s \to \pi^+ K^-) = (5.00 \pm 0.75 \pm 1.00) \times 10^{-6}$ and $A^{cp}_{dir} = 0.39 \pm 0.15 \pm 0.08$. Together with the experimental measurements of $B^0 \to \pi^- K^+$ shown in Table 1, we have

$$\frac{1}{A_{CP}(B^0_s \to \pi^+ K^-)} = 4.2 \pm 2.0 ,$$
$$BR(B^0_s \to K^+\pi^-) = 3.9 \pm 1.0 .$$

We therefore see that, although the errors are still large, the two ratios are equal, and that Eq. (2) is satisfied by the current data.

In addition, the SM predicts that $BR(B^0_s \to \pi^+ K^-) \sim (3 \times 10^{-6}) \times 10^{-6}$ [24], in agreement with measurement. Moreover, recent theoretical calculations within the SM [20] make predictions for $B^0_s \to \pi^- K^+$ which are consistent with the results shown in Table 1. This indicates that pair #1 shows no sign of NP. In other words, it supports the idea that $A^{c,c}_{CP} e^{i\Phi_c^C}$ is small.

Finally, above we showed that present $B \to \pi K$ data predicts a measurable U-spin-violating effect only for pair #3 ($B^0_s \to K^0\pi^0$ and $B^0 \to K^0\pi^0$). Obviously, it will be important to check U-spin violation in all the $B$-decay pairs, but special attention will be paid to this pair, since a nonzero effect is expected. In particular, it will be necessary to measure the branching ratio and direct CP asymmetry in $B^0_s \to K^0\pi^0$. This will be challenging.

In fact, measurements of $B^0_s \to K^0\pi^0$ are quite impracticable at present colliders. Since the $B^0_s$ direction cannot be determined at hadron colliders, the $B^0_s$ decay vertex cannot be reconstructed via $K_s \to \pi^+\pi^-$ ($\pi^0 \to \gamma\gamma$ leaves no track). Instead, with a similar technique to that used for $B_s^- \to K^0\pi^0$ measurements, the (Super) $B$ factory must be used for measurements of this decay. However, the direct CP asymmetry in $K^0\pi^0$ requires measurements of the time-dependent CP asymmetry. For this purpose, $Super B$ must have a much better $\Delta t$ resolution than planned (an improvement of an order of magnitude), so that $\Delta m_{B_s}$ measurements can be performed. It is ironic that, although most $B^0_s$ decays can
be well studied by hadron collider experiments, the best machine for a U-spin test turns out to be the Super B factory with a better $\Delta t$ resolution.

In summary, some time ago it was pointed out that, within the standard model (SM), a relation involving branching ratios and direct CP asymmetries [Eq. 2] holds for two $B$ decays that are related by U-spin. There are six decay pairs to which this applies. If this relation is found not to hold for a given pair, this implies the presence of physics beyond the SM in that pair. In this paper, we follow the experimental indications and assume that this new physics (NP) appears only in $b \to s$ decays ($\Delta S = 1$) but does not affect $b \to d$ decays ($\Delta S = 0$). There are only a handful of NP operators that can affect the $\Delta S = 1$ $B$-decay amplitudes. We have shown that, for most decay pairs, to a good approximation these operators are the same as the three appearing in $B \to \pi K$ decays. (The one exception is a pair that requires the annihilation of the initial quarks. The branching ratios for these decays are consequently expected to be extremely small and so we ignore this decay pair.) The fit to the latest $B \to \pi K$ data shows that only one NP operator is found to be large. As a result, only one decay pair -- $B_d^0 \to K^0\pi^0$ and $B_s^0 \to K^0\pi^0$ -- is expected to exhibit U-spin violation. The measurement of U-spin violation in this $B$-decay pair will thus be a test of NP in $B \to \pi K$ decays.

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[22] For example, see F. Schwab, talk given in http://eurphysflavour06.ifae.es.
[24] For recent CDF measurements of the branching ratio and direct CP asymmetry of $B_d^0 \to \pi^+ K^-$, see G. Punzi [CDF Collaboration], talk at the 4th Workshop on the CKM Unitarity Triangle, December 2006, Nagoya University, Nagoya, Japan.