In-medium properties of kaons in a chiral approach

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The first order self-energy corrections of the kaon in the symmetric nuclear matter are calculated from kaon-nucleon scattering matrix elements using a chiral Lagrangian within the framework of relativistic mean field approximation. It shows that the effective mass and the potential of $K^+$ meson are identical with those of $K^-$ meson in the nuclear matter, respectively. The effective mass of the kaon in the nuclear matter decreases with the nuclear density increasing, and is not relevant to the kaon-nucleon Sigma term. The kaon-nucleus potential is positive and increases with the nuclear density. Our results indicate that $K^-$ meson can not be bound in the nuclei.

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The in-medium properties of kaons have caused more attentions of nuclear physicists for many years. It was predicted 20 years ago that Bose-Einstein condensation of $K^-$ mesons is possible in the nuclear matter with a density up to several times of the normal nuclear matter density, which is called kaon-condensation phenomenon [1]. This prediction had inspired great interests of nuclear physicists in the following years [2, 3, 4, 5]. It’s assumed that there exists kaon-condensation in the high density nuclear matter in the core of neutron stars, and hence it can be utilized to soften the equation of state of beta-stable matter in neutron stars [4, 5].

In 1999, by fitting the $K^-$ atomic data, it was predicted that there can be deeply bound kaonic atoms, i.e., kaonic nuclei [6, 7]. Although the experimental data at KEK and of FINUDA collaboration were interpreted as the evidences that deeply bound kaonic atoms exist [8, 9, 10, 11], whether there can be deeply bound states of $K^-$ meson in the nuclei still is an issue of great controversy [12, 13, 14, 15].

Whether or not there exist deeply bound states of $K^-$ depends upon properties of $K^-$ in the nuclear medium, and is closely relevant to the depth of potential-well of $K^-$ in the nuclear matter. Nevertheless, it can be seen from present results that the interaction between the kaon and the nucleus is mainly model-dependent. By fitting the data of $K^-$ atoms, a strong attractive $K^-$-nucleus potential with the depth of 150-200MeV is obtained [16]. However, the calculation based on the chiral coupling channel approach gives the $K^-$-nucleus potential ranging from 85-140MeV [17, 18, 19], and the chiral unitary theory that starts from the bare $K^-N$ interaction predicts an even more shallower $K^-$-nucleus potential in the range of 50 -70 MeV [20, 21]. The medium properties of kaons have also been studied from mean field theories, built within the framework of chiral Lagrangians [22, 23], based on Weise model extended to incorporate kaons [24, 25], or using explicitly quark degrees of freedom [26]. Moreover, the behavior of the effective masses of kaons at the finite nuclear density has been studied in the SU(3) Nambu-Jona-Lasinio model [27].

Although these models give different $K^-$-nucleus potential depths, most of them predict negative values of the $K^-$-nucleus potential in the nuclear matter, i.e., the $K^-$ meson will feel an attractive force in the nucleus. Moreover, many models predict different behaviors of $K^+$ and $K^-$ mesons in the nuclear matter. The effective energy of $K^+$ meson is higher than its energy in vacuum, and increases with the nuclear density. Contrarily, the effective energy of $K^-$ meson is lower than its energy in vacuum, and decreases with the nuclear density [24, 25, 26]. Therefore, it is easy to draw a conclusion that $K^+$ meson can not be bound in the nucleus and whether $K^-$ meson can be bound in the nucleus or not will be determined by the correct calculation on the $K^-$-nucleus potential. In Ref. [18], T. Waas and W. Weise give an increasing effective mass of $K^+$ meson with the nuclear density. However, some other models give different results.

In our previous papers, according to Wick’s theorem, we have studied the in-medium properties of the photon, the scalar meson and the vector meson [28]. In this paper, we will calculate the self-energy of the kaon in the nuclear matter from kaon-nucleon ($KN$) scattering matrix elements using a chiral Lagrangian, and then study the in-medium properties of kaons in the nuclear matter.

According to the one-boson exchange theory, nucleons interact with each other by exchanging mesons. The Weise model provides nucleon-meson coupling Lagrangian density in the form [29, 30, 31, 32, 33]:

$$\mathcal{L} = \bar{\psi} (i\gamma_\mu \partial^\mu - M_N) \psi - g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi$$

$$+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\sigma^2 \omega_\mu \omega^\mu$$

(1)

with

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} b \sigma^3 + \frac{1}{4} c \sigma^4,$$

(2)
where \( \psi, \sigma \) and \( \omega \) denote field operators of the nucleon, the scalar meson and the vector meson, respectively. The contribution of iso-vector mesons to the equation of state of the symmetric nuclear matter is zero within the framework of relativistic mean-field (RMF) approximation, so we only considered the scalar meson \( \sigma \) and the vector meson \( \omega \) in Eq. 4.

In the relativistic mean-field approximation, the effective mass and effective energy of the nucleon are defined as:

\[
M_N^* = M_N + g_\sigma \sigma_0,
\]

and

\[
\epsilon^{(+)}(\vec{p}) = \sqrt{\vec{p}^2 + M_N^*} + g_\omega \omega_0,
\]

where \( \sigma_0, \omega_0 \) are the expectation values of the scalar meson \( \sigma \) and the vector meson \( \omega \) in the nuclear matter, respectively.

In the chiral expansion for mesons and baryons, the \( KN \) interaction Lagrangian to the next-to-leading order can be written as [34, 35, 36]:

\[
\mathcal{L}_{KN}^{chiral} = g_1 \bar{\psi} \gamma^{\mu} \psi [(\partial_\mu \bar{K}) K - \bar{K} \partial_\mu K] + g_2 \bar{\psi} \gamma^5 \gamma^3 \psi K K + g_3 \bar{\psi} \partial_\mu (\bar{K} \partial^\mu K),
\]

(3)

where

\[
K = \begin{pmatrix} K^+ \\ K_0 \end{pmatrix}, \quad \bar{K} = \begin{pmatrix} K^- \\ \bar{K}_0 \end{pmatrix},
\]

and the coupling constants are:

\[
g_1 = \frac{3i}{8f_K}, \quad g_2 = \frac{\Sigma_{KN}}{f_K}, \quad g_3 \approx \left( \frac{0.33}{m_K} \frac{\Sigma_{KN}}{m_K^2} \right)/f_K^2
\]

with \( f_K \approx 93\text{MeV} \) the decay constant of the kaon and \( \Sigma_{KN} \) the Sigma term of \( KN \) interaction. Because we are studying the properties of the kaon in the symmetric nuclear matter, the nucleon isospin correlating terms have been ignored in the \( KN \) interaction Lagrangian. In Eq. 4, the first term corresponds to Tomozawa-Weinberg vector interaction, the second term is \( KN \) scalar interaction term and the last is the off-shell term.

The nucleon field operator and its conjugate operator can be expanded in terms of a complete set of solutions to the Dirac equation:

\[
\psi(x) = \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(\vec{p})} [A_{\lambda \bar{\lambda}} U(\vec{p}, \lambda) \exp \left( i\vec{p} \cdot \vec{x} - i\epsilon^{(+)}(\vec{p})t \right)],
\]

(4)

and

\[
\bar{\psi}(x) = \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(\vec{p})} [A_{\lambda \bar{\lambda}} U(\vec{p}, \lambda) \exp \left( -i\vec{p} \cdot \vec{x} + i\epsilon^{(+)}(\vec{p})t \right)],
\]

(5)

where \( E^*(\vec{p}) = \sqrt{\vec{p}^2 + M_N^*} \), and \( \lambda \) denotes the spin of the nucleon, \( A_{\lambda \bar{\lambda}} \) and \( A_{\lambda \bar{\lambda}}^\dagger \) are the annihilation and creation operators of the nucleon, respectively. We have assumed that there are not antinucleons in the ground state of nuclear matter, thus only positive-energy components are considered in Eqs. 4 and 5.

The kaon field operators with the fixed momentum \( k \) can be expressed as

\[
K^+(k, x) = a_{K^+}(k)e^{-ik \cdot x} + a_{K^+}^\dagger(k)e^{ik \cdot x},
\]

\[
K^-(k, x) = a_{K^-}(k)e^{-ik \cdot x} + a_{K^-}^\dagger(k)e^{ik \cdot x},
\]

\[
K^0(k, x) = a_{K^0}(k)e^{-ik \cdot x} + a_{K^0}^\dagger(k)e^{ik \cdot x},
\]

\[
\bar{K}^0(k, x) = a_{K^0}(k)e^{-ik \cdot x} + a_{K^0}^\dagger(k)e^{ik \cdot x}.
\]

(6)

With the Legendre transformation of the Lagrangian in Eq. 3, the \( KN \) interaction Hamiltonian can be written as

\[
\mathcal{H}_1(x) = \mathcal{H}_1(x) + \mathcal{H}_2(x) + \mathcal{H}_3(x),
\]

(7)
where

\[
\begin{align*}
\mathcal{H}_1 &= g_1 \bar{\psi} \gamma^i \psi \left[ \vec{K} \partial_i K - (\partial_i \vec{K}) K \right], \\
\mathcal{H}_2 &= -g_2 \bar{\psi} \vec{K} K, \\
\mathcal{H}_3 &= g_3 \bar{\psi} \left( \vec{K} \vec{K} - \theta \partial_i \vec{K} \partial^i K \right).
\end{align*}
\]  

(8)

In order to obtain the self-energies of kaons in the nuclear matter, the first order \(KN\) scattering matrix should be calculated firstly:

\[
\hat{S}_1 = -i \int d^4x T[\mathcal{H}_1(x)]
\]

\[
= -i \int d^4x T\{ \mathcal{H}_1(x) + \mathcal{H}_2(x) + \mathcal{H}_3(x) \}.
\]

(9)

The corresponding Feynman diagram is shown in Fig. 1. At the point of the saturation density of the nuclear matter, the first order self-energy correction is enough for us to study the in-medium properties of kaons.

By substituting Eqs. (4), (5), (6) and (8) into Eq. (9), the first order scattering matrix element for \(K^-\) meson in the nuclear matter can be written as

\[
\langle p\lambda,k|\hat{S}_1|p\lambda,k\rangle
= -i(2\pi)^4\{ -2ig_1 \int \frac{d^3p}{(2\pi)^3} \frac{4\vec{p} \cdot \vec{k}}{2E^*(\vec{p})} \theta(p_F - |\vec{p}|) \\
+ [-g_2 + g_3(\omega_k^2 + \vec{k}^2)] \int \frac{d^3p}{(2\pi)^3} \frac{2M_N^*}{E^*(\vec{p})} \theta(p_F - |\vec{p}|) \}
= -i(2\pi)^4\{ 0 + [-g_2 + g_3(\omega_k^2 + \vec{k}^2)] \rho_s \}.
\]

(10)

Considering the number density of protons equals that of neutrons in the symmetric nuclear matter, the scalar density of nucleons \(\rho_s\) in Eq. (10) should be replaced with:

\[
\rho_s = 4 \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{\sqrt{p^2 + M_N^*}^2} \theta(p_F - |\vec{p}|),
\]

where \(p_F\) is the Fermi momentum of nucleons in the nuclear matter. With the Dyson equation for the propagator of kaons in the nuclear matter

\[
\frac{i}{k^2 - m_K^2 + i\epsilon - \Sigma_K} = \frac{i}{k^2 - m_K^2 + i\epsilon} + \frac{i}{k^2 - m_K^2 + i\epsilon}(-i\Sigma_K) \frac{i}{k^2 - m_K^2 + i\epsilon},
\]

the first order self-energy correction of \(K^-\) meson is obtained as:

\[
-i\Sigma_K = -i[-g_2 + g_3(\omega_k^2 + \vec{k}^2)] \rho_s.
\]

(11)

Evidently, the Tomozawa-Weinberg interaction term in Eq. (9) has no contribution to the first order self-energy correction of \(K^-\) meson in the nuclear matter. The first order self-energy correction of \(K^+\), \(K^0\) and \(K^0\) mesons in the nuclear matter can be obtained similarly, and the results are same as that of \(K^-\) meson in Eq. (11).

Taking into account the on-shell relation of the kaon

\[
\omega_k^2 = \vec{k}^2 + m_K^2,
\]

the effective energy and effective mass of the kaon in the nuclear matter can be written as

\[
\omega^* = (1 - 2g_3\rho_s)^{1/2}\omega_k = \left[ 1 - 2\rho_s \left( \frac{0.33}{m_K} - \frac{\Sigma_K}{m_K^2} \right) \right]^{1/2}\omega_k,
\]

(12)

and

\[
\frac{m_K^*}{m_K^2 - (m_K^2g_2 + g_3)\rho_s}^{1/2} = \left[ m_K^2 - \frac{0.33m_K^2\rho_s}{f_K^2} \right]^{1/2},
\]

(13)
In a chiral approach, we found that the effective energy and effective mass of kaons and anti-kaons in the nuclear matter are identical, respectively. Moreover, the effective mass of the kaon is independent of the \( K\Sigma \) Sigma term \( \Sigma_{KN} \). Such result is different from previous models.

The effective mass of the kaon \( m_{K}^* \) as a function of nucleon density \( \rho \) in the framework of relativistic mean-field approximation is shown in Fig. 2 where \( \rho_0 = 0.15fm^{-3} \) is the saturation density of the normal nuclear matter. The solid line denote the results with NL3 parameters[37], and the dashed line is for the NLSH parameters[38]. The effective kaon mass decreases monotonically with the nucleon density increasing. At the point of the saturation density, the effective kaon mass is 473MeV, about 20MeV less than the corresponding value in vacuum. The results for the two different parameter sets are almost same as each other when \( \rho < 1.5\rho_0 \). At higher densities for \( \rho > 1.5\rho_0 \), the effective mass of the kaon is nearly a constant for NL3 parameters, while the result with NLSH parameters decreases continually.

In Ref. [29], the effective mass of the kaon in the nuclear matter is extracted straightly from the equation of motion of the kaon in the relativistic mean-field approximation:

\[
m_{K}^* = \sqrt{\frac{m_{K}^2 - g_{3}\rho_s}{1 + g_{3}\rho_s}}.
\] (14)

At the low density limit \( g_{3}\rho_s \ll 1 \),

\[
m_{K}^* \approx \sqrt{m_{K}^2 - g_{3}\rho_s} - \frac{m_{K}^2 g_{3}\rho_s}{2}.
\]

The equation of the effective mass of the kaon in Eq. (14) is just in agreement with our result in Eq. (13).

The kaon-nucleus potential is defined as the difference between the effective energy of the kaon in the nuclear matter and its energy in vacuum at the limit of zero momentum,

\[
U_K = \omega^*(\vec{k})|_{\vec{k}=0} - \omega(\vec{k})|_{\vec{k}=0} = (1 - 2g_{3}\rho_s)^{1/2}m_K - m_K
\]

\[
= [1 - 2\rho_s(\frac{0.33}{m_K} - \frac{\Sigma_{KN}}{m_K^2})/f_K^2]^{1/2}m_K - m_K.
\] (15)

The kaon-nucleus potential is relevant to the \( K\Sigma \) Sigma term \( \Sigma_{KN} \). In the original paper[36], the authors choose \( \Sigma_{KN} \approx 2m_\pi \) in accordance with the Bonn model[39], while the value \( \Sigma_{KN} = 450 \pm 30MeV \) is favored according to lattice gauge calculations[40]. The kaon-nucleus potentials \( U_K \) at different nuclear matter densities are illustrated in Fig. 3.

It can be seen clearly that both \( K^+ \) meson and \( K^- \) meson have positive potentials in the nuclear matter. Meanwhile, the kaon-nucleus potential \( U_K \) increases with the nucleon number density \( \rho \). It means the effective energy of the kaon in the nuclear matter is higher than the corresponding energy of the kaon in vacuum, and the the effective energy of the kaon at the fixed momentum increases with the nuclear density. \( U_K \) is relevant to the \( K\Sigma \) Sigma term \( \Sigma_{KN} \). At the saturation density \( \rho = \rho_0 \), \( U_K = 44MeV \) for \( \Sigma_{KN} = 450MeV \), and \( U_K = 6.5MeV \) for \( \Sigma_{KN} = 280MeV \). In the range of \( \rho < 1.5\rho_0 \), the kaon-nucleus potential \( U_K \) is nearly independent of the choice of different RMF parameter sets. As \( \rho > 3\rho_0 \), the increase of \( U_K \) becomes slow. At \( \rho = 3\rho_0 \), \( U_K = 70MeV \) for \( \Sigma_{KN} = 450MeV \) and \( U_K = 8MeV \) for \( \Sigma_{KN} = 280MeV \). The positive values of the kaon-nucleus potential in Fig. 3 imply neither \( K \) meson nor \( \bar{K} \) meson can be bound in the nuclei. Obviously, our results do not support the occurrence of kaon condensation in the high density nuclear matter, either.

If the self-consistency is taken into account in the calculation of \( U_K \), the value of kaon mass \( m_K \) in the effective energy of the kaon \( \omega^*(\vec{k}) \) in Eq. (15) should be replaced with the corresponding effective mass of the kaon \( m_{K}^* \) in the nuclear matter, and then the kaon-nucleus potential in the nuclear matter will take the form:

\[
U_K = (1 - 2g_{3}\rho_s)^{1/2}m_{K}^* - m_K
\]

\[
= [1 - 2\rho_s(\frac{0.33}{m_K} - \frac{\Sigma_{KN}}{m_K^2})/f_K^2]^{1/2}[m_{K}^2 - \frac{0.33m_K\rho_s}{f_K^2}]^{1/2} - m_K
\]

\[
\approx \rho_s(\frac{\Sigma_{KN}}{m_K} - \frac{0.99}{2})/f_K^2.
\] (16)
From Eq. (10) we can see even if the $KN$ Sigma term takes its minimum value $\Sigma_{KN} = 280\text{MeV}$, the kaon-nucleus potential is still positive, i.e., $U_{K_{\text{min}}} = U_K(\Sigma_{KN} = 280\text{MeV}) > 0$. It shows once more that there can not be kaonic bound states in the nucleus.

In this paper, we utilized the $KN$ interacting Lagrangian in a chiral approach to figure out the first order self-energy correction of the kaon in the nuclear matter. The calculation results show the effective mass and the potential of $K$ meson are identical with those of $\bar{K}$ meson in the nuclear matter, respectively. The effective mass of the kaon in the nuclear matter decreases with nuclear density increasing, and is not relevant to the $KN$ Sigma term. The potential of the kaon in the nuclear matter is positive and increases with the nuclear density. In a conclusion, as long as the $KN$ interacting Lagrangian given in Eq. (3) is correct, it's impossible that there exist $K^-$ bound states in the nuclei.

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FIG. 1: Feynman diagram on the first order self-energy of the kaon in the nuclear matter. The dashed lines denote the kaon, and the solid lines denote the nucleon.

FIG. 2: The effective mass of the kaon $m_K$ as a function of nucleon density $\rho$ in units of saturation densities $\rho_0$ of the nuclear matter for different parameter sets, and $\rho_0 = 0.15 fm^{-3}$. The solid line denotes the results with NL3 parameters, and the dashed line is for the NLSH parameters.
FIG. 3: The potential of the kaon $U_K$ in the nuclear matter as a function of nucleon density $\rho$ in units of saturation densities $\rho_0$ of the nuclear matter for different values of the $KN$ sigma term, and $\rho_0 = 0.15 fm^{-3}$. The meanings for the solid and dashed lines are same as those in Fig. 2.