Axion Excitation by Intense Laser Fields

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Abstract

Parametric excitation of axions by two beating electromagnetic fields is considered here. This can be used as a new concept for active experiments using the existing ultra-intense laser systems. Comparison is made of this active experimental concept with the present day passive experiments using static magnetic fields.

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I. INTRODUCTION

The axions were proposed about twenty years ago to explain, in the frame of the standard model for particle physics, the observed charge-parity in strong interactions, or in other words, the strong CP problem [1, 2, 3]. These hypothetical elementary particles would have a very small mass (possibly in the meV range) and would couple very weakly with quarks, leptons and photons. For this reason they could be very good candidates to explain the large amount of dark matter in the universe [4].

Various experiments are presently in operation, with the aim of discovering this new particle [5] and very recently a positive result was reported by the PVLAS experiment [6], which however needs to been confirmed before we can conclude for the existence of this new particle [7]. An alternative explanation was advanced, based on a new vacuum QED effect [8], but the amplitude of the signal cannot be explained by these nonlinear vacuum properties [9]. New results, based on independent experimental configurations are therefore necessary in order to arrive at a definite conclusion concerning the existence of axions.

Here we propose a new experimental approach. In contrast with the existing experimental arrangements, which are purely passive, the present proposal is based on the possible excitation of axions by two intense electromagnetic fields interacting in vacuum. We explore a situation where the static magnetic field of the PVLAS experiment is replaced by the magnetic field of an intense laser beam.

The interaction axions with photons in the presence of a static magnetic field is theoretically well understood [10, 11, 12]. This corresponds to an indirect interaction, mediated by quarks, which is known as the Primakov effect. In this work we consider a generalized Primakov process, and establish the coupled mode equations describing the collective interactions between an axion and two photons, in the absence of any static field. Explicit analytical results are derived, and used to estimate the possible observation of axions with the existing Peta-Watt lasers, or with the proposed Exa-Watt systems [13]. Comparison of the expected results with those recently reported in [6] will be made, and the experimental issues will be discussed in detail.
II. AXION PHOTON COUPLING

Axions are elementary excitations of a pseudo-scalar field $a$. Photons couple to the axion field through the Lagrangian density \[4\]

$$L_{int} = \frac{1}{4} g_{a\gamma} F_{ij} \tilde{F}^{ij} a = g_{a\gamma} (\vec{E} \cdot \vec{B}) a$$ \hspace{1cm} (1)

where $g_{a\gamma}$ is the coupling constant, $F$ is the electromagnetic field tensor and $\tilde{F}$ its dual, with $\vec{E}$ and $\vec{B}$ the electric and magnetic fields. It is obvious that, under a CP transformation, this interaction Lagrangian will remain invariant because $\vec{E}$ is a polar vector and $\vec{B}$ is an axial one, therefore $(\vec{E} \cdot \vec{B})$ is a pseudo-scalar. Equation (1) means that an axion can couple to two photons, or in alternative to one photon in the presence of a static static electric or magnetic field. The first option will be considered here. This can be described by coupled evolution equations that can be derived from the above Lagrangian. In terms of the electric and magnetic fields, the coupled equations are written (using $c = 1$) as

$$\left( \partial_t^2 - \nabla^2 \right) \vec{E} = -g_{a\gamma} \vec{B} \partial_t^2 a$$ \hspace{1cm} (2)

and

$$\left( \partial_t^2 - \nabla^2 + m_a^2 \right) a = g_{a\gamma} (\vec{E} \cdot \vec{B})$$ \hspace{1cm} (3)

where $\partial_t \equiv \partial/\partial t$ and $m_a$ is the axion mass. It is obvious from these equations that two electromagnetic waves (or two different photon modes) $i = 1, 2$ can only interact if their polarization stated are nearly identical or at least not orthogonal, in order to guarantee that $\vec{E}_i \cdot \vec{B}_{j\neq i} \neq 0$. In this case they can couple with the axions and exchange energy with the field $a$. Let us now consider an axion field of the form

$$a(\vec{r}, t) = a_0 \exp(i\vec{k} \cdot \vec{r} - i\omega t) + c.c.$$ \hspace{1cm} (4)

and two electromagnetic waves with fields $\vec{E}_1$ and $\vec{E}_2$, such that the total electric field is

$$\vec{E}(\vec{r}, t) = \sum_{j=1,2} \vec{E}_j \exp(i\vec{k}_j \cdot \vec{r} - i\omega_j t) + c.c.$$ \hspace{1cm} (5)

In the absence of coupling between fields ($g_{a\gamma} = 0$), equations (??) and (3) lead to the dispersion relations
\[ \omega^2 = k^2 + m_a^2, \quad k_j^2 = \omega_j^2 \] (6)

The amplitudes \( a_0 \) and \( \vec{E}_j \) will be constant, and the associated magnetic field amplitudes will be given by \( \vec{B}_j = (\vec{k}_j \times \vec{E}_j) / \omega_j \). However, in the presence of coupling \( (g_{a\gamma} \neq 0) \), these amplitudes will become slowly varying functions of space and time. Using a perturbative approach, which is well justified due to the smallness of the coupling parameter, we can derive from the above equations for the fields, evolution equations for the slowly varying amplitudes, as determined by

\[
\left( \partial_t + \vec{k}_j \cdot \nabla \right) \vec{E}_j = -ig_{a\gamma} \frac{\omega^2}{2\omega_l} \vec{B}_l^* a \exp(i\phi)
\]

for \( j = 1, 2 \) and \( l \neq j \), and

\[
\left( \partial_t + \vec{k} \cdot \nabla \right) a = ig_{a\gamma} \frac{1}{2\omega} (\vec{E}_1 \cdot \vec{B}_2 + \vec{E}_2 \cdot \vec{B}_1) \exp(-i\phi)
\]

The phase function \( \phi \) is defined here as

\[
\phi \equiv \phi(\vec{r}, t) = \Delta \vec{k} \cdot \vec{r} - \Delta \omega t
\]

with \( \Delta \vec{k} = \vec{k} - \vec{k}_1 - \vec{k}_2 \) and \( \Delta \omega = \omega - \omega_1 - \omega_2 \). It can easily be recognized that effective coupling between the axion and the photon fields only occurs for \( \Delta k = 0 \) and \( \Delta \omega = 0 \), otherwise coupling will be damped by phase mixing. This exact phase matching condition \( \phi = 0 \) corresponds to energy and momentum conservation in the axion-two photons decay interaction, as determined by

\[
\vec{k} = \vec{k}_1 + \vec{k}_2, \quad \omega = \omega_1 + \omega_2
\]

(10)

It is important to discuss the physical consequences of such stringent conditions. First, if the axion mass was negligible, \( m_a = 0 \), we would be reduced to a purely one-dimensional problem. The two electromagnetic modes (for instance two laser beams) and the axion field that verify both conservation relations would propagate in same direction. However, the existence of a finite axion mass implies that, given the condition \( \Delta \omega = 0 \), a perfect phase matching such that \( \Delta \vec{k} = 0 \) can only be achieved at a small angle \( \beta \) between the two photon modes. This angle is determined by

\[
\cos \beta = 1 - \frac{m_a^2}{2\omega_1 \omega_2}
\]

(11)
For a small mass such that $m_a \ll \omega_{1,2}$ this is a small angle of the order of $\beta \simeq \pm m_a(\omega_1 \omega_2)^{-1/2}$. In principle we could also study the decay $\omega = \omega_1 - \omega_2$. But in can easily be demonstrated that, in this case, there is no real value for $\beta$ that will satisfy the phase matching condition $\vec{k} = \vec{k}_1 - \vec{k}_2$. This is due to the lack of axion momentum associated with the existence of a finite mass. Therefore, the case of an axion with an energy smaller than that of any of the two photons is strictly forbidden.

### III. AXION EXCITATION

Let us then focus on the plausible physical situation where there is a perfect phase matching between three interacting field modes, as defined by equation (10), which means that there is a small angle $\beta$ between the two photon modes, as determined by equation (11). Neglecting group velocity dispersion, which is valid for a very small axion mass $m_a^2 \ll k^2$, such that the axion velocity (in units $c = 1$) is nearly equal to one, $v_a = k/\omega \simeq 1 - m_a^2/2k^2$, we can use

$$\frac{d}{d\tau} \equiv \partial_t + \vec{k}_1 \cdot \nabla \simeq \partial_t + \vec{k}_2 \cdot \nabla \simeq \partial_t + \vec{k} \cdot \nabla \quad (12)$$

This corresponds to consider a nearly one-dimensional configuration, and stays valid for interaction distances $L$ such that $(k - k_j)L \ll 1$. We can then reduce the coupled equations for the amplitudes (7-8) to a much simpler form

$$\frac{d}{d\tau} E_j = ig_{a} \frac{\omega^2 f_i}{2\omega_i} a E_i^* \quad (13)$$

for $j = 1, 2$ and $i \neq j$, and

$$\frac{d}{d\tau} a = ig_{a} \frac{f_a}{2\omega} E_1 E_2 \quad (14)$$

where the quantities $f_{1,2}$ and $f_a$ appearing in these equations are geometrical factors determined by

$$f_j = (\hat{n}_j \times \hat{e}_j^*) \cdot \hat{e}_j^* \quad , \quad f_a = (\hat{n}_1 \times \hat{e}_2) \cdot \hat{e}_1 + (\hat{n}_2 \times \hat{e}_1) \cdot \hat{e}_2 \quad (15)$$

with the unit vectors $\hat{e}_j = \vec{E}_j/|\vec{E}_j|$ and $\hat{n}_j = \vec{k}_j/|\vec{k}_j|$. Notice that, for this nearly one-dimensional problem we have $f_1 \simeq f_2 = f_a/2 = \sin \theta$, where $\theta$ is the angle between the electric field of one of the photon modes $\vec{E}_1$ and the magnetic field of the other photon mode.
\( \vec{B}_2 \). In practical terms this means that we should consider photon modes with nearly parallel electric field polarization directions, propagating at a very small angle \( \beta \) with respect to each other.

It is now useful to consider the case where one of the photon fields, say \( \vec{E}_2 \) is associated with an intense laser such that its amplitude can be taken as constant during the interaction process. This is particularly valid for the present problem of an extremely weak interaction between modes. It also corresponds to the parametric approximation, well known from nonlinear optics. In this case, the above equations reduce to

\[
\frac{d}{d\tau} E_1 = i w_1 a , \quad \frac{d}{d\tau} a = i w_2 E_2
\]

(16)

with

\[
w_1 = g_\alpha \frac{f_1 \omega^2}{2 \omega^2} E_2^* , \quad w_2 = g_\alpha \frac{f_0 \omega}{2 \omega} E_2
\]

(17)

From this we can easily obtain the solutions,

\[
E_1(\tau) = E_1(0) \cos(\Omega \tau) + \frac{i}{\Omega} w_1 a(0) \sin(\Omega \tau)
\]

(18)

and

\[
a(\tau) = i \frac{\Omega}{w_1} E_1(0) \frac{\Omega}{w_1} \sin(\Omega \tau) + a(0) \cos(\Omega \tau)
\]

(19)

with the quantity \( \Omega \) defined by

\[
\Omega^2 = w_1 w_2 = g_\alpha^2 \frac{\omega}{4 \omega_1} f_1 f_0 |E_2|^2
\]

(20)

We conclude that, for a negligible value of the initial axion field \( |a(0)| \ll |E_1(0)\Omega/w_1| \), the following approximate solutions can be used

\[
E_1(\tau) = E_1(0) \cos(\Omega \tau) , \quad a(\tau) = E_1(0) \frac{\Omega}{w_1} \sin(\Omega \tau)
\]

(21)

This results shows that, the stronger the pump intensity \( |E_2|^2 \), the faster is the growth of the axion field. But the axion amplitude \( a(\tau) \) also depends on the initial value of amplitude of the other field \( |E_1(0)| \). This photon field can called the probe field. In contrast with the pump lasers beam, the probe laser beam can be a very low intensity beam. But the most interesting thing about the solutions (18) is that they show the decrease in the probe beam amplitude, \( E_1(\tau) \), and the increase of the axion field \( a(\tau) \) at the expense of both the pump
and the probe fields. This means that the excitation of axions can be monitored by a slight decrease of the number of probe photons with frequency $\omega_1$.

The above results could be generalized to the case where perfect phase matching $\Delta \vec{k} = 0$ cannot be achieved, for $\Delta \omega = 0$. In this case, the amplitude of the axion field would be decreased by a factor of $\Omega/\sqrt{\Omega^2 + |\Delta \vec{k}|^2}$. But this is not relevant to the axion problem. Because of the very week coupling with the photon field, only the most favorable situation of perfect phase matching should be considered.

An experiment using an intense laser beam, in the Peta-Watt domain, could then be conceived. Due to the smallness of the expected coupling parameter $g_{a\gamma}$, the quantity $\Omega \tau$ will certainly be very small, and we can use the following approximation for the expected relative variation of the probe field

$$\frac{\Delta E_1}{E_1(0)} \simeq -\frac{1}{2}(\Omega \tau)^2$$  \hspace{1cm} (22)

where $\Delta E_1 = E_1(\tau) - E_1(0)$. Assuming that $\omega \simeq 2\omega_1$ and using $\sin \theta \simeq 1$, we can write $\Omega^2 \simeq g_{a\gamma}^2 |E_2|^2$. This is written in a unit system such that $\hbar = 1$ and $c = 1$. Turning now to the SI unit system, this can be explicitly written as

$$\Omega^2 = g_{a\gamma}^2 \hbar c^2 I_2$$  \hspace{1cm} (23)

where $I_2$ is the pump laser intensity. The coupling constant is usually expressed in units of $(GeV^{-1})$, and the laser intensity in units of $Watt$ per $cm^2$, which leads to the final value of

$$\Omega \ (s^{-1}) = 3 \times 10^{-8} \sqrt{I_2 \ (Watt/cm^2)} \ g_{a\gamma} \ (GeV^{-1})$$  \hspace{1cm} (24)

Therefore, for the Peta-Watt laser systems presently in operation, such that $I_2 \simeq 10^{22} \ (Watt/cm^2)$, and for an assumed value of $g_{a\gamma} \sim 3 \times 10^{-6} \ (GeV^{-1})$, we obtain the value of $\Omega \simeq 10^{-2} \ (s^{-1})$. For the proposed Exa-watt laser systems, to be built in the future, this parameter would approach $1 \ (s^{-1})$. The use of the above process of axion excitation is therefore not unconceivable.
IV. STATIC MAGNETIC FIELD

It is now useful to compare the efficiency of the two-photon parametric process with the passive detection of axions with a rotating static magnetic field, as used in the recently reported experiments [6]. This case can also be described by the above mode coupled equations, where the pump laser field is replaced by a rotating static magnetic field. The relevant quantity is the projection of this field in the polarization direction of the probe laser field, which is \( B(t) = B_0 \cos(\omega_0 t) \) where \( \omega_0 \ll \omega_1 \) is the rotation angular frequency. Going back to equations (2-3), and replacing equations (4-5) by the following field decompositions

\[
a(\vec{r}, t) = \sum_n a_n \exp(ik_{an} \cdot \vec{r} - i\omega_n t) + c.c.
\]

where \( n \) is an integer, and

\[
\vec{E}(\vec{r}, t) = \sum_n \vec{E}_n \exp(ik_{n} \cdot \vec{r} - i\omega_n t) + c.c.
\]

where \( \omega_n = \omega_1 + n\omega_0 \), \( k_n = \omega_n \) and \( k_{an} = \sqrt{\omega_n^2 - m_a^2} \). From a perturbative approach, similar to that used above, we then get the following coupled equations for the evolution of the slowly varying amplitudes amplitudes \( a_n \) and \( E_n \),

\[
\left( \partial_t + \vec{k}_n \cdot \nabla \right) E_n = -ig_{a\gamma} \frac{B_0}{4\omega_n} \left[ \omega_{n-1}^2 a_{n-1} \exp(i\phi) + \omega_{n+1}^2 a_{n+1} \exp(-i\phi') \right]
\]

and

\[
\left( \partial_t + \vec{k}_{an} \cdot \nabla \right) a_n = ig_{a\gamma} \frac{B_0}{2\omega} \left[ E_{n-1} \exp(-i\phi) + E_{n+1} \exp(i\phi') \right]
\]

with the phase factors \( \phi = (k_{a(n-1)} - k_n) \cdot \vec{r} \) and \( \phi' = (k_{a(n+1)} - k_n) \cdot \vec{r} \). For order of magnitude estimates, we can again take the one-dimensional approximation, neglect the phase factors and write

\[
\frac{dE_n}{d\tau} = i \frac{g_{a\gamma}}{4} B_0 \omega_n (a_{n-1} + a_{n+1})
\]

and

\[
\frac{da_n}{d\tau} = ig_{a\gamma} \frac{B_0}{2\omega} (E_{n-1} + E_{n+1})
\]

From where we can easily get

\[
\frac{d^2E_n}{d\tau^2} = -\Omega'(2E_n + E_{n-2} + E_{n+2})
\]
where $\Omega'$ is defined by

$$\Omega'^2 = \frac{1}{2^3 g_{a\gamma}^2 B_0^2} \frac{\omega_n}{\omega}$$  \hspace{1cm} (32)$$

This clearly shows that the axion field is coupled to the even sidebands with frequencies $\omega_{2n}$, originating a cascade of sidebands of the probe laser signal that is very similar to that associated with the vacuum QED effects discussed in reference [8, 9], but no odd sidebands $\omega_{2n+1}$ can be excited. An estimate of the signal associated with the first sideband $\omega_2 = \omega_1 + 2\omega_0$ can be obtained from the above equations, as $E_2(\tau) \sim (g_{a\gamma} \tau)^2 B_0^2 E_0/4$. Comparing the relative amplitude variation $\delta_{\text{static}} = E_2/E_0$ with that due to the parametric axion excitation considered before, as determined by equation (22), $\delta_{\text{laser}} = \Delta E_1/E_1(0)$. The result is

$$\delta_{\text{static}}/\delta_{\text{laser}} \simeq \left(\frac{B_0}{B_2}\right)^2$$  \hspace{1cm} (33)$$

where $B_2$ is the magnetic field associated with the intense pump laser. For an experiment such as that report by [6], the magnetic field $B_0$ is of the order of a 5 Tesla, while the magnetic field of a Peta-Watt laser can attain $10^5$ Tesla. This means that the parametric process is more efficient by a large factor in that respect. However, the interaction time $\tau$ (or equivalently the interaction length) is about $10^4$ meters for the static case, where cavity resonators can be used. In contrast, Peta-Watt laser experiments are not compatible with the use of optical cavities in the interaction area, and possible the interaction lengths at most of the order of 1 meter. This means that the combined balance between field strength and interaction distance, makes these two experimental concepts of nearly equal efficiency. A possible Peta-Watt experiment based on the concept of parametric excitation of axions by intense laser fields, could then be a good candidate for an independent assessment of the axion detection problem.

V. CONCLUSIONS

Active interaction of photons with axions was considered here. The parametric coupling of an axion field with intense electromagnetic wave fields was shown to be possible. The case of a very strong electromagnetic wave, associated for instance to an intense laser beam, coupled to the axion field by a second laser beam was described. This process, involving a pump laser field, which can be considered as constant, and a probe laser field, is described
by equations that are very similar to those used in nonlinear optics. Explicit analytical solutions were derived, and order of magnitude estimates were given.

The efficiency of this active approach to the search for axions was compared with currently used passive methods. In particular, static rotating magnetic fields are presently being used in axion motivated experiments. The advantages and disadvantages of the active method based on the use of intense Peta-Watt laser experiments were emphasized. The present work proposes a new experimental approach to the axion problem, which can enlarge the area of research of the existing and future ultra-intense laser systems and, simultaneously, help to solve the ambiguities of the present day experimental findings.