“Meissner effect” and Blandford-Znajek mechanism in conductive black hole magnetospheres

S.S. Komissarov\(^1\) and Jonathan C. McKinney\(^2\)

\(^1\)Department of Applied Mathematics, the University of Leeds, Leeds, LS2 9GT, UK. E-mail: serguei@maths.leeds.ac.uk
\(^2\)Institute for Theory and Computation, CfA, 60 Garden St., Cambridge, MA, 02138. E-mail: jmckinney@cfa.harvard.edu

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ABSTRACT

The expulsion of axisymmetric magnetic field from the event horizons of rapidly rotating black holes has been seen as an astrophysically important effect that may significantly reduce or even nullify the efficiency of the Blandford-Znajek mechanism of powering the relativistic jets in Active Galactic Nuclei and Gamma Ray Bursts. However, this Meissner-like effect is seen in vacuum solutions of black hole electro-dynamics whereas the Blandford-Znajek mechanism is concerned with plasma-filled magnetospheres. In this paper we argue that conductivity dramatically changes the properties of axisymmetric electromagnetic solutions – even for a maximally rotating Kerr black hole the magnetic field is pulled inside the event horizon. Moreover, the conditions resulting in outgoing Poynting flux in the Blandford-Znajek mechanism exist not on the event horizon but everywhere within the black hole ergosphere. Thus, the Meissner effect is unlikely to be of interest in astrophysics of black holes, at least not in the way this has been suggested so far. These conclusions are supported by the results of time-dependent numerical simulations with three different computer codes. The test problems involve black holes with the rotation parameter ranging from \(a = 0.999\) to \(a = 1\). The pure electrodynamic simulations deal with the structure of conductive magnetospheres of black holes placed in a uniform-at-infinity magnetic field (Wald’s problem) and the magnetohydrodynamic simulations are used to study the magnetospheres arising in the problem of disk accretion.

Key words: Relativity – black hole physics – magnetic fields – galaxies: jets

1 INTRODUCTION

The electromagnetic mechanism of extraction of rotational energy of black holes by Blandford and Znajek (1977) is considered as one of the most promising models for powering the relativistic jets of active galactic nuclei (AGN), gamma-ray bursts (GRBs), and galactic black hole binaries. When this mechanism is described in terms of classical physics the black hole horizon is often compared with a rotating magnetized conductor (Damour 1978; Ruffini & Treves 1973). This viewpoint was canonized in the “Membrane paradigm” by Thorne et al. (1986). Such a description puts black holes on the same footing as say the Sun since the Blandford-Znajek mechanism appears not much different from the mechanism of magnetic braking by Weber & Davis (1967). In this respect the so-called “Meissner effect of black hole electrodynamics,” that is the expulsion of magnetic flux from the horizon of rapidly rotating black holes, seems to undermine the role of the Blandford-Znajek mechanism in astrophysics as “the conductor” becomes unmagnetized.

Wald (1974) found the exact steady-state vacuum solution for a rotating black hole placed into a uniform-at-infinity magnetic field aligned with hole’s rotational axis. For this solution, King et al. (1975) computed the total magnetic flux, \(\Phi\), threading the event horizon as a function of hole’s rotation parameter, \(0 \leq a < 1\). They found that \(\Phi \to 0\) as \(a \to 1\) and, thus, for maximally rotating black holes, the magnetic flux is totally expelled from the black hole horizon (see fig. 1). By its appearance the phenomenon is similar to the Meissner effect, that is the expulsion of magnetic field by superconductors. Later Bícak & Janiš (1985) showed that this result held for all axisymmetric steady-state vacuum solutions and concluded that their finding has a detrimental effect on the prospects of the Blandford-Znajek mechanism. Last year this argument was reiterated by Bícak et al. (2006).

and a number of groups have carried out simulations of magnetized flows around Kerr black holes. None has reported observations of the Meissner-like effect and so it is natural to ask why. McKinney & Gammie (2001) suggested that this could be due to a relatively small value of parameter \( a \) in their simulations – indeed, the effect is not strong unless \( a \) is very close to unity. This explanation has also been put forward in Bicak et al. (2006). However there are other reasons that may be even more important. Indeed, the difference between the systems of equations. The two physical factors that are accounted for in MHD but not in vacuum electrodynamics are the conductivity and inertia of plasma. Both of them have been argued to be of crucial importance for powering relativistic outflows from black holes (Blandford & Znajek 1977, Punsly & Coroniti 1990).

In this paper we show that conductivity alone is sufficient to nullify the Meissner effect even for maximally rotating black holes. In particular, we present the results of electrodynamic simulations for the Kerr black hole with the rotation parameter \( a = 1.0 \) placed in an aligned uniform magnetic field. We also present the results of a general relativistic magnetohydrodynamic (GRMHD) simulation of a realistic magnetized accretion flow with black hole spin \( a = 0.999 \).

The GRMHD model confirms that the inclusion of inertia and thermal effects does not help to revive the Meissner effect.

### 2 ELECTRODYNAMICS OF BLACK HOLES

In this study we only consider the astrophysically most relevant case of test fields, that is we assume that the mass-energy of the electromagnetic field is too small to effect the curvature of space-time. In our analysis we employ the 3+1 formulation of black hole electrodynamics developed in Komissarov (2004a). While the better known formulation of Macdonald and Thorne (1982) is adapted to the Boyer-Lindquist foliation of spacetime that introduces a coordinate singularity on the event horizon, the formulation in Komissarov (2004a) is more general and can be used for the Kerr-Schild foliation as well. This is important because there is no coordinate singularity in the Kerr-Schild foliation, which therefore has advantages for both numerical and analytical studies of physical processes in the vicinity of the event horizon. In particular, this allows us to place the inner boundary of the computational domain \( \text{inside} \) the event horizon. The differential equations of this formulation are identical to the equation of classical electrodynamics in matter:

\[
\nabla \cdot \mathbf{B} = 0, \\
\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0, \\
\nabla \cdot \mathbf{D} = \rho, \\
- \partial_t \mathbf{D} + \nabla \times \mathbf{H} = \mathbf{J},
\]

The purely spacial vectors \( \mathbf{B}, \mathbf{D}, \mathbf{E}, \mathbf{H} \) are related to the Maxwell tensor via

\[
B^i = \frac{1}{2} \epsilon_{ijk} F_{jk},
\]

where \( \epsilon_{ijk} \) is the Levi-Civita tensor of space.

The curved space of black hole behaves as an electromagnetically active medium whose electromagnetic properties are described by the following constitutive equations

\[
\mathbf{E} = \alpha \mathbf{D} + \beta \times \mathbf{B}, \\
\mathbf{H} = \alpha \mathbf{B} - \beta \times \mathbf{D},
\]

where \( \alpha \) is the lapse function and \( \beta \) is the shift vector of the spacetime foliation. Following the general principle of relativity it is most convenient to describe local physical processes in the frames of local inertial observers who are instantaneously at rest in the “absolute space” of the foliation (“fiducial observers” or FIDOs). Such an observer sees \( \mathbf{B} \) as the local magnetic field, \( \mathbf{D} \) as the local electric field, and \( \rho \) as the local electric charge density. The local electric current density, \( j \), is related to \( \mathbf{J} \) of the Ampere equation via

\[
\mathbf{J} = \alpha j - \rho \beta.
\]

Neglecting the inertia of charged particle one can write the generalized Ohm law as

\[
j = \sigma_\parallel \mathbf{D}_\parallel + \sigma_\perp \mathbf{D}_\perp + j_d,
\]

where

\[
j_d = \rho \frac{\mathbf{D}_\times \mathbf{B}}{B^2}
\]

is the drift current, \( \sigma_\parallel \mathbf{D}_\parallel \) is the conductivity current parallel to the magnetic field and \( \sigma_\perp \mathbf{D}_\perp \) is the conductivity current perpendicular to the magnetic field (Komissarov 2004a). In a collisionless plasma with a strong magnetic field the cross-field conductivity is highly suppressed so one can safely use \( \sigma_\perp = 0 \). On the contrary, the parallel conductivity is very large, thus resulting in almost vanishing parallel component of the electric field, \( \mathbf{D}_\parallel \ll \mathbf{B} \).

### 3 COMPUTER SIMULATIONS

In our search for insights into the role of conductivity in the Meissner effect of black hole electrodynamics, we first consider the famous Wald problem (Wald 1974). The steady state solution of vacuum electrodynamics to this problem describes not only the magnetic field but the electric field as well. The magnetic part is

\[
B^\phi = B_0(g_{\phi \theta} + 2ag_{\theta \phi})/\sqrt{\gamma}, \\
B^r = -B_0(g_{\phi \theta} + 2ag_{\theta \phi})/\sqrt{\gamma}, \\
B^\theta = B_0(g_{\phi \phi} + 2ag_{\phi \phi})/\sqrt{\gamma},
\]

where \( t, \phi, r, \theta \) are the Kerr-Schild coordinates, \( g_{ij} \) is the metric tensor of spacetime, and \( \gamma \) is the determinant of the metric tensor of space. The electric part is

\[
E^\phi = 0, \\
E^r = -B_0(g_{\phi \theta} + 2ag_{\theta \phi}), \\
E^\theta = -B_0(g_{\phi \phi} + 2ag_{\phi \phi}).
\]
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Figure 1. Left panel: Steady-state numerical solution for the vacuum case. The magnetic flux is expelled from the event horizon thus illustrating the “Meissner effect” of black hole electrodynamics. Right panel: Numerical solution for the conductive case at time $t = 6GM/c^3$. The magnetic flux is pulled back onto the horizon. The unit length corresponds to $GM/c^2$. The thick line shows the black hole ergosphere ($a = 1.0$).

Figure 2. Left panel: Similar to the right panel of figure 1 showing the steady-state numerical solution for the highly conducting case. All of the flux passing through the ergosphere threads the black hole. Right panel: Same model but shows field angular frequency of rotation, where for $x > 0$ there are 11 contours from $\Omega_F = 0$ to $\Omega_F = \Omega_H/2$. This demonstrates presence of cylindrically collimated Poynting jet.

(The components of $B$ and $E$ are given in the non-normalized coordinate bases of Kerr-Schild coordinates.) The emergence of the electric component in this solution is a particular example of the electromagnetic activity of curved space-times. This effect is of great astrophysical importance as it leads to the Blandford-Znajek process.

The equations of conductive electrodynamics of black holes are more complicated than those of vacuum electrodynamics and the analytic approach has not been very successful so far. Fortunately, there are now a number of numerical techniques that one may use. In our study we first used the upwind scheme described in Komissarov (2004a) and set $\sigma_\parallel = 1/\Delta t$ where $\Delta t$ is the computational timestep. Since $\Delta t$ is much smaller than the light crossing time of the magnetosphere, the solution is expected to be fairly close to one with infinite conductivity, and this is confirmed
by the results. We used solution (14–15) as the initial solution of our numerical simulations and to make sure that the Meissner effect is apparent we set $\alpha = 1.0$, that corresponds to a maximally rotating black hole. We carried out two runs: one with vanishing conductivity, $\sigma_\parallel = 0$, and the other with $\sigma_\parallel = 1/\Delta t$, which corresponds to the highly conductive case (except in current sheets). The left panel of figure 1 shows the steady-state numerical solution of vacuum equations. One can see that magnetic flux is expelled from the black hole horizon. In fact, on this plot the numerical solution is indistinguishable from the exact analytic solution. The right panel of figure 1 shows the conductive case at time $t = 6GM/c^3$. Now the magnetic flux is no longer expelled from the event horizon but on the contrary the magnetic field lines are actually attracted to it. By this time the solution starts to develop the dissipative equatorial current sheet (Komissarov 2004a). Unfortunately, the code cannot handle this kind of current sheets where the electric field tends to become stronger than the magnetic field. It turns out that the numerical structure of such current sheets in this code is controlled by numerical resistivity (Komissarov 2006). Thus this scheme gives reliable results only until the ergospheric current sheet is formed.

McKinney (2006a) constructed a different numerical scheme that provides greater control over the influx of energy into such current sheets thus allowing them to evolve to an almost dissipationless state. Figure 2 shows the quasi-steady-state solution to the Wald problem with $\alpha = 1.0$ obtained with this code (a low level unsteady reconnection controlled by numerical resistivity occurs in the equatorial current sheet.) The left panel of figure 2 shows the magnetic field lines and the right panel shows their angular velocity, $\Omega_F = -E_\theta/\sqrt{\gamma} B^\theta$, where $\gamma$ is the determinant of the metric tensor of space. This result is similar to that obtained by Komissarov (2005) for the GRMHD Wald problem with $\alpha = 0.9$ where the reconnection rate in the equatorial current sheet is inhibited by the high thermal pressure of locally heated plasma. The Blandford-Znajek power from the black hole is maximally efficient since much of the magnetosphere rotates with the angular velocity $\Omega_F = \Omega_H/2$, where $\Omega_H$ is the angular velocity of the black hole. The cylindrically collimated Poynting jet is driven by both the black hole and the ergospheric disc.

Finally, we also use the GRMHD code HARM (Gammie et al. 2003; McKinney 2006b) to confirm the results of electrodynamic simulations, to see if the inclusion of plasma inertia and thermal pressure makes any difference, and to study the problem in the astrophysically more realistic context of disk accretion. Here we examine a GRMHD solution of a magnetized accretion onto a black hole with $\alpha = 0.999$ and otherwise similar parameters as described in McKinney & Gammie (2004). Figure 3 shows the GRMHD solution for the field geometry and rest-mass density. We find that, as in the previous simulations with a slower rotating black hole (McKinney 2005), there is no sign of magnetic flux expulsion from the event horizon. In the funnel the magnetic geometry is close to the monopole assumed in the model by Blandford and Znajek (1977). Thus, even for very rapidly rotating black holes the Blandford-Znajek effect can drive magnetized jets similar to those seen in simulations with slower rotating black holes (McKinney 2006b; Hawley & Krolik 2006).

![Figure 3](image.png)

**Figure 3.** Quasi-steady GRMHD magnetized accretion disk solution for a black hole with $\alpha = 0.999$ at time $t = 2000GM/c^3$. Contours show the magnetic field lines and the colour image shows logarithm of the rest-mass density. Amount of flux threading black hole is qualitatively similar to lower black hole spins for similar models. The Meissner effect is not observed.

### 4 DISCUSSION AND CONCLUSIONS

The numerical simulations described here show that the “Meissner effect” is nullified when high conductivity, typical for the magnetospheres of astrophysical black holes, is taken into account. What are the reasons for this result and how general is it? In order to answer these questions it is perhaps helpful to speculate on why one would expect the results of vacuum electrodynamics to be applicable under the conditions where conductivity is undoubtedly very high. Consider for example the case of a uniform magnetic field in flat spacetime. This is an exact steady-state solution of vacuum Maxwell’s equations. Now let us uniformly fill the space with static electrically neutral plasma. Obviously, this does not disturb the equilibrium and the magnetic configuration remains unchanged. The same applies to any other equilibrium magnetostatic configuration (we ignore gravity.).

The difference between this case and the case of rotating black holes is in the fact that strong electric field is necessarily induced in the vicinity of a black hole when it is placed in vacuum magnetic field even if the black hole itself has zero electric charge. In the particular case of the Wald problem this “gravito-rotationally induced” electric field is described by equations (14–15). However, this result is quite generic (Komissarov 2004a). Once plasma is introduced in such an electromagnetic field the charged particles of different signs will move in the opposite directions bringing about screening of the electric component. This screening deeply upsets the equilibrium of vacuum solution as the electric and magnetic fields are tightly coupled via equations (9–10). The effect is strongest in the vicinity of the event horizon where the magnitudes of electric and magnetic fields are comparable. Moreover, it has been shown in Komissarov (2004a) that no static distribution of electric charge can give full screening of the electric component within the black hole.
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ergosphere where under this condition the purely poloidal magnetic field is bound to have lower magnitude than the normal component of the electric field. To achieve marginal screening a poloidal electric current must flow through the ergospheric region thus strengthening the magnetic field by creating the azimuthal component that was absent in the vacuum solution. The hoop stress associated with this component tends to pull the magnetic flux back on to the event horizon.

The discussion of the Meissner effect with connection to the Blandford-Znajek mechanism in Bičák et al. (2006) highlights the widely spread misinterpretation of the electromagnetic mechanism by Blandford-Znajek, namely that the energy is generated by the rotating event horizon. In fact, it is not the rotation of the horizon, or the “stretched horizon” (Thorne et al., 1986), that makes possible the electromagnetic extraction of the rotational energy of black holes. It is much more instructive to consider the space itself as rotating and this rotation is particularly strong within the black hole ergosphere where it results in unavoidable rotation of plasma in the same sense as the space (or the black hole). Punsly and Coroniti (1990), were the first who correctly and clearly identified the ergosphere as the key region for the magnetic energy extraction and constructed a plausible model of “inertially driven wind” based on this plasma rotation within the ergosphere. However, in their criticism of the Blandford-Znajek mechanism they failed to see that this mechanism is also based on the extreme conditions existing within the ergosphere and not on the properties of the event horizon. The signature of space rotation, expressed by the shift vector $\beta$, is present outside of the event horizon, where it provides the additional coupling of the electric and magnetic fields that holds even in steady-state configurations (see eqs.10,11). Screening of the gravito-rotationally induced electric field, that is the field generated via this additional coupling, involves generation of stationary poloidal electric current (Komissarov 2004a) and this current remains non-vanishing even when a steady-state is reached. It is this current, driven by the marginally screened ergospheric electric field, that provides for the energy and angular momentum extraction in the Blandford-Znajek mechanism and as far as the magnetic flux is not expelled from the ergosphere, and it is not expelled even in the vacuum solutions, the electromagnetic mechanism will continue to operate. Thus, even if the Meissner effect did hold in the conductive regime this would not be very important for the Blandford-Znajek mechanism.

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