Gauss-Bonnet brane gravity with a confining potential

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Abstract

A brane scenario is envisaged in which the \( m \)-dimensional bulk is endowed with a Gauss-Bonnet term and localization of matter on the brane is achieved by means of a confining potential. The resulting Friedmann equations on the brane are modified by various extra terms that may be interpreted as the X-matter, providing a possible phenomenological explanation for the accelerated expansion of the universe. The age of the universe in this scenario is studied and shown to be consistent with the present observational data.

1 Introduction

Recent years have been witnessing a phenomenal interest in the possibility that our observable four-dimensional (4\( D \)) universe may be viewed as a brane hypersurface embedded in a higher dimensional bulk space. Physical matter fields are confined to this hypersurface, while gravity can propagate in the higher dimensional space-time as well as on the brane. The most popular model in the context of brane world theory is that proposed by Randall and Sundrum (RS). In the so-called RSI model [1], they proposed a mechanism to solve the hierarchy problem with two branes, while in the RSII model [2], they considered a single brane with a positive tension, where 4\( D \) Newtonian gravity is recovered at low energies even if the extra dimension is not compact. This mechanism provides us an alternative to compactification of extra dimensions. The cosmological evolution of such a brane universe has been extensively investigated and interesting effects such as dark radiation or quadratic density term in Friedmann equations have been found [3].

There is a general belief that Einstein gravity is the low-energy limit of a quantum theory of gravity which is still unknown. A promising candidate is the string theory which suggests that in order to have a ghost-free action, quadratic curvature corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term [4]. This term also plays a fundamental role in Chern-Simons gravitational theories [5]. The appearance of higher derivative gravitational terms can also be seen in the renormalization of quantum field theories in curved space-times [6]. From a geometric point of view, the combination of the Einstein-Hilbert and Gauss-Bonnet term constitutes, for 5\( D \) space-times, the most general Lagrangian producing second-order field equations [7]. In four dimensions, the Gauss-Bonnet combination reduces to a total divergence and is dynamically irrelevant. These facts provide a strong motivation for the study of braneworld theories which include a Gauss-Bonnet term. Interestingly, it has been shown that the graviton zero mode is as localized at low energies in the Gauss-Bonnet brane system as in the RSII model [8]-[10] and that the correction of the Newton’s law becomes less pronounced by including the Gauss-Bonnet term [11].

As the discussion above suggests, it comes as no surprise that brane theories with a Gauss-Bonnet term have been generating so much interest over the recent past [12]. One of the results of these investigations is that the Gauss-Bonnet correction to the Friedmann equations has been

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dominant in the early universe, having the form $H^2 \propto \rho^{2/3}$ at high energies. This term arises from the imposition of the generalized Israel junction conditions [13]. However, it has been argued that such junction conditions may not be unique. Indeed, other forms of junction conditions exist, so that different conditions may lead to different physical results [14]. Furthermore, these conditions cannot be used when more than one non-compact extra dimension is involved. To avoid such concerns, an interesting higher-dimensional model was introduced in [15] where particles are trapped on a 4-dimensional hypersurface by the action of the confining potential $V$. In [16], the dynamics of test particles confined to a brane by the action of such potential at the classical and quantum levels were studied and the effects of small perturbations along the extra dimensions investigated. Within the classical limits, test particles remain stable under small perturbations and the effects of the extra dimensions are not felt by them, making them undetectable in this way. The quantum fluctuations of the brane cause the mass of a test particle to become quantized and, interestingly, the Yang-Mills fields appear as quantum effects. Also, in [17], a braneworld model was studied in which matter is confined to the brane through the action of such a potential, rendering the use of any junction condition unnecessary and predicting a geometrical explanation for the accelerated expansion of the universe.

In brane theories the covariant Einstein equations are derived by projecting the bulk equations onto the brane. This was first done in [18] where the Gauss-Codazzi equations together with Israel junction conditions were used to obtain the Einstein field equations on the 3-brane. The same procedure was subsequently used in [19] to obtain the field equations on the brane in the presence of the Gauss-Bonnet term where to confine matter on the brane, the authors used a delta-function in the energy-momentum tensor. A feature of these works is that the effective field equations are too complicated to be physically transparent because the extrinsic curvature cannot be expressed solely by the energy-momentum tensor on the brane. Still, they are important in assessing the effects of the higher order terms appearing in the action. It would therefore be interesting to investigate the effects of such higher order terms when a confining potential, rather than junction conditions, is used to localize matter on the brane.

In this paper, we consider an $m$-dimensional bulk space in the presence of the Gauss-Bonnet term without imposing the $Z_2$ symmetry [20]. As mentioned above, to localize matter on the brane, a confining potential is used rather than a delta-function in the energy-momentum tensor. The resulting Friedmann equation is modified by the appearance of extra terms that behave like the X-matter; the phenomenological model proposed to fit the data explaining accelerated expansion of the universe. Within the framework of our model, we also calculate the age of the universe and show that it is consistent with the bounds set by the modern observational data. We should emphasize here that there is a difference between the model presented in this work and models introduced in [9, 10] in that in the latter no mechanism is introduced to account for the confinement of matter to the brane.

\section{Geometrical considerations}

Consider the background manifold $\nabla_{m-1}$ isometrically embedded in a pseudo-Riemannian manifold $V_m$ by the map $\mathcal{Y}: \nabla_{m-1} \rightarrow V_m$ such that

$$G_{AB}Y^A_{\mu}Y^B_{\nu} = g_{\mu\nu}, \quad G_{AB}Y^A_{\mu}N^B = 0, \quad G_{AB}N^A_N^B = \epsilon = \pm 1.$$  \hspace{1cm} (1)

where $G_{AB}$ ($g_{\mu\nu}$) is the metric of the bulk (brane) space $V_m(\nabla_{m-1})$ in arbitrary coordinates, $\{Y^A\}$ ($\{x^\mu\}$) is the basis of the bulk (brane) and $N^A$ denotes the vector normal to the brane. The perturbation of $\nabla_{m-1}$ in a sufficiently small neighborhood of the brane along an arbitrary transverse direction $\xi$ is given by

$$Z^A(x^\mu, \xi) = Y^A + (\mathcal{L}_\xi Y)^A,$$  \hspace{1cm} (2)

where $\mathcal{L}$ represents the Lie derivative. By choosing $\xi$ orthogonal to the brane, we ensure gauge independency [21] and have perturbations of the embedding along a single orthogonal extra direction.
\( \mathcal{N} \) giving local coordinates of the perturbed brane as

\[
Z^A_{\mu}(x^\nu, \xi) = Y^A_{\mu} + \xi \tilde{N}^A_{\mu}(x^\nu),
\]

where \( \xi \) is a small parameter along \( \mathcal{N}^A \) that parameterizes the extra noncompact dimensions. One can see from equation (2) that since the vectors \( \tilde{N}^A \) depend only on the local coordinates \( x^\mu \), they do not propagate along the extra dimensions

\[
\mathcal{N}^A(x^\mu) = \tilde{N}^A + \xi[\mathcal{N}, \tilde{N}]^A = \tilde{N}^A.
\]

The embedding equations of the perturbed hypersurface \( \mathcal{V}_{m-1} \) is given by

\[
\mathcal{G}_{AB} Z^A_{\mu} Z^B_{\nu} = \mathcal{G}_{\mu\nu}, \quad \mathcal{G}_{AB} Z^A_{\mu} \mathcal{N}^B = 0, \quad \mathcal{G}_{AB} \mathcal{N}^A \mathcal{N}^B = \epsilon = \pm 1.
\]

Denoting by \( \tilde{K}_{\mu\nu} = -\mathcal{G}_{AB} Y^A_{\mu} \mathcal{N}^B_{\nu} \) the extrinsic curvature of the \( \mathcal{V}_{m-1} \), and using (3), the perturbed metric \( g_{\mu\nu} \) can be written as

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} - 2\xi \tilde{K}_{\mu\nu} + \xi^2 \bar{g}^{\alpha\beta} \tilde{K}_{\mu\alpha} \tilde{K}_{\nu\beta},
\]

with the extrinsic curvature of the perturbed brane given by

\[
K_{\mu\nu} = -\mathcal{G}_{AB} Z^A_{\mu} \mathcal{N}^B_{\nu} = \bar{K}_{\mu\nu} - \xi \tilde{K}_{\mu\alpha} \tilde{K}^{\alpha}_{\nu}.
\]

Comparison of equations (6) and (7) then leads to the York relation, describing the metric evolution with respect to the perturbation parameter \( \xi \)

\[
K_{\mu\nu} = -\frac{1}{2} \frac{\partial \mathcal{G}_{\mu\nu}}{\partial \xi}.
\]

The components of the Riemann tensor of the bulk written in the the embedding \( \{ Z^A, \mathcal{N}^A \} \) lead to the Gauss-Codazzi equations given by [24]

\[
\mathcal{R}_{ABCD} Z^A_{\alpha} Z^B_{\beta} Z^C_{\gamma} Z^D_{\delta} = R_{\alpha\beta\gamma\delta} + 2K_{\alpha[\gamma} K_{\beta]\delta]},
\]

\[
\mathcal{R}_{ABCD} Z^A_{\alpha} \mathcal{N}^B_{\beta} Z^C_{\gamma} Z^D_{\delta} = K_{\beta\gamma;\alpha} - K_{\alpha\gamma;\beta},
\]

together with [19]

\[
\mathcal{R}_{ABCD} Z^A_{\mu} Z^C_{\nu} \mathcal{N}^B_{\alpha} N^D = -\mathcal{L}_\mathcal{N} K_{\mu\nu} + \mathcal{K}_{\mu\alpha} K^{\alpha}_{\nu},
\]

where \( \mathcal{R}_{ABCD} \) and \( R_{\alpha\beta\gamma\delta} \) are the Riemann tensors for the bulk and the perturbed brane respectively and \( \mathcal{L}_\mathcal{N} \) denotes the lie derivative in the \( \mathcal{N} \)-direction. Using this projection, the \( m \)-dimensional Riemann curvature and its contractions (the Ricci tensor and scalar curvature) are described by the \( (m-1) \)-dimensional variables on the brane with the normal \( \mathcal{N}_A \) as [19]

\[
\mathcal{R}_{ABCD} = R_{ABCD} - K_{AC} K_{BD} + K_{AD} K_{BC} - N_A K_{BD;C} + N_A K_{BC;D} + N_B K_{AD;C} - N_B K_{CA;D}
\]

\[
- N_C K_{BD;A} + N_C K_{AD;B} + N_D K_{BC;A} - N_D K_{AC;B}
\]

\[
+ N_A N_C K_{BE;E_D} + N_A N_D K_{BE;E_C} - N_B N_C K_{AE;E_D} + N_B N_D K_{AE;E_C}
\]

\[
- N_A N_C \mathcal{L}_\mathcal{N} K_{BD} + N_A N_D \mathcal{L}_\mathcal{N} K_{BC} + N_B N_C \mathcal{L}_\mathcal{N} K_{AD} - N_B N_D \mathcal{L}_\mathcal{N} K_{AC},
\]

\[
\mathcal{R}_{AB} = R_{AB} - K K_{AB} + 2K_{AC} K^{C}_{B} + N_A \left(K_{B;C}^{C} - K_B^C\right) + N_B \left(K_{A;C}^{C} - K_A^C\right)
\]

\[
+ N_A N_B K_{CD} K^{CD} - \mathcal{L}_\mathcal{N} K_{AB} - N_A N_B g^{CD} \mathcal{L}_\mathcal{N} K_{CD},
\]

\[
\mathcal{R} = R - K^2 + 3K_{CD} K^{CD} - 2g^{CD} \mathcal{L}_\mathcal{N} K_{CD}.
\]

The Einstein-Gauss-Bonnet equation is quasi-linear [11], which means that apart from non-singular terms given by the \( (m-1) \)-dimensional variables, it contains only linear terms of \( \mathcal{L}_\mathcal{N} K_{AB} \) with no quadratic terms appearing.
3 Effective field equations

We consider the total action for space-time \((\mathcal{M}, \mathcal{G}_{AB})\) with boundary \((\Sigma, g_{\mu\nu})\) as

\[
S = \frac{1}{2\alpha^4} \int_{\mathcal{M}} d^m X \sqrt{-g}(\mathcal{R} - 2\Lambda^{(b)} + \beta \mathcal{L}_{GB}) + \int_{\Sigma} d^{m-1} x \sqrt{-g}(\mathcal{L}_{\text{surface}} + \mathcal{L}_m).
\]  

(15)

Variation of the total action gives our basic field equations as

\[
G_{AB}^{(b)} + \Lambda^{(b)} \mathcal{G}_{AB} + \beta \mathcal{H}_{AB}^{(b)} = \alpha^* S_{AB},
\]

(16)

where

\[
G_{AB}^{(b)} = \mathcal{R}_{AB} - \frac{1}{2} \mathcal{G}_{AB} \mathcal{R},
\]

(17)

\[
\mathcal{H}_{AB}^{(b)} = 2 \left[ \mathcal{R} \mathcal{R}_{AB} - 2 \mathcal{R}_{AC} \mathcal{R}_{B} - 2 \mathcal{R}^{CD} \mathcal{R}_{ACBD} + \mathcal{R}_{A}^{CDE} \mathcal{R}_{BCDE} \right] - \frac{1}{2} \mathcal{G}_{AB} \mathcal{L}_{GB},
\]

(18)

and

\[
\mathcal{L}_{GB} = \mathcal{R}^2 - 4 \mathcal{R} \mathcal{R}^{AB} + \mathcal{R}^{ABCD} \mathcal{R}_{ABCD}
\]

(19)

is the Gauss-Bonnet term. In the above equation \(\alpha^* = \frac{1}{M_7^{2-\frac{2}{3}}}\) \(M_7\) is the fundamental scale of energy in the bulk space), \(\beta\) is the Gauss-Bonnet coupling constant with dimension \((\text{length})^2\), \(\Lambda^{(b)}\) is the cosmological constant of the bulk and \(S_{AB}\) consists of two parts

\[
S_{AB} = T_{AB} + \frac{1}{2} \mathcal{V} \mathcal{G}_{AB},
\]

(20)

where \(T_{AB} \equiv -2 \frac{\delta \mathcal{L}_{\text{m}}}{\delta g_{AB}} + g_{AB} \mathcal{L}_m\) is the energy-momentum tensor of the matter confined to the brane through the action of the confining potential \(\mathcal{V}\). We require \(\mathcal{V}\) to satisfy three general conditions: firstly, it has a deep minimum on the non-perturbed brane, secondly, depends only on extra coordinates and thirdly, the gauge group representing the subgroup of the isometry group of the bulk space is preserved by it \[16\]. Substituting relations \((12), (13)\) and \((14)\) into Eq. \((16)\), we find the effective equations on the brane as

\[
P_{\mu\nu} = - \frac{1}{2} P g_{\mu\nu} + K_{\mu\rho} K^\rho - g_{\mu\nu} K^\alpha K^\alpha - \mathcal{L}_N K_{\mu\nu} + g_{\mu\nu} g^{\rho\sigma} \mathcal{L}_N K_{\rho\sigma} + 2 \beta \left( H_{\mu\nu} - P \mathcal{L}_N K_{\mu\nu} + 2 P^\rho \mathcal{L}_N K_{\rho\mu} + 2 P^\nu \mathcal{L}_N K_{\rho\nu} + W_{\mu\nu} \mathcal{L}_N K_{\rho\sigma} \right) + \alpha^* S_{AB} Z_A^A Z_B^B - \Lambda^{(b)} g_{\mu\nu},
\]

(21)

\[
= P + \beta \left( (P^2 - 4 P^\alpha P^\beta + P_{\alpha\gamma\delta} P^{\alpha\beta\gamma\delta}) = -2 \alpha^* S_{AB} N_A^A N_B^B + 2 \Lambda^{(b)},
\]

(22)

where

\[
P_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} - K_{\alpha\gamma} K_{\beta\delta} + K_{\alpha\beta} K_{\gamma\delta},
\]

\[
P_{\alpha\beta} = g^{\rho\sigma} P_{\alpha\rho\beta\sigma} = R_{\alpha\beta} - K K_{\alpha\beta} + K_{\alpha\gamma} K_{\beta}^\gamma,
\]

\[
P = g^{\alpha\beta} P_{\alpha\beta} = R - K^2 + K_{\alpha\beta} K^{\alpha\beta},
\]

(23)

\[
H_{\mu\nu} = P P_{\mu\nu} - 2 (P_{\mu\rho} K_{\nu}^\rho + P_{\mu}^{\rho\sigma} P_{\nu\rho\sigma}) + P_{\mu\rho\sigma} P_{\nu\rho\sigma} + 2 K_{\alpha\beta} K^{\alpha\beta} P_{\mu\nu} + P K_{\mu\rho} K_{\nu}^\rho
\]

\[
- 2 (K_{\mu\rho} K_{\nu}^\rho + K_{\nu}^\rho K_{\mu}^\rho) - 2 K_{\rho\mu} K_{\nu}^\rho K_{\rho\nu} - 2 K_{\rho\nu} K_{\mu}^\rho K_{\rho\mu} - \frac{1}{4} g_{\mu\nu} \left[ P^2 - 4 P_{\alpha\beta} P^{\alpha\beta} + P_{\alpha\beta\gamma\delta} P^{\alpha\beta\gamma\delta} \right]
\]

\[
+ g_{\mu\nu} \left[ -K_{\alpha\beta} K_{\gamma\delta} P_{\alpha\beta\gamma\delta} + 2 P P_{\mu\rho} K_{\nu}^\rho + 2 P P_{\nu\rho} K_{\mu}^\rho \right] = 0,
\]

(24)

\[
W_{\mu\nu}^{\rho\sigma} = P g_{\mu\rho} g^{\rho\sigma} - 2 P_{\mu\rho} g^{\rho\sigma} - 2 g_{\mu\nu} P_{\rho\sigma} + 2 P_{\mu\rho\sigma} g^{\rho\sigma} g^{\beta\sigma}.
\]

(25)
In order to find the effective equations on the brane, we have to replace the terms \( \mathcal{L}_N K_{\mu \nu} \) in Eq. (21) with the \((m - 1)\)-dimensional variables on the brane. Using the decomposition of the Riemann tensor into the Weyl curvature, the Ricci tensor and the scalar curvature

\[
R_{ABCD} = C_{ABCD} - \frac{2}{(m - 2)} \left( G_{A[D} R_{C]B} - G_{B[D} R_{C]A} \right) - \frac{2}{(m - 1)(m - 2)} R (G_{A[C} G_{D]B}), \tag{26}
\]

we obtain

\[
\mathcal{L}_N K_{\mu \nu} - \frac{1}{(m - 1)} g_{\mu \nu} g^{\alpha \beta} \mathcal{L}_N K_{\alpha \beta} = -\frac{(m - 2)}{(m - 3)} \mathcal{E}_{\mu \nu} + K_{\mu \rho} K_{\rho \nu} - \frac{1}{(m - 3)} \left[ P_{\mu \nu} - \frac{1}{(m - 1)} P g_{\mu \nu} \right] - \frac{1}{(m - 1)} g_{\mu \nu} K_{\rho \sigma} K^{\rho \sigma}, \tag{27}
\]

where

\[
\mathcal{E}_{\mu \nu} = C_{ABCD} \mathcal{N}^B \mathcal{N}^D Z^A_{\cdot \mu} Z^C_{\cdot \nu},
\]

is the electric part of the Weyl tensor \( C_{ABCD} \). However, since Eq. (27) is a trace free equation, we cannot fix \( \mathcal{L}_N K_{\mu \nu} \) by it. We have to find \( g^{\alpha \beta} \mathcal{L}_N K_{\alpha \beta} \) from the other independent equation. If we take the trace of our basic Eq. (16), we find

\[
(m - 2) R + \beta (m - 4) L_{GB} = -2 \alpha^* S + 2m \Lambda^{(b)}.
\]

Substituting Eqs. (9)-(11) with Eq. (27) into Eq. (29), we obtain

\[
g^{\alpha \beta} \mathcal{L}_N K_{\alpha \beta} = \frac{P}{2} + K_{\alpha \beta} K^{\alpha \beta} + \frac{(\alpha^* S - m \Lambda^{(b)})}{((m - 2) + \beta P(m - 4))} + \frac{\beta (m - 4) I}{2 ((m - 2) + \beta P(m - 4))}, \tag{30}
\]

where

\[
I = P^2 - 8 P_{\alpha \beta} P^{\alpha \beta} - 12 P_{\rho \sigma} \mathcal{E}^{\rho \sigma}.
\]

From Eq. (27) with Eq. (30), we then obtain

\[
\mathcal{L}_N K_{\mu \nu} = \frac{(m - 2)}{(m - 3)} \mathcal{E}_{\mu \nu} - \frac{1}{(m - 3)} \left[ P_{\mu \nu} - \frac{1}{2} P g_{\mu \nu} \right] + \frac{(\alpha^* S - m \Lambda^{(b)})}{((m - 2) + \beta P(m - 4))} g_{\mu \nu}
-K_{\mu \rho} K_{\rho \nu} + \frac{(m - 4) I}{2 ((m - 2) + \beta P(m - 4))} g_{\mu \nu}.
\]

Substituting Eq. (32) into Eq. (21), we obtain the effective gravitational equations on the brane as

\[
\frac{(m - 2)}{(m - 3)} (P_{\mu \nu} + \mathcal{E}_{\mu \nu}) + \beta \left( H^{(1)}_{\mu \nu} + H^{(2)}_{\mu \nu} \right) - \frac{2 (m - 4)^2 \beta^2 I}{(m - 1) ((m - 2) + \beta P(m - 4))} \left[ P_{\mu \nu} - \frac{1}{4} P g_{\mu \nu} \right] - \frac{P g_{\mu \nu}}{2 (m - 3)} = \alpha^* S_{\mu \nu} - \Lambda^{(b)} g_{\mu \nu} + \beta \frac{4 (m - 4)}{(m - 1) ((m - 2) + \beta P(m - 4))} \left[ \frac{(\alpha^* S - m \Lambda^{(b)})}{(m - 1) ((m - 2) + \beta P(m - 4))} \left( P_{\mu \nu} - \frac{1}{4} P g_{\mu \nu} \right) \right]
- \frac{(m - 2)}{(m - 3)} \mathcal{E}_{\mu \nu} + \frac{1}{(m - 1)} g_{\mu \nu}, \tag{33}
\]

where

\[
H^{(1)}_{\mu \nu} = 2 P_{\alpha \beta \gamma \delta} P_{\nu}^{\alpha \beta} - \frac{4 (m - 2)}{(m - 3)} P_{\rho \sigma} P_{\mu \nu} + \frac{6}{(m - 3)} P P_{\mu \nu} - \frac{4 (m - 2)}{(m - 3)} P_{\mu \rho} P_{\nu}^{\rho},
\]

\[
+ \frac{g_{\mu \nu}}{2 (m - 1) (m - 3)} \left[ (m^2 - 9m + 6) P^2 + 4 (m^2 - 4m + 7) P_{\alpha \beta} P^{\alpha \beta} \right] - \left( m^2 - 5m + 6 \right) P_{\alpha \beta \gamma \delta} P^{\alpha \beta \gamma \delta}, \tag{34}
\]

\[
H^{(2)}_{\mu \nu} = -\frac{4 (m - 2)}{(m - 3)} \left( P_{\mu \rho} \mathcal{E}_{\nu} + P_{\nu \rho} \mathcal{E}_{\mu} + P_{\mu \rho \sigma} \mathcal{E}^{\rho \sigma} \right)
+ \frac{2 (m^2 - 7)}{(m - 1) (m - 3)} g_{\mu \nu} P_{\rho \sigma} \mathcal{E}^{\rho \sigma} + \frac{2 (m - 2)}{(m - 3)} P \mathcal{E}_{\mu \nu}. \tag{35}
\]
As was mentioned in the introduction, localization of matter on the brane is realized in this model by the action of a confining potential. Let us take
\[ \alpha \tau_{\mu \nu} = \frac{\alpha^3 (m-3)}{(m-2)} T_{\mu \nu}, \quad \Lambda = \frac{(m-3)}{(m-1)} \Lambda^{(b)}, \]
where \( \alpha \) is the scale of energy on the brane. Now, we rewrite Eq. (33) as an Einstein-type equation with “correction” terms. From Eqs. (22) and (33) we obtain
\[
G_{\mu \nu} = \beta \left( \tilde{H}^{(1)}_{\mu \nu} + \tilde{H}^{(2)}_{\mu \nu} \right) = \alpha \tau_{\mu \nu} + \Lambda g_{\mu \nu} - \frac{\alpha \tau}{(m-1)} g_{\mu \nu} + Q_{\mu \nu} - \mathcal{E}_{\mu \nu}
+ \frac{\beta (P_{\mu \nu} - \frac{1}{4} P g_{\mu \nu})}{[(m-2) + \beta P(m-4)]} \left[ \frac{4(m-4)}{(m-1)} \alpha \tau + \frac{4m(m-4)}{(m-2)} \Lambda \right],
\]
where
\[
\tilde{H}^{(1)}_{\mu \nu} = \frac{2(m-3)}{(m-2)} P_{\mu \alpha \beta \gamma} P_{\nu}^{\alpha \beta \gamma} - 4 P_{\rho \sigma} P_{\mu \rho \nu \sigma} + \frac{6}{(m-2)} P P_{\mu \nu} - 4 P_{\mu \rho} P_{\nu}^{\rho}
- \frac{9 g_{\mu \rho}}{2(m-1)(m-2)} \left[ (-5m+3) P^2 - 4(2m^2 - 8m + 10) P_{\alpha \beta} P_{\alpha \beta} \right]
+ \left( -2 m^2 + 9 m - 9 \right) P_{\alpha \beta \gamma \delta} P_{\mu \nu \alpha \beta \gamma \delta} - \frac{2 \beta}{[(m-2) + \beta P(m-4)]} \frac{(m-4)^2 (m-3)}{(m-1)(m-2)},
\]
\[
\tilde{H}^{(2)}_{\mu \nu} = -4 \left( P_{\mu \rho} \mathcal{E}_{\nu}^{\rho} + P_{\nu \rho} \mathcal{E}_{\mu}^{\rho} + P_{\mu \rho \sigma} \mathcal{E}_{\nu}^{\rho \sigma} \right)
+ \frac{2(m^2 - 7)}{(m-1)(m-2)} g_{\mu \nu} P_{\rho \sigma} \mathcal{E}_{\rho \sigma} + 2 P \mathcal{E}_{\mu \nu}
+ \frac{24 \beta}{[(m-2) + \beta P(m-4)]} \frac{(m-4)^2 (m-3)}{(m-1)(m-2)} \left( P_{\mu \nu} - \frac{1}{4} P g_{\mu \nu} \right) P_{\rho \sigma} \mathcal{E}_{\rho \sigma}.
\]
Here, \( \Lambda \) is the effective cosmological constant in four dimensions with \( Q_{\mu \nu} \) being a completely geometrical quantity given by
\[
Q_{\mu \nu} = K K_{\mu \nu} - K_{\mu \rho} K^{\rho}_{\nu} + \frac{1}{2} \left( K_{\alpha \beta} K^{\alpha \beta} - K^2 \right) g_{\mu \nu}.
\]
A brief discussion on the energy-momentum conservation on the brane would be in order here. The contracted Bianchi identities in the bulk space \( G^{AB(b)};_{;A} = 0 \) and \( \mathcal{H}^{AB(b)};_{;A} = 0 \), using Eq. (16), imply
\[
\left( T^{AB} + \frac{1}{2} \mathcal{V} G^{AB} \right);_{;A} = 0.
\]
Since the potential \( \mathcal{V} \) has a minimum on the brane, the above conservation equation reduces to
\[
\tau^{\mu \nu}_{;\mu} = 0,
\]
As a check of our calculations so far, we note that Eq. (37) reduces to Eq. (32) given in [17] in a 5-dimensional bulk with no Gauss-Bonnet term, that is \( \beta = 0 \). As can be seen from the definition of \( Q_{\mu \nu} \), it is independently a conserved quantity which, according to [20], may be interpreted as an energy-momentum tensor of a dark energy fluid representing the X-matter, also known as the ‘X-Cold-Dark Matter’ (XCDM). This matter has the most general form of the equation of state which is characterized by the following conditions [22]: first it violates the strong energy condition at the present epoch for \( \omega_x < -1/3 \) where \( p_x = \omega_x \rho_x \), second, it is locally stable i.e. \( c_s^2 = \frac{\delta p_x}{\delta \rho_x} \geq 0 \), and third, causality holds good, that is \( c_s \leq 1 \). Ultimately, we have three different types of “matter” on the right hand side of Eq. (37), namely, ordinary confined conserved matter represented by \( \tau_{\mu \nu} \), the matter represented by \( Q_{\mu \nu} \) which will be discussed later and finally, the Weyl matter represented by \( \mathcal{E}_{\mu \nu} \).
4 Modified Friedmann equations

In this section we will analyze the influence of the trace of $\tau_{\mu\nu}$, the extrinsic curvature terms and higher curvature terms on a FRW universe, regarded as a brane embedded in an $m$-dimensional bulk with a Gauss-Bonnet term. The FRW line element is given by

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right]. \quad (43)$$

The confined source is the perfect fluid given in co-moving coordinates by

$$\tau_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + pg_{\mu\nu}, \quad u_{\mu} = -\delta^0_{\mu}, \quad p = (\gamma - 1)\rho. \quad (44)$$

Let us consider an $AdS_m$, $dS_m$ or flat bulk, so that $E_{\mu\nu} = 0$. For late times this assumption seems reasonable because the effects of such a term is negligible. The Codazzi equations (10) reduce to

$$K_{\alpha\gamma\sigma} - K_{\alpha\sigma\gamma} = 0. \quad (45)$$

Using the Yorks relation, it follows that in the FRW space-time (diagonal metric), $K_{\mu\nu}$ is diagonal. After separating the spatial components, the Codazzi equations reduce to

$$K_{\mu\nu,\rho} - K_{\nu\sigma} \Gamma^\sigma_{\mu\rho} = K_{\mu\rho,\nu} - K_{\rho\sigma} \Gamma^\sigma_{\mu\nu}, \quad (46)$$

$$K_{\mu\nu,0} - K_{\mu\nu} \frac{\dot{a}}{a} = -a \dot{a} \left( \delta^1_\mu \delta^1_\nu + r^2 \delta^2_\mu \delta^2_\nu + r^2 \sin^2 \theta \delta^3_\mu \delta^3_\nu \right) K_{00}. \quad (47)$$

The first equation gives $K_{11,\nu} = 0$ for $\nu \neq 1$, since $K_{11}$ does not depend on the spatial coordinates. After defining $K_{11} = b(t)$, where $b(t)$ is an arbitrary function of $t$, the second equation gives

$$K_{00} = -\frac{1}{a^2} \frac{d}{dt} \left( \frac{b}{a} \right). \quad (48)$$

For $\mu, \nu = 2, 3$ we obtain $K_{22} = b(t)r^2$ and $K_{33} = b(t)r^2 \sin^2 \theta$ and generally $(\mu, \nu \neq 0)$

$$K_{\mu\nu} = \frac{b}{a^2} g_{\mu\nu}. \quad (49)$$

We find from Eq. (40) that

$$Q_{\mu\nu} = -\frac{1}{a^2} \left( \frac{bb}{H} - b^2 \right) g_{\mu\nu}, \quad Q_{00} = \frac{3b^2}{a^4}. \quad (50)$$

Denoting $h = \frac{b}{t}$ and $H = \frac{\dot{a}}{a}$, the components of $Q_{\mu\nu}$ become

$$Q_{\mu\nu} = -\frac{b^2}{a^4} \left( \frac{2H}{H} - 1 \right) g_{\mu\nu}, \quad Q_{00} = \frac{3b^2}{a^4}. \quad (51)$$

It would now be interesting to see how the above geometrical interpretation is compared with the X-matter explanation. To this end we define $Q_{\mu\nu}$ as a perfect fluid and write

$$Q_{\mu\nu} = \frac{1}{\alpha} \left[ (\rho_x + p_x) u_{\mu} u_{\nu} + p_x g_{\mu\nu} \right], \quad p_x = (\gamma_x - 1)\rho_x. \quad (52)$$

Comparison with the components of $Q_{\mu\nu}$ given by Eq. (51) gives

$$P_x = -\frac{1}{\alpha} \frac{b^2}{a^4} \left( \frac{2H}{H} - 1 \right), \quad \rho_x = \frac{3}{\alpha} \frac{b^2}{a^4}. \quad (53)$$
Use of the above equations leads to an equation for \( b(t) \)

\[
\frac{\dot{b}}{b} = \frac{1}{2} (4 - 3\gamma_x(t)) \frac{\dot{a}}{a},
\]  

(54)

It is interesting to note that this equation resembles one of the phenomenological candidates for dark energy, the X-matter \([22]\), but in our case this field has a fundamental geometrical justification for the equation of state, having been derived from the term \( Q_{\mu\nu} \) in the Einstein equation (37), itself a result of the extrinsic curvature. If \( \gamma_x \) is taken as a constant, the solution for \( b(t) \) is

\[
b(t) = b_0 a(t)^{\frac{1}{2}(4 - 3\gamma_x)},
\]  

(55)

where \( b_0 \) is an integration constant. Using Eq. (53) and this solution, the energy density of XCDM becomes

\[
\rho_x = \frac{3b_0^2}{\alpha} a^{-3\gamma_x}.
\]  

(56)

It should be noted that we have considered an \( AdS_m, dS_m \) or flat bulk and consequently \( E_{\mu\nu} = 0 \), therefore Eq. (41) leads to \( H^{(2)}_{\mu\nu} = 0 \).

Now, using these relations and Eq. (37), the Friedmann equations, to first order in \( \beta \) become

\[
3H^2 + f(a) + \beta \left[ H^2 A(a) + \dot{H} B(a) + 18H^4 + 4\dot{H}^2 + 16\dot{H} H^2 + \dot{f}(a) \right] + O(\beta^2) = 0,
\]  

(57)

\[
-2\ddot{H} - 3H^2 + g(a) + \beta \left[ H^2 C(a) + \dot{H} D(a) - 18H^4 - \frac{8}{3}\dot{H}^2 - \frac{56}{3}\dot{H} H^2 + g(a) \right] + O(\beta^2) = 0,
\]  

(58)

where

\[
A(a) = 4\Lambda + \frac{20k}{a^2} - 12b_0^2 a^{-3\gamma_x}(3 - 2\gamma_x) - \alpha \rho_0 a^{-3\gamma}(3\gamma),
\]

\[
B(a) = \frac{16\Lambda}{3} + \frac{8k}{a^2} - 4b_0^2 a^{-3\gamma_x}(4 - 3\gamma_x) - 2\alpha \rho_0 a^{-3\gamma},
\]

\[
f(a) = \Lambda + \frac{3k}{a^2} - 3b_0^2 a^{-3\gamma_x} - \frac{3}{4}\alpha \rho_0 a^{-3\gamma},
\]

\[
\dot{f}(a) = -(4 - 8\gamma_x)\Lambda b_0^2 a^{-3\gamma_x} + 3b_0^4 a^{-6\gamma_x} \left[ +3\gamma_x^2 - 8\gamma_x + 6 \right] + 3\alpha \rho_0 b_0^2 a^{-3(\gamma+\gamma_x)}(\gamma - \gamma_x)
\]

\[
+ 6 \left( \frac{k}{a^2} \right)^2 - \frac{4\Lambda k}{3a^2} + k\alpha \rho_0 \left( 2 - 3\gamma \right) a^{-3\gamma - 2} - kb_0^2(20 - 12\gamma_x)a^{-3\gamma_x - 2};
\]  

(59)

and

\[
C(a) = -4\Lambda - \frac{52k}{3a^2} + 4b_0^2 a^{-3\gamma_x} \left( 9 - 7\gamma_x \right) - \alpha \rho_0 \gamma a^{-3\gamma},
\]

\[
D(a) = -\frac{8\Lambda}{9} - \frac{40k}{3a^2} + 4b_0^2 a^{-3\gamma_x} \left( \frac{14}{3} - 2\gamma_x \right) - \frac{2}{3}\alpha \rho_0 a^{-3\gamma},
\]

\[
g(a) = -\Lambda - \frac{k}{a^2} + 3b_0^2 a^{-3\gamma_x} \left( 1 - \gamma_x \right) - \frac{\alpha \gamma}{4}\rho_0 a^{-3\gamma},
\]

\[
\dot{g}(a) = -b_0^4 a^{-6\gamma_x} \left[ 6\gamma_x^2 - 28\gamma_x + 18 \right] + \alpha \rho_0 b_0^2 a^{-3(\gamma+\gamma_x)}(\gamma - \gamma_x) + \Lambda b_0^2 a^{-3\gamma_x} \left( 4 - \frac{4\gamma_x}{3} \right)
\]

\[
- 2 \left( \frac{k}{a^2} \right)^2 - \frac{28\Lambda k}{9a^2} + k\alpha \rho_0 \left( \frac{2}{3} - \gamma \right) a^{-3\gamma - 2} + kb_0^2 \left( \frac{52}{3} - 20\gamma_x \right) a^{-3\gamma_x - 2}.
\]  

(60)

Equations (57) and (58) now constitute our Friedmann equations, having been modified by a number of extra terms. Such terms may be used to offer an explanation for the X-matter discussed earlier. In the next section, we discuss the implications of such terms on the cosmology of our model.
5 Cosmological implications

Recent observational data indicate that our universe is expanding with a positive acceleration. This acceleration is explained in terms of the so-called dark energy which could result from some exotic form of matter with negative pressure. The nature of such dark energy constitutes an open and tantalizing question connecting cosmology and particle physics. The simplest form of dark energy is the vacuum energy (cosmological constant). However, this scenario faces the two well known cosmological and coincidence problems [25]. Another possible form of dark energy is provided by scalar fields. Dark energy can be attributed to the dynamics of a scalar field, called the quintessence [26]. Also a phenomenological explanation based on current observational data is given by the X-matter model which is associated with an exotic fluid characterized by an equation of state \( p_x = w_x \rho_x \). In this section, following the method developed in [17], we show that within the context of the present model, it is possible to have a universe exhibiting accelerated expansion without the need to resort to a cosmological constant term or a scalar field or any other kind of dark energy.

Let us proceed by taking equations (57) and (58) from which, the Hubble parameter can be written as

\[
H^2 = \frac{1}{210} \left[ 2\Lambda - \frac{210k}{a^2} + 210b_0^2a^{-3\gamma_x} - 3 \left( 7 - \frac{27\gamma_x}{2} \right) \alpha \rho_0 a^{-3\gamma_x} \right] - \frac{3}{70\beta} \pm \frac{3}{70\beta} \left\{ 1 + 2\beta \left[ -8\Lambda + \left( \frac{4\gamma_x + 7}{3} \right) \alpha \rho_0 a^{-3\gamma_x} \right] \right\}^{\frac{1}{2}}.
\]

We note that for \( \beta \to 0 \) the negative sign yields a singular term in the above equation and, therefore, we select the positive sign. A qualitative classification of the solutions on the basis of different values of the parameter \( \gamma_x \) can be achieved without solving the equation. If we define the effective potential

\[
V(a) = k - b_0^2a^{-3\gamma_x} + \frac{\alpha \rho_0}{70} \left( 7 - \frac{27\gamma_x}{2} \right) a^{-3\gamma_x+2} + \frac{3a^2}{70\beta} - \frac{3a^2}{70\beta} \left\{ 1 + \frac{2}{3} \beta \left( 4\gamma_x + 7 \right) \alpha \rho_0 a^{-3\gamma_x} \right\}^{\frac{1}{2}},
\]

we then have

\[
\dot{a}^2 + V(a) = 0.
\]

Now, it is possible to deduce the qualitative behavior of the scale factor \( a(t) \), keeping in mind that \( \dot{a}^2 \) should be positive. This behavior is much dependent on the range of the values that \( \gamma_x \) can take. We distinguish the following possibility for having an accelerating universe [23]

\[
0 < \gamma_x < \frac{2}{3}.
\]

We also note that to avoid imaginary values for the effective potential, \( \beta > 0 \). The behavior of the potential for \( k = 0, \pm 1 \), is illustrated in figure 1. This figure shows that, within the context of the present model, the universe exhibits accelerated expansion in the case of a vanishing cosmological constant with \( \beta > 0 \). This case is what we expect from the slope of the expansion in heterotic string theory. It should be mentioned that Gauss-Bonnet theories with a negative coupling constant have been suggested [28] in the past where cosmic acceleration can be achieved, but this is not well motivated by string theory and has problems with stability [29].

The above equation cannot be solved in closed form. However, in two extreme cases corresponding to small and large \( a(t) \), exact solutions can be easily found

\[
a(t) = \left( \frac{3}{2} \right)^{\frac{2}{3}} \left( \frac{130 \rho_0}{140} \right)^{\frac{1}{3}} t^{\frac{1}{3}}, \quad \text{for small } a,
\]

for large \( a \).
Figure 1: Left, the behavior of the potential $V(a)$ for $\gamma = 1$, $\gamma_x = -\frac{1}{3}$ and $k = 0$ middle, $k = 1$ top and $k = -1$ bottom and right, the behavior of the scale factor as a function of $t$ for the same parameters and $k = 0$.

and

$$a(t) = \frac{b_0^2}{4} t^2, \quad \text{for large } a.$$  \hfill (66)

The first solution is of the Einstein de-Sitter type, while the second represents an evidently inflationary of the power-law type. This means that in our model the universe starts as decelerating and finally ends up as accelerating. In the simplest FRW cosmological models with a one-component fluid filling up the universe such behavior is not possible. In the next section we discuss the observational parameters of our model.

6 Age of the universe

The age of the universe in FRW models is given by

$$t_0^E = \frac{1}{H_0} \int_0^1 \frac{dx}{[\Omega_m x + (1 - \Omega_m)]^{\frac{1}{2}}},$$  \hfill (67)

where $H_0^{-1} = 9.8 \times 10^9 h^{-1}$ years and the dimensionless parameter $h$, according to modern data, is about 0.7. Hence, in the flat matter dominated universe with $\Omega_{\text{total}} = 1$ the age of the universe would be only 9.3 Gyr, whereas the oldest globular clusters yield an age of about 12.5 with an uncertainty of 1.5 Gyr [30]. With the assumption of the cosmological constant, the WMAP data [31] yields $t_0 = 13.7 \pm 0.2$ Gyr, and the best estimate of the dynamical age of the universe, coming from CMB data, is $t_0 = 14 \pm 0.5$ Gyr [32].

It would now be interesting to compare the above results and those predicted by the present work to the results obtained in [17] where the same mechanism for localization of matter is used in the absence of the Gauss-Bonnent term and the age of the universe is given by

$$t_0^B = \frac{1}{H_0} \int_0^1 \frac{dx}{(\Omega_m x + \Omega_x x)^{\frac{1}{2}}},$$  \hfill (68)

for a flat, matter dominated universe with $\Omega_m = 0.3$ and $\Omega_x = 0.7$. This leads to a prediction for the age of the universe of about 12 Gyr. Here, with the GB term present, we introduce the dimensionless cosmological parameters

$$\Omega_m = \frac{\alpha \rho_m}{3H_0^2}, \quad \Omega_x = \frac{\alpha \rho_x}{3H_0^2}, \quad \Omega_{\text{GB}} = \beta H_0^2,$$  \hfill (69)

where the subscript zero refers to the present values of the cosmological quantities. The modified Friedmann equation (61) with $\Lambda = 0$ and $k = 0$ is then given by

$$\frac{H^2}{H_0^2} = \Omega_x \left( \frac{a}{a_0} \right)^{-3\gamma_x} - 3 \left( 7 - \frac{27\gamma}{2} \right) \Omega_m \left( \frac{a}{a_0} \right)^{-3\gamma} - \frac{3}{70\Omega_{\text{GB}}} \left( \frac{a}{a_0} \right)^{-3\gamma} \left[ 1 - \left[ 1 + 2(4\gamma + 7) \Omega_{\text{GB}} \Omega_m \left( \frac{a}{a_0} \right)^{-3\gamma} \right]^\frac{1}{2} \right].$$  \hfill (70)
This equation leads to the constraint equation that should be satisfied amongst the cosmological parameters defined above when \( a = a_0 \), that is

\[
1 = \Omega_x - \frac{3}{70} \left( 7 - \frac{27\gamma}{2} \right) \Omega_m - \frac{3}{70\Omega_{GB}} \left[ 1 - \frac{1}{2} \left( 1 + 2(4\gamma + 7)\Omega_{GB}\Omega_m \right) \right],
\]

(71)

which reduces the number of independent \( \Omega \) parameters. We therefore find the age of the universe by direct integration of the Friedmann Eq. (70)

\[
t_0^{GB} = \frac{1}{H_0} \int_0^1 \frac{dx}{\left\{ \Omega_x x^{-3\gamma_x+2} - \frac{3}{70}(7 - \frac{27\gamma}{2})\Omega_m x^{-3\gamma+2} - \frac{3x^2}{70\Omega_{GB}} \left[ 1 - \frac{1}{2} \left( 1 + 2(4\gamma + 7)\Omega_{GB}\Omega_m x^{-3\gamma} \right) \right] \right\}^{\frac{1}{2}}},
\]

(72)

The results are summarized in table 1. It shows that the age of the universe in our model is longer than the FRW model and brane models without the Gauss-Bonnet term in the bulk action [17]. Noting that the dynamical age of the universe depends on the rate of the expansion, we realize that in the presence of the GB term, the universe accelerates faster than brane models without the GB term. We have plotted the age of the universe in three models as a function of the energy density parameter \( \Omega_m \) in figure 2. Figure 3 shows the age of the universe as a function of \( \Omega_m \), corresponding to the values of \( \gamma_x \) and \( \gamma \). In figure 4, we have plotted the age as a function of two parameters, \((\gamma_x,\Omega_m)\) and \((\gamma,\Omega_m)\). Note that the age of the universe increases with decreasing \( \gamma_x, \gamma \) and \( \Omega_m \).

<table>
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<th>( \gamma_x )</th>
<th>( t_0H_0 )</th>
<th>( t_0 ) (Gyr)</th>
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<tr>
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<td>1.061</td>
<td>14.54</td>
</tr>
<tr>
<td>0.2</td>
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<td>14.21</td>
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<tr>
<td>0.3</td>
<td>1.010</td>
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<td>0.4</td>
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</tr>
<tr>
<td>0.5</td>
<td>0.942</td>
<td>12.91</td>
</tr>
<tr>
<td>0.6</td>
<td>0.901</td>
<td>12.34</td>
</tr>
</tbody>
</table>

Table 1: Age of the universe for various values of \( \gamma_x \).

At this point, it would be appropriate to compare these results with other brane world models where the Gauss-Bonnet term is absent. A very noticeable and popular scenario in this connection is the Randall-Sundrum brane model where the effects of the brane parameters and dark energy on the age of the universe have been studied. It has been shown that the effect of the quadratic term \( \rho^2 \), resulting from the imposition of the Israel junction conditions, in the energy density term is to lower significantly the age of the universe. The quadratic term contribution in the energy density even for a small negative pressure contributes effectively as the positive pressure, and makes brane models less accelerating. This problem can be avoided if we accept the dark energy \( p = -\frac{1}{3}\rho \) (phantom matter) on the brane, since it has a very strong influence on increasing the age [33].

Another scenario which has attracted a considerable amount of attention in recent years is the so-called DGP model, proposed by Dvali, Gabadadze and Porrati (DGP) [34] and generalized to cosmology by Defayet [35]. This proposal explains the observed late time acceleration of the expansion of the universe through a large scale modification of gravity coming from the “leakage” of gravity at large scales into an extra dimension, without requiring a non-vanishing cosmological constant [36]. In other words, the bulk gravity sees its own curvature term on the brane as the cosmological constant and accelerates the universe. From the observational viewpoint, it has been shown that such models are in agreement with the most recent cosmological observations. For example, in [36], the authors show that constraints from the SNe Ia+CMB data require a flat universe with \( \Omega_m = 0.3 \) and
Figure 2: The age of universe with the GB term (dot-dashed line), without the GB term (dashed line) and as predicted by the FRW model (solid line), as a function of $\Omega_m$ for $\gamma = 1$, $\gamma_x = \frac{1}{3}$ and $\Omega_{GB} = 10^{-3}$.

Figure 3: Left, the age of universe as predicted by the present model as a function of $\Omega_m$ for different values of $\gamma_x = 0.5, 0.3, 0.1$, bottom, middle and top curves respectively with $\gamma = 1$ and $\Omega_{GB} = 10^{-3}$ and right, the age of universe as a function of $\Omega_m$ for different values of $\gamma = 2, 1.5, 1$, bottom, middle and top curves respectively with $\gamma_x = \frac{1}{3}$ and $\Omega_{GB} = 10^{-3}$.

$\Omega_{r_c} = 0.12$, where $\Omega_{r_c}$ is the density parameter associated with the crossover distance between the $4D$ and $5D$ gravities. Also, the age of the universe in the context of these models have been studied and shown that for a fixed value of $\Omega_m$, the predicted age is longer for larger values of $\Omega_{r_c}$ and therefore these models are more efficient in addressing the problem of estimating the age of the universe [37].

7 Conclusions

In this paper, we have studied the Gauss-Bonnet brane world cosmology where the matter is confined to the brane through the action of a confining potential, rendering the use of any junction condition redundant. The modified Friedmann equations on the brane were obtained and shown to be modified by various terms. As a notable consequence, it was shown that dark energy can emerge as a result of the extrinsic curvature in the Gauss-Bonnet brane world. Therefore, this scenario features the

Figure 4: Age of the universe as predicted by the present model as a function of $\gamma_x$ and $\Omega_m$, left and as a function of $\gamma$ and $\Omega_m$, right. We have taken $\Omega_{GB} = 10^{-3}$. 

possibility of an accelerated expansion at the late stage of the cosmic evolution without the need to invoke either a cosmological constant or a quintessence component. It turns out that if the Gauss-Bonnet coupling is positive, the universe undergoes an accelerated expansion. The addition of the Gauss-Bonnet term has resulted in an estimate for the age of the universe that is compatible with modern observational data, improving on our estimates in a previous work where the same mechanism was used to localized the matter on the brane in the absence of the Gauss-Bonnet term.

References


