Isospin effect in the statistical sequential decay

W. D. Tian,¹ Y. G. Ma,¹ X. Z. Cai,¹ D. Q. Fang,¹ W. Guo,¹ W. Q. Shen,¹ K. Wang,¹ H. W. Wang,¹ and M. Veselsky²

¹Shanghai Institute of Applied Physics, Chinese Academy of Sciences, P. O. Box 800-204, 201800, Shanghai, China
²Institute of Physics, Slovak Academy of Sciences, Dubravska cesta 9, Bratislava, Slovakia

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Isospin effect of the statistical emission fragments from the equilibrated source is investigated in the frame of sequential binary decay implemented into GEMINI code, isoscaling behavior is observed and the dependences of isoscaling parameters α and β on emission fragments size, source size, source isospin asymmetry and excitation energies are studied. Results show that (1) α and β do not depend on fragment size for light and intermediate mass fragments but depend on the size for the residue; (2) No obvious source size dependence of α and β are found; (3) α and β of the light and intermediate mass fragments exhibit linear dependence on the inverse of temperature T; (4) all calculation results satisfy the relationship α = 4C_{sym}[(Z_s/A_s)^2 - (N_s/A_s)^2]/T and β = 4C_{sym}[(N_s/A_s)^2 - (N_s/A_s)^2]/T for light and intermediate mass fragments from different isospin asymmetry sources; and (5) symmetry energy coefficient C_{sym} extracted from simulation results is about 19 ~ 23 MeV which includes both the volume and surface term contributions, of which the surface effect seems to play a significant role in the symmetry energy. The isospin compositions of the bulk residues in sequential decay process show different picture from the emitted light and intermediate mass fragments.

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I. INTRODUCTION

The growing interest in isospin effects in nuclear reactions is motivated by an increasing awareness of the importance of the symmetry term in the nuclear equation of state. The availability of beams with large neutron-to-proton ratio, N/Z, provides the opportunity to explore the symmetry energy in very isospin-asymmetric nuclear systems. In such reactions, isospin degree of freedom has prominent roles and can be served as a valuable probe of the symmetry energy term of the nuclear equation of state. The isotopic composition of the nuclear reaction products contains important information on the role of the isospin on the reaction process. N/Z degree of freedom and its equilibration, as well as the isospin asymmetry dependent terms of the nuclear equation of state (EOS) [32, 33, 34] have motivated detailed measurements of the isotopic distributions of reaction products.

One important observable in heavy-ion collisions for determining the symmetry energy experimentally is the fragment isotopic composition investigated with the recently developed isoscaling approach [3, 7]. The isoscaling approach attempts to isolate the effects of the nuclear symmetry energy in the fragment yields, thus allowing a direct study of the symmetry energy term in the nuclear binding energy during formation of hot fragments. Isoscaling refers to a general exponential relation between the yield ratios of given fragments between two reactions which differ only in their isospin asymmetry (N/Z). In particular, if two reactions, 1 and 2, lead to primary fragments having approximately same temperature but different isospin asymmetry, the ratio R_{21}(N, Z) of the yields of a given fragment (N, Z) from these primary fragments exhibits an exponential dependence on the neutron number N and the atomic number Z by following form:

\[ R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C \exp(\alpha N + \beta Z), \]

where α and β are two scaling parameters and C is an overall normalization constant. This scaling behavior has been observed in a very broad range of reactions [9, 10, 11, 12, 13, 14, 15, 16, 17, 18] and theoretical calculations [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

The aim of the present paper is to investigate the isospin effect in sequential binary decay implemented into GEMINI code by isolating formation of the excited composite system, to learn how sensitive the isoscaling parameters are with respect to the observables characterizing the source’s state, such as N/Z distribution and isotopic characteristics of the evaporated light, intermediate clusters and heavy residues. Sequential binary decay model GEMINI [32, 33, 34] has been successfully used to describe the light particle evaporation, complex fragment emission, and N/Z distribution of the equilibrated compound source.

The article is organized as follows. Section I makes a simple review on the sequential binary decay implemented into GEMINI code, where a brief description of the symmetry energy adopted in the binding energy calculation and the method to extract temperature are given. In Section II isoscaling phenomenon and its dependence on compound source size, excitation energy and isospin asymmetry are presented, the sensitivities of the isoscaling parameters α and β to observables characterizing the state of the source are discussed in detail. Section IV shows the N/Z distribution of the emitted fragments.

*Electronic address: tianwendong@sinap.ac.cn
The contribution from surface effect on the symmetry energy term is discussed in Section VII. The different isospin exhibitions between heavy residue and light evaporation fragment, binary fission fragment are analyzed in following Section VII. Finally a summary is given.

II. MODEL OVERVIEW

GEMINI model [32, 33] calculates the decay of compound nuclei by modes of sequential binary decays. All possible binary divisions from light-particle emission to symmetric division are considered. The model employs a Monte Carlo technique to follow the decay chains of individual compound nuclei through sequential binary decays until the resulting products are unable to undergo further decay.

The decay width for the evaporation of fragments with \( Z \leq 2 \) is calculated using the Hauser-Feschbach formalism [35]. For the emission of a light particle \((Z_1, A_1)\) with spin \( J_1 \) from a system \((Z_0, A_0)\) with excitation energy \( E^* \) and spin \( J_0 \), leaving the residual system \( Z_2, A_2 \) with spin \( J_2 \), the decay width is given by:

\[
\Gamma(Z_1, A_1, Z_2, A_2) = \frac{2J_1 + 1}{2\pi\rho_0} \sum_{l=|J_0-J_2|}^{J_0+J_2} \int_0^{E^*-B-E_{rot}(J_2)} T_1(\varepsilon) \rho_2(U_2, J_2) d\varepsilon.
\]  

\( (2) \)

In this equation \( l \) and \( \varepsilon \) are the orbital angular momentum and kinetic energy of the emitted particle, \( \rho_2(U_2, J_2) \) is the level density of the residual system with thermal excitation energy

\[
U_2 = E^* - B - E_{rot}(J_2) - \varepsilon, \quad (3)
\]

where \( B \) is the binding energy, \( E_{rot}(J_2) \) is the rotation plus deformation energy of the residual system, \( \rho_0 \) is level density of the initial system and \( T_1 \) is the transmission coefficients.

For binary divisions corresponding to the emission of heavier fragment, the decay width is calculated using the transition state formalism of Moretto [36]

\[
\Gamma(Z_1, A_1, Z_2, A_2) = \frac{1}{2\pi\rho_0} \int_{E^* - E_{sad}(J_0)}^\infty \rho_{sad}(U_{sad}, J_0) d\varepsilon, \quad (4)
\]

where \( U_{sad} \) and \( \rho_{sad} \) are the thermal energy and level density of the conditional saddle-point configuration, respectively,

\[
U_{sad} = E^* - E_{sad}(J_0) - \varepsilon, \quad (5)
\]

where \( E_{sad} \) is the deformation plus rotation energy of the saddle-point configuration and \( \varepsilon \) now is the kinetic energy of the translational degree of freedom.

The symmetry energy term due to the neutron-proton excess is presented in calculating the masses of nuclei. For heavy systems \((Z>12)\), the masses of the initial and residual systems are obtained from the Yukawa-plus-exponential model of Krappe, Nix and Sierk [37] without the shell correction, pairing correction term for odd-odd nuclei is included. The parameters for this model are taken from the fit to experimental masses of Möller and Nix [38]. For very light systems \((A\leq12)\), masses of the nuclei are calculated from the experimental ones.

Taking into account the effect of a predicted increase in the symmetry energy associated with the temperature dependence of effective nucleon mass in the surface of the nucleus [10], the kinetic part of the symmetry energy is related to the level spacing at the Fermi surface and so it is also related to the level density parameter \( a_T \), the following temperature-dependent kinetic symmetry energy was therefore included to calculate the temperature dependent level density parameter \( a_T \) in the GEMINI simulations [34, 39]

\[
E_{sym}^{kin}(T) = 0.82247 \frac{1}{a_T^0} - \frac{1}{a_T}(N - Z)^2, \quad (6)
\]

where \( a_0 = (1.64 + 1.8A^{2/3})/15.5. \)

The effective thermal excitation energy of the equilibrated system before light particle evaporation and saddle-point configuration before heavy fragment emission can be got by

\[
E_{ex}^{ther} = U_2 - E_{sym}^{kin}, \quad (7)
\]

or

\[
E_{ex}^{ther} = U_{sad} - E_{sym}^{kin}. \quad (8)
\]

Then the nuclear system temperature is approximately

\[
T = \sqrt{E_{ex}^{ther}/a_T}. \quad (9)
\]

III. ISOSCALING BEHAVIORS

To make a systematic study of source parameters which might influence on the isoscaling behavior, in our

FIG. 1: (Color online) Isoscaling parameters \( \alpha \) (positive values) and \( \beta \) (negative values) as a function of the fragment proton number \( Z \) or neutron number \( N \) from source pair \((Z_s = 75, A_s = 150)\) and \((Z_s = 75, A_s = 168)\) at various excitation energies: \( E_{ex} = 2 \) (solid squares), 3 (open squares), 4 (solid circles), 5 (open circles), 6 (up triangles) MeV/nucleon.
present work several pairs of equilibrated sources are considered at various initial excitation energies $E_{\text{ex}} = 2, 3, 4, 5$ and 6 MeV/nucleon. To avoid possible effects of different magnitudes of Coulomb interaction on isotopic distributions, we first consider pairs of sources with the same proton numbers $Z_s$ but different mass numbers $A_s$. The equilibrated source pairs are chosen at different mass region and system isospin asymmetry $N/Z$, which were divided into three groups according to their atomic number, namely (1) heavy source pairs ($Z_s = 75$, $A_s = 150, 168, 186$ and 204), (2) intermediate mass source pairs ($Z_s = 50, A_s = 100, 110, 120$ and 130) and (3) light mass source pairs ($Z_s = 30, A_s = 60, 66, 72$ and 78), respectively. Couple source pairs with fixed mass numbers $A_s = 100$ or 120 but different isospin asymmetry at excitation energy $E_{\text{ex}} = 5$ MeV/nucleon are also studied, respectively, in order to compare with the source pairs calculation with fixed proton number. Two group source pairs with fixed mass number are investigated: (1) ($A_s = 100, Z_s = 35, 40, 45$ and 50) and (2) ($A_s = 120, Z_s = 45, 50, 55$ and 60). We adopt the widely used convention to denote with the index "2" the more neutron-rich system and with the index "1" the more neutron-poor system. In this situation the value of $\alpha$ is always positive because more neutron-rich clusters will be produced by the neutron-richer source and the value of $\beta$ is always negative. The yield ratios $R_{21}(N, Z)$ are calculated and the corresponding isoscaling behaviors are discussed over all possible decayed fragments.

Fig. 2 plots the isoscaling parameters $\alpha$ and $\beta$ as a function of fragment proton number $Z$ and neutron number $N$ for two different size source pairs at excitation energies 2, 3, 4, 5 and 6 MeV/nucleon: left panel for the source pair: ($Z_s = 30, A_s = 60$ ($N_s/Z_s = 1.0$) and $A_s = 66$ ($N_s/Z_s = 1.2$)) and right panel for the source pair: ($Z_s = 50, A_s = 100$ ($N_s/Z_s = 1.0$) and $A_s = 110$ ($N_s/Z_s = 1.2$)), respectively. We also plot the isoscaling parameters $\alpha$ and $\beta$ for two group source pairs with the fixed mass number in Fig. 3, in which the left panel is for the fixed mass number sources ($A_s = 120, Z_s = 45, 50, 55$ and 60) and the right panel for the fixed mass number sources ($A_s = 100, Z_s = 35, 40, 45$ and 50) at excitation energies 5 MeV/nucleon, respectively.

### A. Dependence of $\alpha$ and $\beta$ on emitted fragment size

Fig. 2 shows that the values of isoscaling parameters $\alpha$ and $\beta$ are essentially flat with the fragment proton number $Z$ or neutron number $N$ except a rising $\beta$ for the large size fragment region at $E_{\text{ex}} = 5$ MeV/nucleon. Average values of isoscaling parameters $\alpha$ and $\beta$ can be calculated over the flat region to discuss the dependence of $\alpha$ and $\beta$ on the properties of emission source, such as excitation energy, source size and source asymmetry of the isospin. The average $\alpha$ is calculated over the range $Z \leq 40$ and the average $\beta$ is calculated over the range $N \leq 42$, which keep both averages are calculated over on the same fragments region.
In Fig. 2 both \( \alpha \) and \( \beta \) for the source pairs of \( Z_s = 30 \) and of \( Z_s = 50 \) are plotted as a function of proton number \( Z \) and neutron number \( N \) of the emitted fragments, respectively, which shows a different trend from Fig. 1. For the light and intermediate mass fragments, namely \( Z \leq 20 \) or \( N \leq 20 \) in the \( Z_s = 30 \) source, \( Z \leq 30 \) or \( N \leq 30 \) in the \( Z_s = 50 \) source (dash line in the figure indicates the location of the turning point), \( \alpha \) and \( \beta \) are basically located in a flat range, and exhibit the excitation energy dependence (in the following subsection we will calculate \( \alpha \) and \( \beta \) over this flat region to discuss the excitation energy dependent roles). However, \( \alpha \) increases with \( Z \) monotonically for heavy fragments (right part of the dash line in fig. 2), and its excitation energy dependence tends disappeared. In contrast, \( \beta \) of the heavy fragments for both \( Z_s = 30 \) and 50 source pairs still exhibit clear excitation energy and neutron number dependences, i.e. \( |\beta| \) presents monotonically decreasing with both \( N \) and \( E_{ex} \). This trend can be possibly attributed to different mobility of neutrons and protons, neutrons are free to emit above their binding energy and dominate the emission channel, while the emission of protons or light charge particles evolves gradually with increasing excitation energy.

In Fig. 3 \( \alpha \) and \( \beta \) are plotted as a function proton number \( Z \) and neutron number \( N \) of emitted fragments, respectively, for emission source pairs with the fixed mass number \( A_s \) but different isospin asymmetry \( N_s/Z_s \). The left panel is for the source mass number \( A_s = 120 \) and the right panel for \( A_s = 100 \). \( \alpha \) and \( \beta \) of emitted light and intermediate mass fragments show similar behavior as in Fig. 2 (for examples, \( Z \) (or \( N \)) \( \leq 30 \) in the left panel and \( Z \) (or \( N \)) \( \leq 25 \) in the right panel), they keep almost constant. \( \alpha \) values of the top group (up-triangles) exhibit slight increase with \( Z \), but the increasing slope is slight comparing with the decreasing slope of the heavy fragments on the right part of the dash line. The top up-triangle symbols in the left panel correspond to the source pair with \( (A_s = 100, Z_s = 45 \) and 60) and the largest isospin asymmetry difference ( \( \Delta(N_s/Z_s) = 0.67 \) ), in the right panel top up-triangle symbols are the results from \( (A_s = 100, Z_s = 45 \) and 60) with its isospin asymmetry difference \( \Delta(N_s/Z_s) = 0.86 \). The slight increase of \( \alpha \) in the left of dash line can be apparently attributed to the too large isospin asymmetry difference. Though \( \alpha \) values of the heavy emission fragments decrease with fragment \( Z \), \( \beta \) values of the light and intermediate fragments still locate on the same plateau.

It can be concluded that for the light and intermediate mass emission fragments (on the left of dash lines in Fig. 1 2 and 3), \( \alpha \) and \( \beta \) display the similar behavior, i.e. they are on the flat region, which can make us to calculate the average \( \alpha \) and \( \beta \) values to discuss the dependence of \( \alpha \) and \( \beta \) on excitation energy, source size and source isospin asymmetry difference in the following two subsections 4.1.

**B. Dependence of \( \alpha \) and \( \beta \) on excitation energy**

The excitation energy and temperature dependences of \( \alpha \) and \( \beta \) are shown in the left and right panels of Fig. 4 for different size and isospin asymmetry source pairs, respectively. Temperature of the initial source are tabulated in Table I, in which the level density parameter \( a_T \) is derived from the Eq. 8 and the temperature are calculated from Eq. 9.

One can see that in the left panel both parameters \( \alpha \) and \( |\beta| \) have a monotonic dependence on the excitation energy and their absolute values decrease with the excitation energy. There is a significant sensitivity to temperature at low excitation energy, \( |\beta| \) generally have higher value than \( \alpha \). In right panel of Fig. 4 \( \alpha \) and \( |\beta| \) show linear dependence on \( 1/T \), that evidences the relationship of \( \alpha = \Delta \mu_n/T \) and \( \beta = \Delta \mu_p/T \) [21]. The slope of the relation between \( \alpha (|\beta|) \) and \( 1/T \) should be the free neutron (proton) chemical potential difference \( \Delta \mu_n (|\Delta \mu_p|) \), the linear dependence of \( \alpha (|\beta|) \) on \( 1/T \) also evidences the constant of free neutron (proton) chemical potential difference \( \Delta \mu_n (|\Delta \mu_p|) \) between two initial sources with asymmetry isospin, which are independent of the excitation energy or temperature.

We can also notice that in all cases of Fig. 4 the slope of \( \alpha \) versus \( 1/T \) is generally greater than that of \( \beta \) versus \( 1/T \), i.e. \( |\Delta \mu_p| > |\Delta \mu_n| \), since \( \Delta \mu_n < 0 \), \( \Delta \mu_p > 0 \), it leads to \( \Delta \mu_p < \Delta \mu_n \). In the isospin symmetry system, like the sources in our calculation in Fig. 4 \( (Z_s = 75, A_s = 150) \), \( (Z_s = 50, A_s = 100) \), \( (Z_s = 30, A_s = 60) \), the free neutron chemical potential equals to the free proton chemical potential, namely \( \mu_{n0} = \mu_{p0} \). Hence in the isospin
asymmetry sources with isospin asymmetry parameter $a_s$ and corresponding temperature $T$ for different source systems and excitation energy.

<table>
<thead>
<tr>
<th>Source</th>
<th>$E_{ex}$ (MeV)</th>
<th>$a_s$</th>
<th>$T$ (MeV) $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_s=75$</td>
<td>2.0</td>
<td>15.3/16.9</td>
<td>4.4/4.5</td>
</tr>
<tr>
<td>$A_s=150/168$</td>
<td>3.0</td>
<td>14.6/16.1</td>
<td>5.5/5.6</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>14.3/15.8</td>
<td>6.5/6.5</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>14.1/15.6</td>
<td>7.3/7.3</td>
</tr>
<tr>
<td></td>
<td>6.0</td>
<td>14.0/15.5</td>
<td>8.0/8.1</td>
</tr>
<tr>
<td>$Z_s=75, A_s=186/204$</td>
<td>5.0</td>
<td>17.1/18.6</td>
<td>7.3/7.3</td>
</tr>
<tr>
<td>$Z_s=50$</td>
<td>2.0</td>
<td>11.0/11.8/12.8</td>
<td>4.2/4.3/4.3</td>
</tr>
<tr>
<td>$A_s=100/110/120$</td>
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<td>10.4/11.3/12.2</td>
<td>5.3/5.4/5.4</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>10.1/10.9/11.8</td>
<td>6.2/6.3/6.3</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>9.9/10.7/11.6</td>
<td>7.1/7.1/7.2</td>
</tr>
<tr>
<td></td>
<td>6.0</td>
<td>9.8/10.6/11.5</td>
<td>7.8/7.9/7.9</td>
</tr>
<tr>
<td>$Z_s=50, A_s=130$</td>
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<td>12.4</td>
<td>7.1</td>
</tr>
<tr>
<td>$Z_s=30$</td>
<td>2.0</td>
<td>7.1/7.1/8.3</td>
<td>4.1/4.3/4.2</td>
</tr>
<tr>
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<td>5.1/5.2/5.2</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>6.6/7.1/7.7</td>
<td>6.1/6.1/6.1</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>6.4/7.0/7.5</td>
<td>6.9/6.9/6.9</td>
</tr>
<tr>
<td></td>
<td>6.0</td>
<td>6.3/6.8/7.4</td>
<td>7.6/7.7/7.6</td>
</tr>
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<td>8.0</td>
<td>6.9</td>
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</tr>
<tr>
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<td>7.1/7.1</td>
<td></td>
</tr>
<tr>
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<td>11.6/11.6</td>
<td>7.0/7.1</td>
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<td></td>
</tr>
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</table>

$^a$refers to the saddle-point configuration temperature

C. Dependences of $\alpha$ and $\beta$ on isospin asymmetry and source size

To explore the origin of isoscaling behavior and the dependence of isoscaling parameters $\alpha$ and $\beta$ on the system size and the isospin composition, we performed calculations on source systems with different sizes and asymmetry ($N_s/Z_s$) values. Symmetry energy coefficient $C_{sym}$ is dependent on not only the nuclear density $\rho$, but also the system temperature [13]. In GEMINI investigation, the nuclear density is set to be around the saturation density $\rho_0$ so that the system density influences on the extraction of symmetry energy coefficient can be neglected. In this case, the symmetry energy coefficient $C_{sym}$ can be seen only temperature dependent. Since the systems have different sizes, the temperatures of the system are slightly different even though the excitation energy is fixed at 5 MeV/nucleon. In this case, in Fig. 5 the temperature is served as a correction factor, we made the temperature correction by $\alpha \times T$ and $\beta \times T$. Fig. 6 depicts $\alpha + T$ as a function of $(Z_s/A_s)_{12}^2 - (Z_s/A_s)_{11}^2$ of the initial source pair (left panel) and $\beta + T$ as a function of $(N_s/A_s)_{12}^2 - (N_s/A_s)_{11}^2$ of the initial source pair (right panel). The source pairs we simulated are either kept constant source charge numbers, namely $Z_s = 75$ pairs ($Z_s = 75$: $A_s = 150, 168, 186$ and 204) (solid squares), $Z_s = 50$ pairs ($A_s = 50$: $A_s = 100, 110, 120$ and 130) (open squares), $Z_s = 30$ pairs ($Z_s = 30$: $A_s = 60, 66$ and 78) (solid circles) or kept constant source sizes, namely $A_s = 100$ pairs ($A_s = 100$: $Z_s = 35, 40, 45$ and 50) and $A_s = 120$ pairs ($A_s = 120$: $Z_s = 45, 50, 55$ and 60) (solid uptriangles). All these systems with different source sizes and isospin asymmetries lie along one line, which illustrates that the isoscaling parameters $\alpha$ and $\beta$ are not sensitive to the system size and charge for the light and intermediate mass fragments. The linear fits of the cal-

![Graph showing the dependence of $\alpha$ and $\beta$ on isospin asymmetry and source size](image-url)
calculation points in Fig. 4 are printed on the figure, the slopes in the left and right panels are 88.15 and 79.66, respectively. This approximate linear relationship has been also observed in many experimental data [12, 13] as well as other model calculations with EES, SMM model [21], AMD [12] and IQMD model [26].

It has been shown in the framework of the grand-canonical limit of the statistical multifragmentation model [21] and in the expanding-emitting source model [13],

\[
\alpha = 4 \frac{C_{sym}}{T} \left( \left( \frac{Z_s}{A_s} \right)_1^2 - \left( \frac{Z_s}{A_s} \right)_2^2 \right) \tag{10}
\]
or

\[
\beta = 4 \frac{C_{sym}}{T} \left( \left( \frac{N_s}{A_s} \right)_1^2 - \left( \frac{N_s}{A_s} \right)_2^2 \right) \tag{11}
\]

Above equations have been proved to be a good approximation by many calculations and experimental data, and are generally adopted to constrain the symmetry energy coefficient \(C_{sym}\) in experiment. Here they are verified by the results in Fig. 5, i.e. the slope of \(\alpha \times T\) with respect to \(\Delta(Z_s/A_s)^2\) or \(\beta \times T\) with respect to \(\Delta(N_s/A_s)^2\). By the fits with Eq. (10) and (11) one can constrain the symmetry energy coefficient, which is 22.06 MeV from \(\alpha\) (in left panel) and 19.92 MeV from \(\beta\) (in right panel). Both are comparable with the values from other models and experiments [12, 13, 15, 21].

IV. \(N/Z\) DISTRIBUTION OF THE DECAYED FRAGMENTS

The isotopic composition of the emission fragments contains important information on the equilibrated source system and the role of the isospin in decay process. Isotopic yields and \(N/Z\) of the emission fragments can achieve knowledge of the isotopic degree of freedom for decay process as well as the isotope asymmetry dependent terms of the nuclear equation of state.

Charge distributions of fragments with the fixed mass numbers \(A\), as well as mass distributions for the fixed \(Z\), are approximately Gaussian, have been reproduced by the grand canonical approximation [13], canonical [21] and micro-canonical models [21, 27]. With a Gaussian distribution for an observable \(X\) (mass, charge or neutron number), \(Y(X) \propto \exp([- (X - \langle X \rangle)^2 / 2 \sigma^2])\), the ratio of this observable for two different systems is given by

\[
\frac{Y_2(X)}{Y_1(X)} = \exp \left[ \text{cst} + X \left( \frac{X_1}{\sigma_1^2} - \frac{X_2}{\sigma_2^2} \right) - \frac{X^2}{2} \left( \frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2} \right) \right], \tag{12}
\]

where \(X_1, X_2\) and \(\sigma_1, \sigma_2\) are the mean values and variances for the two systems, respectively. Therefore \(N/Z\) distribution of the emission fragments may reveal the migration of the isotope freedom in decay process as well as the origin of isoscaling parameter dependence.

Fig. 6 (a) shows the \(N/Z\) centroid of emission fragments \(N/Z\) distribution from sources (\(Z_s = 50, A_s = 150, 168, 186\) and 204) at various excitation energies. There are four group symbols which can be seen clearly in panel (a), from bottom to top in order, they are results of emission fragments from sources \(A_s = 150, 168, 186\) and 204, respectively. Solid line in the figure is the linear fits of \(N/Z\) from source \(Z_s = 75\) and \(A_s = 168\), which approximates to the evaporation attractor line (EAL) in Ref. [41], since this source locates around the EAL. For the source far from the EAL, the centroid of \(N/Z\) distribution for the emission fragments moves forward to the EAL with respect to the source \(N/Z\). For sources locating near the EAL, the centroid of \(N/Z\) distribution for the emission fragments is source temperature independent. For instance, for the \((Z_s = 75, A_s = 168)\) case in panel (a), its centroid of \(N/Z\) distribution shows temperature independent. In contrary, for sources far from EAL, its centroid of emission fragments is excitation energy dependent, i.e. source temperature dependent, the increasing of the excitation energy drives the emission fragments \(N/Z\) towards the EAL, which is evidenced by \(A_s = 150\) and 186 sources in Fig. 6(a). The dashed and dashed-dot lines represent linear fits to the \(N/Z\) centroid of emission fragments from sources \((Z_s = 75, A_s = 150\) and 186) at \(E_{ex} = 5\) MeV/nucleon. The slopes of the linear fits show a dependence of \(N/Z\) of the source, which increases with the \(N/Z\) of source. In Fig. 6 (b) and (c), the variances \(\sigma_{N/Z}\) of \(N/Z\) distribution for the emission fragments from sources \(A_s = 150\) and 168 are plotted, the variance \(\sigma_{N/Z}\) drops exponentially with the emission fragment charge number \(Z\), and is independent of excitation energy.

Fig. 7 (a) and (b) demonstrates the centroid of \(N/Z\) distribution for the emission fragments from sources \((Z_s = 50, A_s = 100, 110\) and 120) as a function of fragment atomic number \(Z\). To illustrate and compare clearly, Fig. 7 (a) shows the results from sources \(Z_s = 50\) with \(A_s = 100\) and 110, and Fig. 7 (b) shows the results from sources \(Z_s = 50\) with \(A_s = 110\) and 120 in order to illustrate and compare these three sources clearly. The solid line in the figure is EAL obtained in Ref. [41], we notice that the centroid of \(N/Z\) distribution for the emission fragments from source \((Z_s = 50, A_s = 110)\) locates around this line for the fragments in left of the vertical dashed line in panel (a) and (b), corresponding to the light and intermediate mass fragments, the vertical dashed line in panel (a) and (b) corresponds to the vertical dashed line in the right panel of Fig. 2 where the turning point separates the flat region of \(\alpha\) and \(\beta\) from the region of enhanced \(\alpha\) and \(\beta\) around \(Z \approx 30\).

On the left part of the dashed line in Fig. 7, the EAL can reproduce the decayed fragments from \((Z_s = 50, A_s = 110)\) source, since this source is nearby to the EAL. Even though the sources of \((Z_s = 50, A_s = 100)\) and \((Z_s = 50, A_s = 120)\) are quite distant from the EAL, the centroid of decayed fragments \(N/Z\) approaches to the EAL much comparing with the source \(N/Z\). For instance, \(N/Z\)
of the source ($Z_s=50, A_s=100$) is 1.0, but the $N/Z$ centroid of its decayed fragments $>1.0$. The $N/Z$ of source ($Z_s=50, A_s=120$) is 1.4, but the centroid of its decayed fragments $N/Z < 1.4$. For the source ($Z_s=50, A_s=110$), the $N/Z$ centroid of the emission fragments from this source lies very close to the EAL and it is excitation energy (source temperature) independent. In contrary, the centroid of emission fragments from source pair ($Z_s=50, A_s=100$ and $A_s=120$) is quite distant from the EAL and presents excitation energy (source temperature) dependent, i.e. higher excitation energy drives the centroid of $N/Z$ distribution for the emission fragments towards the EAL.

However, on the right of the dashed line in Fig. 7 ($Z>30$), the simulation results deviate the EAL significantly. The $N/Z$ centroid of heavy fragments does not lie around the EAL any more, they begin to drop rapidly after some turning points, contrary to the increasing of $N/Z$ with $Z$ of the light and intermediate mass fragments. No matter the source is on the EAL or far from the EAL, its centroid of $N/Z$ for heavy fragments is excitation energy dependent, i.e. higher excitation energy leads to smaller centroid of $N/Z$. Therefore the turning of the $\alpha$ and $\beta$ around $Z (or) N = 30$ from the flat region to the raising region may be attributed to the changing behavior of the centroid of $N/Z$ around $Z = 30$.

Left panel of Fig. 8 shows the centroid of $N/Z$ distribution for the emission fragments from four different isospin asymmetry sources with fixed $Z_s=50$ but different mass number $A_s=100, 110, 120$ and 130 at $E_{ex} = 5$ MeV/nucleon as a function of proton number $Z$. Solid line is the EAL in Ref. [11], other three lines are linear fits of the centroid of $N/Z$ distributions for $Z \leq 30$. The centroids of $N/Z$ distributions from four sources present the similar shape. Right panel of Fig. 8 shows the variance of $N/Z$ distribution for the emission fragments from four different isospin asymmetry sources at $E_{ex} = 5$ MeV/nucleon, the exponential-like declining slope of $\sigma_{N/Z}$ as a function proton number $Z$ increases with the isospin asymmetry $N/Z$ of the sources.

The centroid of $N/Z$ distributions for the emission fragments from four fixed mass number equilibrated sources ($A_s=120, Z_s=45, 50, 55$ and 60) at $E_{ex} = 5$ MeV/nucleon as a function of fragment proton number $Z$ are also plotted in the left panel of Fig. 8. Solid line is the EAL in Ref. [11], other four lines from bottom to top represent the linear fits of the centroid of $N/Z$ for $Z \leq 30$. Within this region, the centroid of $N/Z$ and its linear fit shows similar shape as in Fig. 7. The right panel is the variance of $N/Z$ distribution for the emission fragments from the same source as in left panel, they present similar shape and isospin asymmetry dependence as in right panel of Fig. 8.

We can find that in Fig. 7 and 8 the $N/Z$ centroid of emission fragments raises linearly, which can be fitted by a linear function within the region that corresponds to the flat region of isoscaling parameters $\alpha$ and $\beta$ in Fig. 7 and 8. It can be observed from Fig. 2 and 8 $\alpha$ and $\beta$ are enhanced or suppressed in the heavy fragments region, we can also find that the centroid of $N/Z$ of the heavy fragments does not keep its linear raising...
by changing its proton number $Z$. However, when the source mass number $N$ is fixed while neutron number $N$ varies, this can be observed more clearly in Fig. 8. The bending point on the centroid of $N/Z$ distribution becomes isospin asymmetry dependent, it moves to higher proton number $Z$ for the lower isospin asymmetry source. While the dropping speed of $N/Z$ centroid with respect to the fragment atomic number $Z$ becomes isospin asymmetry related, i.e. it is enhanced with the increasing of isospin asymmetry. The difference of the $\alpha$ and $\beta$ between light, intermediate mass fragments and the heavy fragments can thus be attributed to the distinct properties of $N/Z$

![FIG. 8: (Color online) Left panel: the centroid of $N/Z$ distribution for the decay fragments from source $Z_s=50$ with $A_s=100$ (open squares), 110 (solid squares), 120 (open circles) and 130 (solid circles) (from bottom to top), respectively at $E_{ex}=5$ MeV/nucleon. The solid line is EAL from Ref. 41, the dash, dot, and dash dot lines are linear fits to the data point for $Z \leq 30$. Right panel: Gaussian fitting width of $N/Z$ distribution for the emission fragments from sources $Z_s=50$, $A_s=100$ (open squares), 110 (solid squares), 120 (open circles) and 130 (solid circles).

FIG. 9: (Color online) Left panel: the centroid of $N/Z$ distribution for the emission fragments from source $A_s=120$ with $Z_s=60$ (open squares), 55 (solid squares), 50 (open circles) and 45 (solid circles) (from bottom to top), respectively at $E_{ex}=5$ MeV/nucleon. The solid line is EAL from 41, the dash, dot, dash-dot and dash-dot-dot lines are linear fits of the data points for $Z \leq 30$. Right panel: Gaussian fitting width of $N/Z$ distribution for the emission fragments from source $A_s=120$ with $Z_s=60$ (open squares), 55 (solid squares), 50 (open circles) and 45 (solid circles) at $E_{ex}=5$ MeV/nucleon.

The trend as the light and intermediate mass fragments show in Fig. 7, 8 and 9 which drops rapidly with the fragment atomic number $Z$. The heavy fragments are generally the residues after light particle emission from the equilibrated source, therefore the reminiscent of the excited hot source demonstrates distinct $N/Z$ distributions from the evaporation products. The bending points on the centroid of $N/Z$ distribution do not appear in Fig. 6 so that $\alpha$ and $\beta$ keep constant for all emission fragments. Similarly, the bending points of the centroid of $N/Z$ distributions appear in Fig. 7, 8 and 9 then the calculated $\alpha$ and $\beta$ present the similar bending point too. In Fig. 7 the bending point is temperature dependent, the low temperature case moves the bending point to higher atomic number $Z$. Meanwhile, the dropping speed of the centroid of $N/Z$ with respect to the fragment atomic number $Z$ is temperature independent, which makes $\alpha$ of the heavy fragment excitation energy independent in Fig. 8. The bending point is isospin asymmetry independent when the source atomic number $Z_s$ is fixed while neutron number $N_s$ varies, this can be observed more clearly in Fig. 8. However, when the source mass number $A_s$ is fixed by changing its proton number $Z_s$ and neutron number $N_s$, like the case in Fig. 9 the bending point becomes isospin asymmetry dependent, it moves to higher proton number $Z$ for the lower isospin asymmetry source. While the dropping speed of $N/Z$ centroid with respect to the fragment atomic number $Z$ becomes isospin asymmetry related, i.e. it is enhanced with the increasing of isospin asymmetry. The difference of the $\alpha$ and $\beta$ between light, intermediate mass fragments and the heavy fragments can thus be attributed to the distinct properties of $N/Z$ distributions. For heavy fragments, $\alpha$ is enhanced when proton number $Z_s$ of the source pair is fixed. In contrary, it is suppressed when the mass number $A_s$ of the source pair is fixed.

V. SURFACE CONTRIBUTION TO ISOSPIN EFFECT

GEMINI code involves the isospin effect or symmetry energy in calculating the binding energy. For heavy systems ($Z_s>12$), the mass excess of the initial and residual systems of a spherical nucleus is given by following equation [37, 38].
\[ M_{\text{macro}}^{(0)} = M_n N + M_p Z - a_s (1 - k_v I^2) A + a_s (1 - k_s I^2) \]
\[ \times \left\{ A^{2/3} - 3 \left( \frac{a_1}{r_0} \right)^2 + \left( \frac{r_0}{a} A^{1/3} + 1 \right) \right\} \left[ 2A^{2/3} + 3 \frac{a}{r_0} A^{1/3} + 3 \left( \frac{a}{r_0} \right)^2 \right] e^{2r_0 A^{1/3}/a} \]
\[ + \frac{3 e^2}{5 r_0} \left[ Z^2 A^{1/3} - \frac{5}{2} \left( \frac{b}{r_0} \right)^2 Z^2 A - \frac{5}{4} \left( \frac{3}{2\pi} \right)^{2/3} Z^4 A^{1/3} \right] + W(|I| + d) - a_{cl} Z^{2.39} + \begin{cases} \Delta - \frac{1}{2} \delta, & \text{N and Z odd} \\ \frac{1}{2} \delta, & \text{N or Z even} \\ - (\Delta - \frac{1}{2} \delta), & \text{N and Z even} \end{cases} \] (13)

FIG. 10: (Color online) Top panel: mass excess calculated from Eq. (14) as a function of nuclear mass number, curves from left to rightcorrespond to the atomic number \(Z\) of isotopes series from \(Z = 5\) to \(Z = 60\) in order; Bottom panel: numerical calculated derivative of symmetry energy \(E_{\text{sym}}\) with respect to neutron number \(N\) in Eq. (13) as a function of \((Z/A)^2\) (solid marks) and the linear fit of the numerical calculation, fitted result is printed on the Figure.

where \(I = (N - Z)/A\), parameters are taken from [38], \(M_n = 8.071431, M_p = 7.289034, e^2 = 1.4399764, b = 0.99, W = 36, a_{cl} = 1.433c^{-5}, r_0 = 1.16, a = 0.68, a_s = 21.13, k_s = 2.3, a_v = 15.9937, k_v = 1.927, \Delta = 12/\sqrt{A}, \delta = 20/A\). Correction arising from single-particle effect is added to Eq. (13), which includes Coulomb energy for diffusive surface, proton form factor and charge asymmetry term [37, 38, 42]. In the top panel of Fig. 10 the calculated mass excess by Eq. (13) is plotted as a function of mass number \(A\), the isotopes with different charge number \(Z\) from 5 to 60 are presented by lines from left to right.

In Eq. (13) the dominant symmetry energy term arises from two parts: the volume symmetry energy part \(a_s k_s I^2 A\) and the surface diffuseness term related with the isospin asymmetry \(-a_s k_s I^2 \{ A^{2/3} + \ldots \} \). As derived in other models and theoretical frame [8, 21], isotope yield ratio is dominantly determined by the symmetry term in the binding energy for two equilibrium sources with comparable mass and temperature but different isospin degree \(N_e/Z_e\). In this case, the isoscaling parameter \(\alpha\) can be achieved by following approximate form

\[ \alpha = -\Delta s_n/T, \] (14)

where \(\Delta s_n\) is the difference in neutron separation energy between the two sources, considering the dominant term in separation energy is symmetry term, which is calculated in GEMINI simulation by Eq. (13). The symmetry term taken from Eq. (13) can be expressed alone by

\[ E_{\text{sym}} = c_v I^2 A - c_s I^2 \left\{ A^{2/3} + \ldots \right\}, \] (15)

the first term in Eq. (15) is the volume term of isospin asymmetry part, the second term is the surface effect of isospin asymmetry. \(c_v = a_s k_s = 15.9937 \times 1.927 = 30.8199\) MeV which is the generally used symmetry energy coefficient at saturate nuclear density \(r_0 = 0.16 f m^{-3}, c_s = a_s k_s = 21.13 \times 2.3 = 48.599\) MeV. The difference in neutron separation energy between two sources can be approximately obtained by taking the derivatives of the symmetry energy of Eq. (15) with respect to \(N\) and \(Z\) odd. Numerical calculation of the derivatives of Eq. (15) with respect to \(N\) (i.e. \(\Delta E_{\text{sym}}/\Delta N\)) is performed, and approximate linear function on \((Z/A)^2\) is observed, shown in the bottom panel of Fig. 10. The linear fits give \(\Delta E_{\text{sym}}/\Delta N = 22.26019 - 0.5205(Z/A)^2 \approx 22.630125(1 - 4(Z/A)^2)^2\), which is consistent with the symmetry energy coefficient \(C_{\text{sym}}\) derived from isoscaling parameters \(\alpha\) and \(\beta\) very well, namely \(C_{\text{sym}} = 22.01\) MeV derived from \(\alpha\) or \(C_{\text{sym}} = 19.92\) MeV from \(\beta\). As we know, the derivatives with respect to \(N\) of the volume term in Eq. (15) is \(c_v(1 - 4(Z/A)^2)^2 = 30.8199 - 123.2796(Z/A)^2 = 30.8199(1 - 4(Z/A)^2)^2\), hence the contribution from the surface effect is approximately \(22.630125(1 - 4(Z/A)^2)^2 - 30.8199(1 - 4(Z/A)^2)^2 = -8.55971(1 - 4(Z/A)^2)^2\).
with the experimental results and other model calculation, which indicates that the surface effect term of the symmetry energy influences on the isoscaling parameter \( \alpha \) and \( \beta \) strongly.

VI. ISOSCALING EFFECT OF THE HEAVY EVAPORATION RESIDUES

Isoscaling phenomena of the light and intermediate mass fragments in the statistical sequential decay are well observed in the GEMINI model as shown in section IIII however, the heavy fragments display different behavior. Isoscaling parameters \( \alpha \) and \( \beta \) of the light and intermediate mass fragments are independent on the fragments size, however, isoscaling parameters \( \alpha \) and \( \beta \) of the heavy fragments are dependent on the fragments size, either enhanced when the source pairs are fixed with atomic number \( Z_s \) or suppressed when the source pairs are fixed with mass number \( A_s \) in the present work. It is interesting to note that the isoscaling parameters \( \alpha \) and \( \beta \) of the fragments, from the heaviest source pairs \( Z_s = 75 \) in the present simulation, show no enhancement or suppression on the fragments size. In order to compare the origin of the different behaviors between these source pairs, the normalized fragment size \( Z/Z_s \) dependence of \( \alpha \), in three different size source pairs \( Z_s = 75, 50 \) and 30, is plotted in the upper panel of Fig. 11 where \( Z \) is the fragment charge number, \( Z_s \) is the source charge number, \( Z/Z_s \) illustrates the relative fragment size comparing with the source size. Results from three different size source pairs show the same behaviors below \( Z/Z_s \leq 0.6 \), where \( \alpha \) keeps constant, is independent on the fragments size. For \( Z/Z_s > 0.6 \), the enhancement appears for the heavy fragments from lighter sources \( Z_s = 30 \) and 50. However, the dominant decay process may be the binary fission followed by light fragment evaporations, no heavy residues are left for the case of source \( Z_s = 75 \) in comparison with the case of lighter sources, such as \( Z_s = 30 \) and 50 where very heavy residues are remained after evaporation and fission. The relative yields of the final fragments for different size sources are shown in the bottom of Fig. 11 heavy residues can be seen for the light source \( Z_s = 30 \) and \( Z_s = 50 \).

Residues of the evaporation process do not follow the isoscaling rule, which can be attributed to different isoscaling effect of heavier fragments from light and intermediate mass fragments by the evaporation and fission. Isoscaling effects of the residues from the source pairs with the fixed charge number \( Z_s = 50 \) and 30, and from the source pairs with the fixed mass number \( A_s = 100 \) and 120, show different pictures in Fig. 2 and 3 and 4 the dominant difference lies on the heavy residues. The centroid of \( N/Z \) distribution for bulk residues after evaporation neither follows the track of EAL nor lies around a line parallel to the EAL, but always locates under the \( N/Z \) line of fission or evaporation products and decreases with the residue size when it is close to the source size. It reveals that the more neutron-rich the source is, the more neutron-rich fragment emitted, which leads to the residue more neutron-deficient. In the context of multi-fragmentation and liquid-gas phase transitions, two component system can be observed, namely the more neutron-rich gas phase and more neutron-deficient liquid phase. In our present calculation, light and intermediate mass fragments exhibit more neutron-rich component like the gas phase, bulk residues exhibit more neutron-deficient component like the liquid phase, this can be regarded as an evidence of the so-called isospin distillation.

VII. SUMMARY

The sequential decay process of the equilibrated source has been successfully investigated by the GEMINI model. The model constrains the source and fragments density at saturate nuclear density \( \rho_0 \) and the decay starts from an equilibrated system which is separated from any dynamical process. All these characters simplify the reaction mechanism and process. In our work we apply it to survey the evolution of isospin degree of freedom. Isoscaling effects on the decayed fragments from different source sizes, isospin asymmetries, and excitation energies have been systematically investigated. Isoscaling phenomena are observed for the emission fragments of light and intermediate mass fragments, but isoscaling behavior is destroyed for the heavy residues. Isoscaling parameters \( \alpha \) and \( |\beta| \) decrease with the increasing
of excitation energy, they generally show linear dependence on the inverse of the system temperature $T$, and are independent on the source size. From the linear function between $\alpha$ versus $1/T$ or $\beta$ versus $1/T$, the extracted linear slope is the free neutron (proton) chemical potential difference $\Delta \mu_n$ and $\Delta \mu_p$ between two systems. If the isospin symmetry source is included, then the difference of neutron and proton chemical potential $\Delta \mu_n - \Delta \mu_p$ can be determined, thus the slope difference between $\alpha$ and $\beta$ versus $1/T$ can be offered as a signal of the neutron and proton chemical potential difference. After considering system temperature, simple relationship $\alpha = 4C_{sym}[(Z/A_1)^2 - (Z_2/A_2)^2]/T$ and $\beta = 4C_{sym}[(N_1/A_1)^2 - (N_2/A_2)^2]/T$ can be well reproduced for various sources. Only when the surface effect term in the symmetry energy is taken into accounted, the above linear relationship between $\alpha$ and $\Delta(Z/A)_2$, or $\beta$ and $\Delta(N/A)_2$ can give consistent symmetry energy coefficients $C_{sym}$ with experimentally proposed results and other model results, which illustrates that the surface effect plays a significant role in the symmetry energy term. In comparison to the light and intermediate fragments, isospin composition of the bulk evaporation residues does not follow the track of light and intermediate fragments, the centroid of $N/Z$ distribution for residues are smaller than the expected.

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[33] R. J. Charity, computer code GEMINI, see http:// wunmr.wustl.edu/pub/gemini


