SGR1806-20: evidence for a superstrong Magnetic Field from Quasi Periodic Oscillations

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ABSTRACT

Fast Quasi-Periodic Oscillations (QPOs, frequencies of $\sim 20 - 1840$ Hz) have been recently discovered in the ringing tail of giant flares from Soft Gamma Repeaters (SGRs), when the luminosity was of order $10^{41} - 10^{41.5}$ erg/s. These oscillations persisted for many tens of seconds, remained coherent for up to hundreds of cycles and were observed over a wide range of rotational phases of the neutron stars believed to host SGRs. Therefore these QPOs must have originated from a compact, virtually non-expanding region inside the star’s magnetosphere, emitting with a very moderate degree of beaming (if at all). The fastest QPOs imply a luminosity variation of $\Delta L/\Delta t \simeq 6 \times 10^{43}$ erg s$^{-2}$, the largest luminosity variation ever observed from a compact source. It exceeds by over an order of magnitude the usual Cavallo-Fabian-Rees (CFR) luminosity variability limit for a matter-to-radiation conversion efficiency of 100%. We show that such an extreme variability can be reconciled with the CFR limit if the emitting region is immersed in a magnetic field $\gtrsim 10^{15}$ G at the star surface, providing independent evidence for the superstrong magnetic fields of magnetars.

Subject headings: stars: magnetic fields — stars: neutron — stars: individual(SGR 1806-20)
1. Introduction

Soft Gamma Repeaters are a small class of galactic sources of X and soft gamma radiation. They have spin periods of $\sim 5 \div 10$ s, display a secular spin-down with timescales of $\sim 10^4 \div 10^5$ yr and do not possess a companion. Unlike radio pulsars, the rotational energy loss of SGRs is a factor of $10 \div 100$ too small to explain their persistent emission, typically $\sim 10^{33} \div 10^{34}$ erg/s (see e.g. Woods & Thompson (2006)). Like Anomalous X-ray Pulsar (AXPs, Mereghetti & Stella (1995)), with whom they share a number of properties, SGRs are believed to host magnetars, neutron stars the emission of which is powered by the decay of their superstrong (internal) magnetic field ($B > 10^{15}$ G, Duncan & Thompson (1992); Thompson & Duncan (1993)).

The name-defining characteristic of SGRs is that they show periods of activity in which recurrent short bursts are emitted, with peak luminosities of $\sim 10^{38} \div 10^{41}$ ergs s$^{-1}$ and sub-second durations. The characteristics of the three giant flares observed so far in about 30 yr of monitoring are much more extreme. Their initial, tenths-of-seconds-long spike releases enormous amounts of energy, $\sim 10^{44}$ ergs in the 1979 March 5 event from SGR 0526-66 (Mazets et al. 1979) and the 1998 August 27 event from SGR 1900+14 (Hurley et al. 1999; Feroci et al. 1999) and as much as $5 \times 10^{46}$ erg in the 2004 December 27 event from SGR1806-20 (Hurley et al. 2005; Mereghetti et al. 2005; Palmer et al. 2005; Terasawa et al. 2005). After the initial spike, giant flares display a very bright ringing tail lasting hundreds of seconds and releasing a total energy of about $\sim 10^{44}$ ergs. The emitted spectrum is roughly thermal, with a blackbody equivalent temperature of $\sim 5$ keV in the case of SGR1806-20 (Hurley et al. 2005).

The highly super-Eddington luminosities of the recurrent bursts of SGRs and especially of their giant flares make models involving accretion energy not viable. According to the magnetar model, the emission of SGRs (and AXPs) draws from their extremely high magnetic fields (Thompson & Duncan 1995, 1996, 2001). Within this model the neutron star interior is characterized by a wound-up, mainly toroidal magnetic field configuration with $B_r > 10^{15}$ G. A less intense (mainly poloidal) field emerges out of the star magnetosphere, causing spin-down via rotating dipole losses at the observed rate (Thompson & Duncan 1993, 2001) (dipole B-field strength of $B_d \sim 7.8 \times 10^{14}$ G). Energy propagates to the neutron star magnetosphere through Alfvén waves driven by local “crust- quakes” and giving rise to recurrent bursts with a large range of amplitudes. Large-scale rearrangements of the internal magnetic field or catastrophic instabilities in the magnetosphere are invoked to explain the sudden release of very large amounts of energy that occurs in giant flares (Thompson & Duncan 2001; Lyutikov 2003). A fireball of plasma expanding at relativistic speeds breaks out of the star’s magnetosphere, causing the initial sub-second spike of giant flares. The ringing tail that follows results most likely from the part of the fireball that remains trapped in the star’s magnetosphere. The energy release in the ringing tail yields a limit for the external field of magnetars ($\gtrsim 10^{14}$ G) in agreement with the values inferred from spin-down.
dipole losses (Thompson & Duncan 1995, 2001), while an analysis of the initial time scales gives evidence in favor of the crustal cracking mechanism (Schwartz et al. 2005).

To confirm this model it is thus essential to measure the surface magnetic field, which complements the measurement of the dipole component (Woods et al. 2002), to include higher order multipoles. In this paper we present an interpretation of the QPOs observed in SGR 1806-20 which provides a lower limit \( B \gg B_q \approx 4.4 \times 10^{13} \, G \) to the surface field. The argument is based upon a seldom-used constraint ((Cavallo & Rees 1978; Fabian & Rees 1979; Fabian 1979)) that puts a very strong upper limit on the time-scale on which significant luminosity variations can take place. In the next section we discuss the relevant observations; in Section 3 we will re-derive the limit and show that it is largely violated by QPOs in SGR 1806-20; after vainly trying to circumvent it, we will show that it can be reconciled with observations only if \( B \gg B_q \).

### 2. Quasi Periodic Oscillations in Giant Flares of SGRs

Recent studies led to the discovery that the X-ray flux of the ringing tail of SGRs’ giant flares is characterized by fast Quasi Periodic Oscillations, QPOs (Israel et al. 2005). Different QPO modes were detected, some of which were excited simultaneously. The ringing tail of the December 2004 event from SGR 1806-20 displayed clear QPO signals at about 18, 30, 93, 150, 625 and 1840 Hz (Strohmayer & Watts 2006a). Similarly, QPOs around frequencies of 28, 54, 84 and 155 Hz were detected during the ringing tail of the 1998 giant flare of SGR 1900+14 (Strohmayer & Watts 2005), while hints for a signal at \( \sim 43 \) Hz were found in the March 1979 event from SGR 0526-66 (Barat et al. 1983). These QPOs show large variations of the amplitude with time and, especially, of the phase of the spin modulation in the giant flare’s tail.

The similarity in some of the QPO modes and frequencies across different SGRs suggests that the production mechanism is the same. A likely interpretation involves the excitation of neutron star oscillation modes, whose expected eigenfrequencies match some of the observed QPOs peaks ((Duncan 1998; Israel et al. 2005; Piro 2005)). Infact, if giant flares result from large scale fracturing of the crust induced by instabilities of the internal magnetic fields, then the excitation of crustal and (possibly) global neutron star modes is to be expected ((Levin 2006)). Regardless of the exact mechanism driving these oscillations we are concerned here with the extremely large and fast luminosity variations of the QPOs.

We concentrate on the signals with the largest luminosity time derivative, i.e. the 625 and 1840 Hz QPOs from SGR 1806-20 (Strohmayer & Watts 2006a). The power spectrum peaks through which these QPOs are revealed, are a few Hz wide, testifying that their signal remained coherent for hundreds of cycles. The signal shape must be close to sinusoidal, as evidenced by
the absence of detectable harmonic signals. The 625 and 1840 Hz QPOs were detected only in a \( \sim 50 \) s long interval of the ringing tail, about 200 s after the initial spike, and were especially prominent over a \( \sim 140 \) deg interval in rotational phase. The QPO amplitude reached a maximum in this phase interval over two consecutive rotation cycles: for both signals the rms amplitude was \( a_{\text{rms}} \sim 18\% \). Approximating the QPOs with sinusoids, we estimate their highest luminosity derivative as \( \Delta L/\Delta t = 2^{3/2}\pi L a_{\text{rms}}\nu_{\text{QPO}} \), with \( \nu_{\text{QPO}} \) the QPO frequency. Here \( L \sim 10^{41} \text{ ergs s}^{-1} \) is the luminosity in the relevant section of the ringing tail (for the likely source distance of 15 kpc). This gives \( \Delta L/\Delta t \sim 1 \times 10^{44} \) and \( 3 \times 10^{44} \text{ ergs s}^{-2} \) for the 625 and 1840 Hz QPOs, respectively. The effects of beaming might decrease these values somewhat, but not by a very large factor. In fact, these QPO signals were observed over a large interval of rotational phases (about \( \sim 140 \) deg), translating into approximately the same azimuthal range of emission angles. It is natural to assume a comparably large angular spread in latitude (unless the neutron star rotation axis is very close to our line of sight, which is unlikely given the large amplitude of the rotation modulation and the size of the emission region, see below). This gives a solid angle of order \( \sim \pi \) ster. Adopting this beaming factor the luminosity derivatives above reduce to \( \sim 2 \times 10^{43} \) and \( 6 \times 10^{43} \text{ ergs s}^{-2} \); this are the values that we adopt in the following discussion.

We stress here, because this is essential to our argument (to be presented shortly), that, together with the QPOs, in the ringing tail a strong modulation at the star’s spin period is clearly present, similar in relative amplitude and shape to the modulation observed when the source is in its quiescent state: this (together with the lack of significant amounts of beaming) indicates that the emission in the ringing tail originates from a region that remains stably anchored to the star’s magnetosphere, and thus that relativistic bulk motions are not present at this stage of the flare.

The blackbody temperature and luminosity in the ringing tail translates into a lower limit on the size of the emitting region of about \( \sim 30 \text{ km} \), i.e. substantially larger than the neutron star. On the other hand the black body-like spectral shape testifies that the emitting region is optically thick (or at least effectively thick), implying a scattering optical depth \( \gg 1 \). We remark that the size estimate, \( \approx 30 \text{ km} \), will play an important role in the following.

### 3. The Cavallo-Fabian-Rees Variability Limit

There is a well–known limit on the rate of change of the luminosity of any given source, which we briefly summarize here (Cavallo & Rees (1978); Fabian & Rees (1979); Fabian (1979), see also Lawrence (1980); Hoshi (1989)). Suppose a source undergoes a large luminosity variation on a time-scale \( \Delta t \), and there is a luminosity variation, over this time-scale, \( \Delta L \). The total energy
released within $\Delta t$ is related to the total mass within the source dimension $R$ by

$$\Delta L \Delta t = \frac{4\pi}{3} R^3 n m_p c^2,$$  \hspace{1cm} (1)

where $n$ is the average baryon density, and $\eta$ is the energy extraction efficiency. The time-scale $\Delta t$ must obviously exceed the time over which the photons manage to diffuse out of the source:

$$\Delta t > \frac{R}{c} (1 + \tau_T)$$  \hspace{1cm} (2)

where $\tau_T = \sigma_T n R$ is the Thomson optical depth and $\sigma_T$ the Thomson cross-section. Eliminating $R$ from the first two equations, we find

$$\Delta t > \frac{3}{8\pi} \frac{\sigma_T}{m_p c^4} \frac{\Delta L (\tau_T + 1)^2}{\eta \tau_T}.$$  \hspace{1cm} (3)

Regarded as a function of $\tau_T$, the above has a minimum for $\tau_T = 1$, which yields the limit

$$\Delta t > \frac{3}{2\pi} \frac{\sigma_T}{m_p c^4} \frac{\Delta L}{\eta}.$$  \hspace{1cm} (4)

The above limit is remarkable in that it is independent of both $R$ and $n$, or any combination thereof: only the dependence on $\Delta L$, a directly observable quantity, is left. The Cavallo-Fabian-Rees, CFR, limit thus writes

$$\Delta L / \Delta t < \eta 2 \times 10^{42} \text{erg s}^{-2}. \hspace{1cm} (5)$$

The 625 and 1840 Hz QPO signals from SGR1806-20 exceed the CFR limit by about an order of magnitude; the largest value found in the previous section is $\Delta L / \Delta t = 6 \times 10^{43} \text{erg s}^{-2}$, which is a whole factor $30/\eta$ larger than the CFR’s limit.

In order to appreciate how hard it is to circumvent this limit, notice the *in situ* re-acceleration of electrons does not help, because it does not change the energies reached by protons; if protons were to escape, leaving electrons behind to be re-accelerated at will, Coulomb forces would quickly make the escape of protons impossible. Nor will having relativistic protons help, as one might think, since this would imply energies per electron to be radiated $\approx \gamma m_p c^2 \gg \eta m_p c^2$, because the energy transfer from protons to electrons is too slow. To show this, let us consider what happens when protons transfer promptly to the electrons their energy gain: protons may then be relativistic, in which case the maximum energy which can be extracted from each of them is $\gamma m_p c^2$, rather than

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1We note that also the slower $\sim 90-150\, \text{Hz}$ QPOs signals in the giant flares’ tail of SGR1806-20 and SGR1900+14, exceed the CFR limit, though by a smaller factor.
the more sedate $\eta m_p c^2$. This however appears like an unlikely way out, because, even admitting that the electrons radiate very promptly their internal energy, the time-scale on which protons manage to transfer to the electrons their internal energy is much longer than the above limit. To see this, we idealize the situation as one where electrons are cold (i.e. Newtonian) as a result of their short cooling timescales, while the protons are still relativistic. The energy transfer rate is given by the usual formula

$$-\frac{dE_p}{dx} = \frac{2\pi e^4 n_e}{m_e v^2} \left( \ln\left( \frac{2m_e v^2 W}{\hbar^2 \omega_p^2} \right) + 1 - 2 \frac{v^2}{c^2} \right),$$

(6)

where $n_e$ is the electron number density, $\omega_p = \sqrt{4\pi e^4 n_e / m_e}$ the plasma frequency, $v \approx c$ the proton speed, and $W \approx 2E_p/(m_e c^2)$ is the maximum energy transfer in the not too extreme limit, $\gamma_p \lesssim m_p / m_e$ (but considering the opposite limit, $\gamma \gtrsim m_p / m_e$ would change the argument very little since $W$ only appears as an argument to a logarithm).

The energy transfer timescale is of course $t_e \equiv E_p / (c \, dE_p/dx)$, which, for this whole idea to work, must be shorter than $\Delta t$. The condition $t_e < \Delta t$ can also be rewritten as, in the limit $v^2 \approx c^2$:

$$\frac{\gamma}{\ln\left( \frac{2m_e v^2 W}{\hbar^2 \omega_p^2} \right) - 1} < \frac{3\sigma_T \, L \, e^4 n_e}{m_pc^3 \eta m_pm_ec^3}.$$  

(7)

Inserting the numerical values for the luminosity of SGR 1806-20 during the ringing tail and for the typical magnetospheric density for a pulsar with a normal magnetic field we find

$$\gamma \lesssim 10^{-8} \frac{1}{\eta} \frac{L}{3 \times 10^{41} \text{ ergs}^{-1}} \frac{n_e}{10^{19} \text{ cm}^{-3}}.$$  

(8)

In order to reconcile the CFR limit with observations, we need $\gamma \approx 30$, corresponding to a magnetospheric density $n \approx 3 \times 10^{19} \text{ cm}^{-3}$. While it is certainly true that plasma outside pulsars need not be charge-separated, still this density would exceed the minimum (i.e., charge separated) value by more than 9 orders of magnitude, making it unlikely that such a plasma may exist.

This argument must be modified in the presence of a pair plasma, where pair creation processes can easily lead to a strong increase in $n_e \propto T^3$. Still, we know that astrophysical pair plasmas are thermally regulated (Pietrini & Krolik 1995), so that their temperatures always lie around $T \lesssim 1 \text{ MeV}$. At these temperatures, $n_e \approx 10^{31} \text{ cm}^{-3}$, but we also know that $E_p \ll m_p c^2$, so that the limit of eq. 4 still holds, provided the total energy density is still dominated by the rest mass of the baryons. When instead the total energy density is dominated by the pairs (in other words, when little or no baryons are admixed), eq. 2 obviously still holds (with $\tau_T$ now indicating the total optical depth due to the pairs and possibly to photons as well), while eq. 1 must be rewritten as

$$\Delta L \Delta t = \eta \frac{4\pi}{3} R^3 n m_e c^2$$

(9)
where we used the fact that $\gamma \approx 2$ for the electrons. The new limit for the variability is

$$\Delta t > \frac{3}{2\pi} \frac{\sigma_T}{m_e c^4} \frac{\Delta L}{\eta} = 134 s \frac{1}{\eta} \frac{\Delta L}{3 \times 10^{41} \text{erg s}^{-1}},$$

(10)

which is even more stringent than eq. 4.

The CFR limit might fail if coherent phenomena are involved, but we are unaware of any coherent mechanism working in the X-ray region of the spectrum where QPOs are observed, rather than the usual radio band.

The classic way to circumvent the CFR limit is by means of relativistic effects. After all, GRBs’ light-curves often display millisecond variability, and even when they don’t they turn on on timescales of a second or less, reaching $L \approx 10^{50}$ erg s$^{-1}$: this clearly violates eq. 4 by eight to ten orders of magnitude (for $\eta = 1$). This is due to a combination of relativistic aberration, blueshift and time contraction, which can drastically increase the luminosity and decrease the variability timescale in the observer’s frame. Also BL Lac objects and Quasars have luminosity derivatives that exceed by orders of magnitude the CFR limit (see e.g. (Bassani et al. 1983)), even after their luminosity is corrected for beaming. This explanation is probably correct also for the giant flare of SGR 1806-20, where values as high as $\Delta L/\Delta t \sim 10^{47}$ ergs s$^{-2}$, for isotropic ejection are observed: the observation of a radio halo expanding at relativistic speed testifies that the observed variability in the initial spike of the giant flare is most likely magnified by relativistic effects as well.

However, this explanation is not suitable for the QPOs in the ringing tail of the same flare: in fact, the presence of modulations at the spin period, with similar pulse shape and amplitude to those during quiescent periods, makes the existence of relativistic bulk motions very unlikely during this phase. Also, one should remember that the photosphere size is estimated to be $\approx 30 \text{ km}$ at all times during the ringing tail, which implies that emission during this phase is due to material stably anchored to the pulsar. Lastly, the observed temperatures ($T \approx 5 \text{ keV}$) testify to the gas having thermal speeds much below the escape velocity at that radius. For these three reasons, we deem relativistic effects an unlikely explanation for the violation of eq. 4.

The last way in which the CFR limit can fail is the one we propose: the scattering cross-section may differ from Thomson’s because of the presence of a strong magnetic field $B$, exceeding the quantum value $B_q = m^2 c^2/(\hbar e) = 4.4 \times 10^{13}$ G. In this case, the scattering cross section for the ordinary (O) and extraordinary (E) modes, and for the conversion of photons into the other state, are given by Meszaros (1992), when the dielectric tensor is dominated by vacuum polarization effects, as:

$$d\sigma_{O\rightarrow O} = \frac{3}{4} \sigma_T \sin^2 \theta \sin^2 \theta' d\cos \theta'$$

\(^2\)Beaming in the ejection, which is presently unknown, could decrease this value substantially.
\[
\frac{d\sigma_{O\rightarrow E}}{d\cos \theta} = \frac{3}{8} \sigma_T \left( \frac{eB_q}{m_e c^2 B} \right)^2 \cos^2 \theta \cos \theta'
\]
\[
\frac{d\sigma_{E\rightarrow O}}{d\cos \theta} = \frac{3}{8} \sigma_T \left( \frac{eB_q}{m_e c^2 B} \right)^2 \cos^2 \theta' \cos \theta'
\]
\[
\frac{d\sigma_{E\rightarrow E}}{d\cos \theta} = \frac{3}{8} \sigma_T \left( \frac{eB_q}{m_e c^2 B} \right)^2 \cos \theta',
\]

where \(\epsilon\) is the photon energy, and \(\theta\) and \(\theta'\) are the angles between the photon momentum before and after the diffusion, respectively, with the direction of the magnetic field.

This equation shows immediately that, for photons emitted in the extraordinary mode, the cross section is reduced, with respect to the Thomson value, by a factor \(\approx \left( \frac{\epsilon B_q}{m_e c^2 B} \right)^2\). Thus, to bring the observed value, \(\Delta L/\Delta t = 6 \times 10^{43} \text{ erg s}^{-1}\), in agreement with eq. (4), we just need to have \(\left( \frac{\epsilon B_q}{m_e c^2 B} \right)^2 \lesssim \eta/30\); here we take for \(\epsilon\) the value \(\epsilon \approx 14 \text{ keV}\), which is the peak of the Planck distribution for the observed temperature \(T = 5 \text{ keV}\). So our conclusion is that the QPOs’ luminosity variation agrees with the CFR limit provided

\[
B \gtrsim 1.5 B_q \left( \frac{0.1}{\eta} \right)^{1/2} \approx 6.6 \times 10^{13} \text{ G}.
\]

A technical comment is in order at this point: the description of radiation transfer in terms of separate modal propagation is not always adequate (Lai & Ho (2003)), because of mode collapse. However this effect seems to be mostly relevant for even higher fields \((B \gtrsim 7 \times 10^{13} \text{ G})\) than those derived here, which means that our naive treatment is probably justified.

We now remark that this lower limit applies to the field close to, but not at the surface of SGR 1806-20, because, as discussed in the previous section, emission from the ringing tail is generated within 30 km. At this distance from the star surface, the dipole field inferred from pulsar spin-down \((B = 7.8 \times 10^{14} \text{ G}, \text{Woods et al.}\ (2002))\), is \(2.9 \times 10^{13} \text{ G}(R_{ns}/10 \text{ km})^3\), which is smaller than the limit just derived, as expected if higher order multipoles are relevant in the star vicinity.

Given the rapid decrease of the dipole field (and \textit{a fortiori} of the other multipoles) with distance from the star, the surface magnetic field must certainly satisfy

\[
B \gtrsim 1.8 \times 10^{15} G \left( \frac{10 \text{ km}}{R_{ns}} \right)^3 \left( \frac{0.1}{\eta} \right)^{1/2},
\]

where a dipole–like radial dependence has been assumed between 30 km and the star surface, located at \(R_{ns}\), we shall refrain from making more elaborate hypothesis about the structure of the magnetic field within \(R = 30 \text{ km} \) (Thompson & Duncan (2001)), because our aim is simply to provide a lower limit to the surface field, for which this minimum hypothesis (pure dipole) is fully adequate.
4. Discussion

Our final result is the lower limit in eq. 13. This value is close to that of the dipole field, as inferred from pulsar spin-down, \( B = 7.8 \times 10^{14} \) G, but our estimate includes higher order multipoles, at least at 30 km. It is thus completely independent of the estimates from pulsar spin-down. We note that the limit on \( B \) derived by requiring that the magnetic field close to the star can prevent the escape of the "trapped fireball" includes also the contribution of higher multipole components (Thompson & Duncan (2001)), but gives a substantially lower magnetic field (\( B > 10^{14} \) G) than the limit derived above.

There is an easy way to test our interpretation of the failure of the CFR limit. The O- and E-mode photospheres of the trapped fireball are located at different heights in the star’s magnetosphere, as a result of the different electron scattering cross sections, which therefore sample regions of different B-field strengths and orientations. The polarization fraction and angle of the emerging X-ray flux should thus be modulated with the phase of the QPO signal as a result of the varying (relative) intensity of the O- and E-mode photon component. This in principle gives a clear test for the correctness of our interpretation, which might perhaps become verifiable in the future.

Photons in the O-mode suffer a strong Comptonization (Thompson & Duncan (2001)). If the atmosphere were due to pure scattering, the ensuing photon distribution would differ from a blackbody at low photon frequency, \( E_\nu \propto \nu^3 \) instead of \( \propto \nu^2 \), a result due to photon number conservation. At first sight, one might think that there are many processes which may lead to photon absorption and emission in a strong magnetic field, like photon splitting or pair creation via \( \gamma + B \) (resulting then in pair annihilation, and photon energy downgrading via Compton recoil), which make photon number conservation unlikely. At the same time, there are important radiation transfer effects taking place, which obviously tend to favor flatter spectra at low energies; also, all rates for photon emission and absorption depart from their values in the absence of magnetic field. Detailed computations (Lyubarsky 2002) show that the ensuing spectrum is flatter, not steeper, than a blackbody; and we remark here that, despite many calibration uncertainties, fits to the spectrum seem to favor spectra flatter than a blackbody, with a preference for thermal bremsstrahlung of temperature \( T = 30 \) keV.

We have presented our argument by stressing its independence from the estimates of the dipole field from pulsar spin-down, and from Thompson and Duncan’s (Thompson & Duncan 1993, 1995, 1996) model, but it should be obvious that the limit in eq. 13 is perfectly consistent with the previous measurements, and the model itself.

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REFERENCES

Hoshi, R. 1989, PASJ, 41, 217

Piran, T., 2005, Rev. Mod. Phys., 76, 1143


