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LINEAR BETATRON COUPLING MEASUREMENT AND COMPENSATION IN THE ISR

by

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ABSTRACT

The vertical response of a beam subject to a horizontal transverse excitation defines a coupling beam transfer function. From the equations of the coupled motion, an analytical expression is found, which directly relates the complex coupling coefficient to the transfer function. The accuracy and sensitivity of the method have allowed a fine dynamic compensation of betatron coupling on "physics" stacks, as well as studies on coupling sources.

1. INTRODUCTION

In storage rings with a horizontal crossing angle, minimization of the vertical beam dimension is essential to maximize the luminosity and minimize the "background" associated with large amplitude particles.

In the case of the CERN-ISR, the horizontal to vertical transverse emittance ratio is of the order of 4 for "physics" beams; an accurate control of the linear betatron coupling is thus necessary to avoid a transfer of oscillatory energy to the vertical motion.

Important betatron coupling sources are:

- random tilts of focusing magnets;
- axial field associated with analyser magnets (for the ISR, the I1 Solenoid, the I8 Open Axial Field Magnet);
- vertical orbit displacements in sextupoles, resulting from controlled closed orbit bumps or distortions.

Some of these sources vary from one run to another; their influence was found to be significant, requiring a dynamic control of the betatron coupling for optimum performances.

A coupling-meter working on bunched beams was developed¹⁾ and used during machine development sessions to measure the static component of betatron coupling in view of its compensation by means of skew quadrupoles²⁾.

The principle presented in this paper allows a very accurate dynamic compensation of betatron coupling based on the measurement of the beam transfer function between the two transverse planes. It is shown to be closely related to the complex coupling coefficient of the linear coupling resonance.

2. PRINCIPLE

The coupling transfer function is obtained by correlating a horizontal transverse perturbation imposed on a beam to the resulting vertical coherent motion (Figure 1). The set-up used is a configuration of a general one for beam transfer function measurement³⁾. The transfer function obtained (Figure 2) reflects the general properties of coupled oscillators; its detailed behaviour is related to the distance to the linear coupling resonance and to the machine coupling vector.

Fig. 1 Principle

Fig. 2 A coupling transfer function

3. ANALYTICAL EXPRESSION OF THE COUPLING TRANSFER FUNCTION

An analytical expression can be found under the following simplifying assumptions, which will be discussed later: the coupling magnetic fields are assumed to be uniformly distributed around a sinusoidal machine; the coupling forces are small compared to the external excitation.

The time domain equations of the coupled motion may then be expressed:

$$\ddot{x} + Q_x^2 \Omega^2 x = G_H e^{-i\omega t}, \quad (1)$$

$$\ddot{z} + Q_z^2 \Omega^2 z = -\Omega^2 Q_0 c_r \dot{x} + \Omega c_i \dot{x}, \quad (2)$$

where Q_x, Q_z are the betatron numbers; Q_0 the average Q value of the particle distribution; $\Omega/2\pi$ the revolution frequency; $c = c_r + ic_i$ the coupling vector as defined in⁴⁾:

$$c_r \sim \frac{\partial B_x}{\partial x}, \quad c_i \sim B_{axial};$$

$G_H e^{-i\omega t}$ is an external excitation.

The betatron frequency spread is small compared to Q_0 ;

$$\text{hence } \dot{x} \approx -i Q_0 \Omega x. \quad (3)$$

Rewriting and combining (1), (2) and (3) yields

$$\ddot{z} + \Omega^2 (Q_x^2 + Q_z^2) z + \Omega^4 Q_x^2 Q_z^2 z = -\Omega^2 Q_0 c G_H e^{-i\omega t}.$$

An observer at phase advance θ of the exciter will observe a wave $z = z_0 e^{in\theta} e^{-i\omega t}$; using the derivative $d/dt = i(n\Omega - \omega)$, the transfer function of a line density of particles may be found:

$$\frac{z_0}{G_H} = \frac{-\Omega^2 Q_0 c e^{-in\theta}}{[\Omega(n - Q_x) - \omega][\Omega(n + Q_x) - \omega][\Omega(n - Q_z) - \omega][\Omega(n + Q_z) - \omega]}.$$

The transfer function of a distribution $f(\omega_\beta)$ of particles is found by integration; the calculation is done e.g. in the fast wave approximation ($\omega \approx (n + Q) \Omega$):

$$\frac{\langle z_o \rangle}{G_H \text{ FW}} = \frac{e^{-in\theta}}{4Q_0} c \left\{ \frac{1}{\Delta\omega_\beta} \left[\frac{f(\omega_\beta) d\omega_\beta}{\omega - \omega_\beta} - \frac{f(\omega_\beta) d\omega_\beta}{\omega - \omega_\beta - \Delta\omega_\beta} \right] \right\} \quad (4)$$

with $\omega_\beta = \Omega (n + Q_z)$; $\Delta\omega_\beta = \Omega (Q_x - Q_z) = \Omega\Delta$;

Δ is taken to be constant for all particles.

The coupling transfer function is thus simply proportional to the complex coupling coefficient c whilst its dependance on Δ (or $\Delta\omega_\beta$) is more elaborate.

The term in bracket is a finite difference of the vertical dispersion integral; qualitatively, it explains the reason why the beam response is large for frequencies corresponding to the distribution edges whilst it is low for the middle (Figure 2), for a nearly flat distribution.

4. GENERALIZATION TO A REAL MACHINE

In a real machine where the coupling sources are not uniformly distributed, the coupling vector of the linear resonance measured at position θ_o is given by

$$c(\theta_o) = \frac{1}{4\pi R} \int_{\theta_o}^{\theta_o+2\pi} \sqrt{\beta_x \beta_z} \left[K + \frac{1}{2} MR \left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{\beta_z} \right) - \frac{i}{2} MR \left(\frac{1}{\beta_x} + \frac{1}{\beta_z} \right) \right] \exp \left\{ i [\Delta\mu - \Delta\mu_{\theta_o} - \Delta (\theta - \theta_o)] \right\} d\theta^4, \quad (5)$$

with $K(\theta)$ skew gradient coefficient; $M(\theta)$ axial field coefficient; $\Delta\mu = \mu_x - \mu_z$; μ, β, α have their usual meanings. If we assume small correlations between coupling fields and betatron phase advance, it is possible to find θ_o for which the average of the exponential factor is unity. In that position the sinusoidal model is valid:

$$\langle \Delta\mu - \Delta\mu_{\theta_o} - \Delta (\theta - \theta_o) \rangle = 0$$

$$\Delta\mu_{\theta_o} - \theta_o \Delta = \langle \Delta\mu - \Delta\theta \rangle = 0 \quad (5)$$

The behaviour of this function is illustrated on figure 3, where it is shown that condition (5) is fulfilled at a number of places around the ISR machine.

Fig. 3

Coherent oscillations give rise to electromagnetic fields which, through the transverse wall impedances^{5),6)} introduce an additional perturbing acceleration; this effect however does not alter the proportionality between the coupling vector and the transfer function.

The complete expression for the transfer function becomes:

$$\frac{\langle z_o \rangle}{G_H} \text{FW} = \frac{ce^{-in\theta}}{4Q_o} I_{zz} \left[1 - i 2 S \Omega Z_H I_H \right]^{-1} \left[1 + i \frac{S\Omega}{2Q_o} \frac{Z_V I_{zz}}{1 - i 2 S \Omega Z_H I_H} \right]^{-1}$$

with I_{zz} : bracketed expression in (4); Z_H, Z_V : transverse wall impedances; I_H horizontal dispersion integral; S frequency spread. Its complexity excludes a direct absolute measurement of c !

5. MEASUREMENT OF THE COUPLING VECTOR

The measured beam transfer function is expressed as a set of pairs amplitude/phase $[A(\omega); \phi(\omega)]$.

According to the model:

$$A(\omega) \sim \sqrt{c_r^2 + c_i^2}; \text{ this is equally true for } \int A(\omega) d\omega$$

$$\phi(\omega) = \text{Arctg} \frac{c_i}{c_r} + \phi(I_{zz}) - n\theta$$

For an excitation frequency outside of the beam's natural frequencies, the phase rotation introduced by the distribution $\phi[I_{zz}]$ is either 0 or 2π ; θ can be measured or calculated; hence c_i/c_r can be evaluated from a single measurement.

The proportionality factor relating the amplitude to the coupling modulus can be measured if a known coupling increment is added via skew quadrupoles. The linearity of this relationship was found to be good (Figure 4). In practice, the proportionality factor does not vary significantly from one beam to another. As can be seen on Figure 4, the accuracy of the measurement is better than 10% on a realistic range of the c values. The sensitivity is large, allowing coupling compensation to $\sim 10^{-4}$.

Fig. 4

6. APPLICATION TO THE ISR

The method described is used operationally to minimize the residual coupling which varies from run to run for the reasons given in the introduction.

Fine coupling compensation is most efficient in reducing the hourly luminosity decay and the "background" measured by the physics detectors (Figure 5).

Fig. 5 An example of coupling compensation

Coupling measurement has been used to study the effect of vertical bumps (Figure 6) and of localized axial fields.

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Fig. 6 Effect of bumps on coupling

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