Higher order correction to the neutrino self-energy in a medium and its astrophysical applications

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We have calculated the $1/M^4$ ($M$ the vector boson mass) order correction to the neutrino self-energy in a medium. The possible application of this higher order contribution to the neutrino effective potential is considered in the context of the Early Universe hot plasma and of the cosmological Gamma Ray Burst fireball. We found that, depending on the medium parameters and on the neutrino properties (mixing angle and mass square difference) the resonant oscillation of active to active neutrinos is possible.

I. INTRODUCTION

The particle propagation in the presence of a finite temperature and density medium has been studied extensively because of its importance in various astrophysical and cosmological scenarios. The vacuum dispersion relation is no longer respected in the medium and the properties of the particle are modified. This has been studied in great detail for photon and charged particles\(^1\). The neutrino propagation in a medium was first studied by Wolfenstein, who determined the refractive index of $\nu_e$ from the forward scattering amplitude of $\nu_e$-electron scattering\(^2\). Using the finite temperature field theory method, the neutrino propagation has been studied in various environments\(^3\)\textsuperscript{–}\textsuperscript{7}.

In the medium, the test neutrino or anti-neutrino will interact with the background particles through both charged current (CC) and neutral current (NC) interactions depending on the nature of the background. For example, the electron neutrino will interact both by CC and NC interaction with the background of normal matter, whereas the muon and tau neutrinos will interact only through NC interaction in the normal matter, because of the absence of muons and taus in the medium. However, exotic environments like, early Universe hot plasma and core of the supernovae have a substantial amount of neutrinos, trapped within it, which can contribute to the potential due to $\nu - \bar{\nu}$ scattering. The leading order contribution to the neutrino potential is proportional to the difference of the number densities of particles. So in an almost CP-symmetric medium (e.g. early Universe) the leading order ($1/M^4$) contribution to the potential is very small and one has to go beyond the leading order (to order $1/M^6$)\(^3\)\textsuperscript{,}\textsuperscript{4}.

The early Universe must possess a tiny excess of matter over antimatter i.e. $N_B/N_\gamma \approx 5 \times 10^{-10}$ to avoid total annihilation of matter-antimatter and charge neutrality implies such an excess of electrons over positrons, but total lepton asymmetry may not be comparable to baryon asymmetry\(^7\). In fact larger lepton asymmetry is not being excluded by any theoretical reasons. As it is well known up to present, neutrino oscillation in the early Universe can have dramatic consequences on BBN. The active-sterile neutrino oscillation in the early Universe can create a large asymmetry between $\nu_e$ and $\bar{\nu}_e$ which could have a direct effect on the nuclear reactions and finally the abundances in the light elements\(^8\)\textsuperscript{–}\textsuperscript{12}. Similar situation may occur, in the cosmological Gamma Ray Burst (GRB) fireball, where radiation and electron positron plasma is formed in a compact region with extremely high optical depth, so that, photons can not escape from it and they split into electron positron pairs. This fireball may be contaminated by baryons both from the central engine and the surrounding medium\(^13\)\textsuperscript{–}\textsuperscript{18}. Neutrinos from the progenitor as well as from the inverse beta decay process within the fireball will propagate in the fireball. Depending on the nature of the fireball (e.g. temperature), there may be resonant oscillation of active neutrinos within the fireball\(^19\).

In this paper we calculate the neutrino effective potential up to order $1/M^4$ and consider its effect on neutrino oscillation in different environments like early Universe hot plasma\(^3\)\textsuperscript{,}\textsuperscript{4} and the GRB fireball. We found that, depending on the parameters of the fireball, there can be resonant oscillation from active to active neutrino. On the contrary, the anti-neutrino conversion is very much suppressed.

The paper is organized as follows: In sec.2 we give a detailed calculation of the neutrino self-energy, from different diagrams that contribute to it. Sec.3 is devoted to the calculation of neutrino effective potential in different backgrounds. The oscillation of neutrinos in the early Universe hot plasma and the GRB fireball environment are discussed in sec. 4 and a brief conclusion is drawn in Sec. 5.

II. NEUTRINO SELF ENERGY

In the medium, the Dirac equation for the neutrino is given by $[\not{p} - \Sigma(p)] \psi = 0$, where $p$ is the neutrino four momentum and $\Sigma(p)$ is its self-energy. The neutrino self-energy can be calculated from the three diagrams given in
The contribution due to the W-exchange Fig[1](a) is given by

$$-i\Sigma_W(k) = \frac{g_W^2}{2} R \int \frac{d^4 p}{(2\pi)^4} \left[ \gamma_\mu S_e(p) \gamma_\nu D^{\mu\nu}(q) \right] L,$$

where we have defined the four momentum $q = k - p$, $R$ and $L$ are the right and left projection operators defined as

$$R, L = \frac{1}{2} \pm \frac{\gamma_5}{2}$$

and $S_e(p)$ is the electron propagator given by

$$S_e(p) = (\not{p} + m) \left( \frac{1}{(p^2 - m^2)} + i\Gamma_e(p) \right).$$

The vector boson propagator up to order $1/M^4$ is given by

$$D^{W,Z}_{\mu\nu}(p) \approx \frac{1}{M_{W,Z}^2} \left( g_{\mu\nu} + \frac{g_{\mu\nu} p^2}{M_{W,Z}^2} - \frac{p_\mu p_\nu}{M_{W,Z}^2} \right).$$

In the electron propagator

$$\Gamma_e(p) = 2\pi \delta(p^2 - m^2)\eta(p.u),$$

and

$$\eta(p.u) = \Theta(p.u)f(p.u) + \Theta(-p.u)f(-p.u),$$

where $u = (1, 0)$ is the four velocity of the medium, considered at rest, and $f(p.u)$ is the fermion distribution function.

The real part of Eq.(2.1) is given by

$$\text{Re}\Sigma_W(k) = -\frac{g_W^2}{2M_W^2} \int \frac{d^4 p}{(2\pi)^4} \delta(p^2 - m^2)\eta(p.u)R\gamma_\mu(\not{p} + m)\gamma_\nu L \times \left( g_{\mu\nu} + \frac{g_{\mu\nu} q^2}{M_W^2} - \frac{q_\mu q_\nu}{M_W^2} \right).$$

For simplicity let us define $\text{Re}\Sigma_W = \bar{R}\bar{\Sigma}_W L$ and by doing the $p_0$ integral, we get

$$\bar{\Sigma}_W(k) = \frac{g_W^2}{2M_W^2} \int \frac{d^3 p}{(2\pi)^3} E \left[ ((\not{p} - 2m)f(p.u) - (\not{p} + 2m)f(p.u) \right.$$

$$+ \frac{1}{M_W^2} \left\{ q^2(\not{p} - 2m) + \frac{q^\mu q^\nu}{2} \gamma_\mu(\not{p} + m)\gamma_\nu \right\} f(p.u)$$

$$- \left\{ q^2(\not{p} + 2m) + \frac{q^\mu q^\nu}{2} \gamma_\mu(\not{p} - m)\gamma_\nu \right\} \bar{f}(p.u) \right].$$
In general $\tilde{\Sigma}_W$ is a function of the external four momentum $k^\mu = (\omega, \mathbf{k})$ and the four velocity $u^\mu$ of the heat bath. Then this can be expressed as

$$-\tilde{\Sigma}_W = a_W k + b_W \mu, \quad (2.8)$$

where $a_W$ and $b_W$ are Lorentz scalars and can be calculated from,

$$T_k = -\frac{1}{4} Tr [j k \tilde{\Sigma}_W] = a_W k^2 + b_W \omega, \quad (2.9)$$

and

$$T_u = -\frac{1}{4} Tr [\mu k \tilde{\Sigma}_W] = a_W \omega + b_W. \quad (2.10)$$

For a test neutrino of energy $E_\nu = k \cdot u$ we obtain,

$$a_W = \frac{E_\nu}{k^2} T_u - \frac{T_k}{k^2}, \quad (2.11)$$

and

$$b_W = T_u - a_W E_\nu. \quad (2.12)$$

For massless neutrinos $k^2 = 0$ which implies $E_\nu = |\mathbf{k}|$. Using Eq. (2.7) in Eqs. (2.9) and (2.10) we obtain

$$T_k = \frac{g_0^2}{2M_W^2} \int \frac{d^3p}{(2\pi)^3} \left[ E_\nu \left( 1 + \frac{3m^2}{2M_W^2} \right) (f(p\cdot u) - \tilde{f}(p\cdot u)) - \frac{2E_\nu^2}{3M_W^2 E} (4E_\nu^2 - m^2)(f(p\cdot u) + \tilde{f}(p\cdot u)) \right], \quad (2.13)$$

and

$$T_u = -\frac{g_0^2}{M_W^2} \int \frac{p^2 dp}{(2\pi)^2 E_\nu} \left[ E_\nu(f(p\cdot u) - \tilde{f}(p\cdot u)) + \frac{1}{M_W^2} \left( \frac{3m^2 E_\nu}{2} + E_\nu^2 \right) (f(p\cdot u) - \tilde{f}(p\cdot u)) - \frac{1}{M_W^2} (2E_\nu^2 E_\nu + m^2 E_\nu) (f(p\cdot u) + \tilde{f}(p\cdot u)) \right], \quad (2.14)$$

with $E_e$ the electron energy and $m$ the electron mass. In Eqs. (2.13) and (2.14) we have three different types of terms which are proportional to $f(p\cdot u)$, $E_e f(p\cdot u)$ and $f(p\cdot u)/E_e$ and also proportional to corresponding anti-fermion distribution functions. The number densities of fermions and anti-fermions are defined as,

$$N_f = g \int \frac{d^3p}{(2\pi)^3} f_f(p\cdot u) \quad \text{and} \quad \tilde{N}_f = g \int \frac{d^3p}{(2\pi)^3} \tilde{f}_f(p\cdot u), \quad (2.15)$$

respectively, where $g = 2$ for $e, p$ and $n$ and one for neutrinos. For $E_e \gg \mu$, the fermion distribution function can be written in terms of a series, as follows

$$f(E_e) = \frac{1}{e^{E_e/\mu} - 1} = \sum_{l=1}^{\infty} (-1)^{l+1} e^{\alpha_l} e^{-\beta E_e}, \quad (2.16)$$

where $\mu$ is the chemical potential and $\alpha_l = \beta \mu l$. Using Eq. (2.16) the number density can be written as

$$N_f = gm^3 \sum_{l=0}^{\infty} (-1)^l e^{\alpha_l} \int_1^{\infty} t dt \sqrt{1 - te^{-\alpha_l}} = gm^3 \sum_{l=1}^{\infty} (-1)^{l+1} e^{\alpha_l} \frac{K_2(a_l)}{a_l}, \quad (2.17)$$

where $a_l = \beta ml$ and $K_i, \ i = 1, 2, \ldots$ are the modified Bessel functions of integer order. Similarly, we have

$$J_2 = \int \frac{d^3p}{(2\pi)^3} f(E_e) E_e = \frac{gm^3}{2\pi^2} \sum_{l=1}^{\infty} (-1)^{l+1} e^{\alpha_l} \frac{K_1(a_l)}{a_l}, \quad (2.18)$$
and

\[ J_3 = g \int \frac{d^3 p}{(2\pi)^3} E_c f(E_c) = \frac{g m^4}{2 \pi^2} \sum_{l=1}^{\infty} (-1)^{(l+1)} e^{\alpha_l} \times \]

\[ \times \left[ \frac{3}{a_l^2} K_0(a_l) + \frac{K_1(a_l)}{a_l} \left( 1 + \frac{6}{a_l^2} \right) \right], \tag{2.19} \]

which gives

\[ N_f - \tilde{N}_f = g \frac{m^3}{\pi^2} \sum_{l=1}^{\infty} (-1)^{(l+1)} \sinh(\alpha_l) \frac{K_2(a_l)}{a_l}, \tag{2.20} \]

\[ \Phi_2 = J_2 + \tilde{J}_2 = g \frac{m^2}{\pi^2} \sum_{l=1}^{\infty} (-1)^{(l+1)} \cosh(\alpha_l) \frac{K_1(a_l)}{a_l}, \tag{2.21} \]

and

\[ \Phi_3 = J_3 + \tilde{J}_3 = g \frac{m^4}{\pi^2} \sum_{l=1}^{\infty} (-1)^{(l+1)} \cosh(\alpha_l) \left[ \frac{3}{a_l^2} K_0(a_l) + \frac{K_1(a_l)}{a_l} \left( 1 + \frac{6}{a_l^2} \right) \right]. \tag{2.22} \]

Inserting these expressions in Eqs. (2.13) and (2.14), we obtain

\[ T_k = - \frac{g^2 W}{2 M_W^2} \left[ E_\nu \left( 1 + \frac{3 m^2}{2 M_W^2} \right) (N_e - \tilde{N}_e) - \frac{8 E_\nu^2}{3 M_W^2} \Phi_3 + \frac{2 E_\nu^2}{3 M_W^2} \Phi_2 \right], \tag{2.23} \]

and

\[ T_u = - \frac{g^2 W}{4 M_W^2} \left[ \left( 1 + \frac{3 m^2}{2 M_W^2} + \frac{E_\nu^2}{M_W^2} \right) (N_e - \tilde{N}_e) - \frac{2 E_\nu^2}{M_W^2} \Phi_3 - \frac{m^2 E_\nu^2}{M_W^2} \Phi_2 \right]. \tag{2.24} \]

In a heat bath one has to consider the thermal average of the quantities. So we have to replace \( E_\nu \) by \( \langle E_\nu \rangle \) and \( E_e \) by \( \langle E_e \rangle \) in the medium. Using the above definitions, Eqs. (2.13) and (2.14) can be written as

\[ T_k = - \frac{g^2 W}{4 M_W^2} \left[ E_\nu \left( 1 + \frac{3 m^2}{2 M_W^2} \right) (N_e - \tilde{N}_e) - \frac{8 E_\nu^2}{3 M_W^2} \langle E_e \rangle N_e + \langle E_e \rangle \tilde{N}_e + 2 m^2 E_\nu^2 \left( \frac{1}{E_e} \right) N_e + \frac{1}{E_e} N_e \right], \tag{2.25} \]

and

\[ T_u = - \frac{g^2 W}{4 M_W^2} \left[ \left( 1 + \frac{3 m^2}{2 M_W^2} + \frac{E_\nu^2}{M_W^2} \right) (N_e - \tilde{N}_e) - \frac{2 E_\nu^2}{M_W^2} \langle E_e \rangle N_e + \langle E_e \rangle \tilde{N}_e - \frac{m^2 E_\nu^2}{M_W^2} \left( \frac{1}{E_e} \right) N_e + \frac{1}{E_e} N_e \right]. \tag{2.26} \]

Now for massless neutrino Eqs. (2.11) and (2.12) give,

\[ a_W = - \sqrt{2} \frac{G_F}{M_W^2} \left[ E_\nu (N_e - \tilde{N}_e) + \frac{2}{3} \langle E_e \rangle N_e + \langle E_e \rangle \tilde{N}_e \right] - \frac{5 m^2}{3} \left( \frac{1}{E_e} \right) N_e + \frac{1}{E_e} N_e \right], \tag{2.27} \]
and

\[ b_W = -\sqrt{2}G_F \left[ \left( 1 + \frac{3}{2} \frac{m_e^2}{M_W^2} \right)(N_e - \bar{N}_e) - \frac{8}{3} \frac{E_{\nu}}{M_W^2} \langle E_{\nu} \rangle N_e + \langle E_{\bar{\nu}} \rangle \bar{N}_e \right] + \frac{2}{3} \frac{m^2 E_{\nu}}{M_W^2} \left( \frac{1}{E_{\nu}} N_e + \frac{1}{E_{\bar{\nu}}} \bar{N}_e \right), \]  

(2.28)

where \( g_W^2/4M_W^2 = \sqrt{2}G_F \), and \( G_F \) is the Fermi coupling constant. Compared to \( b_W \), \( a_W \) is suppressed by a factor \( 1/M_W^2 \). Now we have to calculate the neutrino self-energy due to Z-boson exchange. The Z-coupling to the fermions is

\[ \mathcal{L}_Z = -i \frac{g}{2 \cos \theta_W} \bar{f} \gamma^\mu \left( C_V^f - C_A^f \gamma^5 \right) f Z^\mu, \]  

(2.29)

where \( C_V^f \) and \( C_A^f \) denote the vector and axial vector couplings, which are given by

\[ C_V^f = \begin{cases} \frac{1}{2} + 2 \sin^2 \theta_W & e \\ \frac{1}{2} - 2 \sin^2 \theta_W & \nu \\ -1 & p, \end{cases} \]

(2.30)

and

\[ C_A^f = \begin{cases} \frac{3}{2} & \nu, p \\ -1 & e, n. \end{cases} \]

(2.31)

The contribution from the Z-exchange diagram Fig.II(b), can be obtained from the W-exchange diagram (a) by changing \( M_W \rightarrow M_Z \). The background here is made of neutrinos and anti-neutrinos instead of electrons and positrons. Making the above substitutions we obtain

\[ a_Z = -\sqrt{2}G_F \frac{M_Z^2}{M_W^2} \left[ \langle E_{\nu} \rangle (N_{\nu} - \bar{N}_{\nu}) + \frac{2}{3} \langle E_{\bar{\nu}} \rangle (N_{\bar{\nu}} + \bar{N}_{\bar{\nu}}) \right], \]

(2.32)

and

\[ b_Z = -\sqrt{2}G_F \left[ (N_{\nu} - \bar{N}_{\nu}) - \frac{8}{3} \frac{E_{\nu}^2}{M_Z^2} (N_{\nu} + \bar{N}_{\nu}) \right]. \]

(2.33)

Also \( a_Z \) is suppressed by a factor \( 1/M_Z^2 \) compared to \( b_Z \). Finally, we have to calculate the tadpole diagram due to Z-coupling Fig.II(c). The contribution to the self-energy due to this diagram is given by

\[ -i \Sigma_{Z_t}(k) = \frac{-ig}{2 \cos \theta_W} \gamma^\mu \left( C_V^\nu - C_A^\nu \gamma^5 \right) iD_{\mu\nu}(q = 0) \int \frac{d^4 p}{(2\pi)^4} (-1) \times \]

\[ \times \text{Tr} \left[ \frac{-ig}{2 \cos \theta_W} \gamma^\nu \left( C_V^f - C_A^f \gamma^5 \right) iS_F(p) \right], \]  

(2.34)

where \( Z_t \) corresponds to the tadpole diagram due to Z-boson coupling. The real part of this diagram is given by

\[ \text{Re}\Sigma_{Z_t}(k) = \frac{g^2}{M_Z^2 \cos^2 \theta_W} R \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E} \frac{1}{C_V} \hat{p}(f(p,u) - \bar{f}(p,u)). \]

(2.35)

Finally this gives

\[ a_{Z_t} = 0 \quad \text{and} \quad b_{Z_t} = -2\sqrt{2}G_F \sum_f \frac{C_V^f}{g} (N_f - \bar{N}_f). \]  

(2.36)

This result implies that we have to consider the tadpole diagram contribution for different backgrounds obtaining

\[ b_{Z_t}(e) = -\sqrt{2}G_F \left( \frac{1}{2} + 2 \sin^2 \theta_W \right) (N_e - \bar{N}_e), \]

\[ b_{Z_t}(\nu) = -\sqrt{2}G_F \sum_{\nu}(N_\nu - \bar{N}_\nu), \]

\[ b_{Z_t}(p) = -\sqrt{2}G_F \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) (N_p - \bar{N}_p), \]

\[ b_{Z_t}(n) = \frac{\sqrt{2}}{2} G_F (N_n - \bar{N}_n), \]  

(2.37)
for a background made of electrons, neutrinos, protons and neutrons respectively. For matter containing electrons, nucleons and neutrinos, the tadpole contribution will give

\begin{equation}
 b_{Z_t} = -\sqrt{2}G_F \left[ \left( -\frac{1}{2} + 2 \sin^2 \theta_W \right) (N_e - \bar{N}_e) + \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) (N_p - \bar{N}_p) - \frac{1}{2} (N_n - \bar{N}_n) + (N_{\mu} - \bar{N}_{\mu}) + (N_{\nu} - \bar{N}_{\nu}) \right].
\end{equation}

If the heat bath is charge neutral then,

\begin{equation}
 (N_p - \bar{N}_p) = (N_e - \bar{N}_e),
\end{equation}

and this gives

\begin{equation}
 b_{Z_t} = \frac{G_F}{\sqrt{2}} \left[ (N_n - \bar{N}_n) - 2 \sum \nu (N_\nu - \bar{N}_\nu) \right].
\end{equation}

The test particle is moving in a heat bath and the particles (fermions) in the heat bath have average energy

\begin{equation}
 \langle E \rangle = \frac{g \int \frac{d^3p}{(2\pi)^3} f(E) E}{N_f},
\end{equation}

where $N_f$ is the fermion number density already defined in Eq.(2.15). In the non-relativistic limit ($\beta m \gg 1$), Eqs. (2.17) to (2.19) reduces to

\begin{equation}
 N_f = g \left( \frac{m}{2\pi\beta} \right)^{3/2} e^{-\beta(m-\mu)},
\end{equation}

\begin{equation}
 J_2 = \frac{N_f}{\langle E \rangle} \quad \text{and} \quad J_3 = \langle E \rangle N_f.
\end{equation}

But for the relativistic fermions $E \simeq |p|$ and $N_f$ can be written as

\begin{equation}
 N_f = 2 \int \frac{d^3p}{(2\pi)^3} f(E) = \frac{1}{\pi^2} \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} E^2 dE e^{(n+1)\beta E} = \frac{3}{2\pi^2} \xi(3) T^3,
\end{equation}

and similarly for photon, this is given by

\begin{equation}
 N_\gamma = 2 \int \frac{d^3p}{(2\pi)^3} f_\gamma(E) = \frac{2}{\pi^2} \xi(3) T^3,
\end{equation}

where $\xi(3) = 1.20206$. This gives, $N_e/N_\gamma = 3/4$ and $N_\nu/N_\gamma = 3/8$. Then proceeding in the same way as in Eq.(2.43) we obtain

\begin{equation}
 2 \int \frac{d^3p}{(2\pi)^3} f(E) E = \frac{1}{\pi^2} \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} E^3 dE e^{(n+1)\beta E} = \frac{21}{4\pi^2} \xi(4) T^4,
\end{equation}

where $\xi(4) = \pi^4/90$ and we obtain the quantities

\begin{equation}
 \langle E_f \rangle = \frac{7}{2} \xi(4) T,
\end{equation}

and

\begin{equation}
 \langle \frac{1}{E_f} \rangle \simeq \frac{1}{\langle E_f \rangle}.
\end{equation}

One can define the asymmetry parameter for the particle species $\alpha$ as

\begin{equation}
 L_\alpha = \frac{N_\alpha - \bar{N}_\alpha}{N_\gamma},
\end{equation}
where $\alpha$ can be lepton or baryon. Let us assume that the average particle and anti-particle energies are the same. Then using these we can write

$$a_W = -\sqrt{2} \frac{G_F N_c}{M_W^2} \left[ \frac{\xi(4)}{2 \xi(3)} T(1 + L_e) - \frac{5 \xi(4) m^2}{7 \xi(3) T} \right].$$

(2.49)

and

$$b_W = -\sqrt{2} G_F N_\gamma \left[ \left(1 + \frac{3 m^2}{2 M_W^2}\right) L_e - \left(\frac{\xi(4)}{\xi(3)}\right)^2 \left(\frac{T}{M_W^2}\right)^2 + \frac{m^2}{M_W^2} \right].$$

(2.50)

Similarly Eqs. (2.32) and (2.33) can be written as

$$a_Z = -\sqrt{2} G_F N_\gamma \left[ \left(1 + \frac{3 m^2}{2 M_Z^2}\right) L_e - \left(\frac{\xi(4)}{\xi(3)}\right)^2 \left(\frac{T}{M_Z^2}\right)^2 + \frac{m^2}{M_Z^2} \right].$$

(2.51)

and

$$b_Z = -\sqrt{2} G_F N_\gamma \left[ L_{\nu_e} - \frac{1}{2} \left(\frac{\xi(4)}{\xi(3)}\right)^2 \frac{T^2}{M_Z^2} \right].$$

(2.52)

Finally the tadpole diagram contributions to the matter containing $e, p, n$ and $\nu$ is

$$b_{Z_t} = -\sqrt{2} G_F N_\gamma \left[ \left(\frac{1}{2} + 2 \sin^2 \theta_W\right) (L_e - L_p) - \frac{L_n}{2} + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} \right].$$

(2.53)

For the charge-neutral matter we have $L_e = L_p$, which gives

$$b_{Z_t} = \sqrt{2} G_F N_\gamma \left[ \frac{L_n}{2} - (L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}) \right].$$

(2.54)

We have calculated the quantities $a$ and $b$ as given in Eq (2.32) for all the three diagrams in Fig.1 and using these one can calculate the neutrino effective potential, which is shown in the next section. Also depending on the nature of the medium, there can be contributions from different backgrounds to $a$ and $b$.

### III. NEUTRINO EFFECTIVE POTENTIAL

We have discussed earlier that, the Dirac equation for the neutrino is modified in the medium and this will give the dispersion relation

$$\det ((1 + a) \not{k} + b \not{\mu}) = 0,$$

(3.1)

where the $a$ and $b$ are the same as given in Eq (2.32) and $\det$ is the determinant. From this equation we obtain,

$$(1 + a) \omega + b = \pm (1 + a) |\not{k}|.$$

(3.2)

As we have seen earlier, $a$ is much smaller than $b$, so we can neglect the $a$ contribution to the dispersion relation. Then the dispersion relation for the particle (positive energy solution) is

$$\omega - |\not{k}| \simeq -b.$$

(3.3)

From this we can see that, $-b$ is the effective potential which will be experienced by the neutrino when propagating through a medium. So we have the effective potential

$$V_{eff} \simeq -b.$$

(3.4)

For example, let us consider an electron neutrino propagating in a (i) background of only electrons and positrons and (ii) only neutrinos and anti-neutrinos. In the first case the contribution to $b$ will be given by

$$b = b_W + b_{Z_t}(e),$$

(3.5)
and in the second case

\[ b = b_Z + b_{Z_r}(\nu_e) + b_{Z_r}(\nu_\mu) + b_{Z_r}(\nu_\tau). \] (3.6)

Let us consider two different physical situations which can be realized in the cosmological and astrophysical scenarios, the early universe hot plasma, before the BBN\textsuperscript{3,4} and secondly the GRB fireball\textsuperscript{13,14,15}. Just before the nucleosynthesis when temperature was much above the electron mass but below the muon mass i.e. \( m \ll T \ll m_\mu \). In this situation, the contributions will come from \( e^\pm \), \( n \), \( p \) and all the neutrinos. Then the effective potential experienced by a electron neutrino is given by

\[
V_{\nu_e} \simeq \sqrt{2} G_F N_{\gamma} \left[ \left( \frac{1}{2} + 2 \sin^2 \theta_W \right) L_e + \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) L_\mu - \frac{1}{2} L_n + 2 L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} \right] - \left( 1 + \frac{1}{2} \cos^2 \theta_W \right) \left( \frac{7 \xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2}. \] (3.7)

If the test particle is \( \bar{\nu}_e \), in the above Eq.\textsuperscript{3.7}, the particle anti-particle asymmetry \( L_i \) will be replaced by \(-L_i\). Secondly, let us consider the GRB fireball as a thermalized radiation and electron-positron plasma of temperature about 2 to 10 MeV. We also consider the situation that, the fireball is contaminated by baryons\textsuperscript{16,17,18}. Thus \( \nu_e \) propagating through it will experience an effective potential

\[
V_{\nu_\mu, \bar{\nu}_e} = \sqrt{2} G_F N_{\gamma} \left[ \pm \mathcal{L} - \left( \frac{7 \xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2} \right], \] (3.8)

where

\[
\mathcal{L} = \left( \frac{1}{2} + 2 \sin^2 \theta_W \right) L_e + \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) L_\mu - \frac{1}{2} L_n. \] (3.9)

If the fireball is charge neutral \emph{i.e.} electron asymmetry is the same as the proton asymmetry (\( L_e = L_p \)) then

\[
V_{\nu_\mu, \bar{\nu}_e} = \sqrt{2} G_F N_{\gamma} \left[ \pm L_p + \frac{1}{2} L_n - \left( \frac{7 \xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2} \right]. \] (3.10)

The effective potential for \( \nu_\mu \) and \( \nu_\tau \) can also be calculated accordingly.

**IV. NEUTRINO OSCILLATION**

Here we consider the oscillation of neutrinos in the above mentioned two scenarios. The evolution equation for propagation of neutrinos in the medium is given by

\[
i \begin{pmatrix} \nu_x \\ \nu_y \end{pmatrix} = \begin{pmatrix} V - \Delta \cos 2\theta & \frac{\Delta}{2} \sin 2\theta \\ \frac{\Delta}{2} \sin 2\theta & 0 \end{pmatrix} \begin{pmatrix} \nu_x \\ \nu_y \end{pmatrix} \] (4.1)

where \( x \) is active and \( y \) can be active or sterile, \( \Delta = (m_e^2 - m_\mu^2)/2E_\nu = \delta m^2/2E_\nu \), \( V \) is the potential difference between \( V_x \) and \( V_y \), \( \text{i.e.} V = V_x - V_y \), \( E_\nu \) is the neutrino energy and \( \theta \) is the mixing angle. The conversion probability for the above processes is given by

\[
\mathcal{P}_{x \rightarrow y}(t) = \frac{\Delta^2 \sin^2 2\theta}{(V - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta} \times \sin^2 \left( \frac{\sqrt{(V - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}}{2} t \right). \] (4.2)

In the above equation, the resonance condition is given by

\[
V = \frac{\delta m^2}{2E_\nu} \cos 2\theta. \] (4.3)
So the resonance condition crucially depends on the sign of the potential. In the early universe case, if we consider the $\nu_e \leftrightarrow \nu_\mu$ oscillation, then the effective potential is

$$V = V_{\nu_e} - V_{\nu_\mu} = \sqrt{2} G_F N_\gamma \left[ L_e + L_{\nu_e} - L_{\nu_\mu} - \left( \frac{7 \xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2} \right].$$

(4.4)

It is obvious from the above equation that the active-active oscillation is independent of the baryonic contribution to the potential, whereas, if it is active-sterile oscillation, then the potential does depend on the baryonic component. Let us further assume that, $L_{\nu_e} = L_{\nu_\mu}$. This assumption is probably reasonable, because, during this epoch all the neutrinos are massless and equally populated. If this was the case, then $L_e > 6.14 \times 10^{-9} T_6^2$ ($T_6$ is in units of $10^6$ eV) is needed to have resonant conversion along with the condition Eq. (4.3). As already discussed by Enqvist et. al., the above oscillation can proceed equally from both sides and then it will not create any asymmetry in electron and muon neutrinos. The anti-neutrinos oscillation will be very much suppressed, due to the change in sign of the potential. But oscillation of active neutrinos to sterile ones will create asymmetry in $\nu$, $\bar{\nu}$ and this oscillation will depend on both lepton asymmetry and the baryon asymmetry in the medium.

The cosmological gamma ray bursts release about $10^{52}$ erg energy in a few second and make them the most luminous object in the universe. The sudden release of a large amount of gamma-ray photons into a compact region, can lead to an opaque photon-lepton fireball through the production of electron-positron pairs. It is believed that, the initial temperature of the fireball is in the range 3 to 10 MeV, so pair creation takes place and it forms a lepton-photon fireball with a typical radius of $10^7 - 10^8$ cm. The fireball is contaminated by baryons both from the source as well as from the surrounding environment. Neutrinos produced within the fireball due to inverse beta decay, or neutrinos from the progenitor, can propagate through the fireball. These propagating neutrinos can oscillate resonantly or non-resonantly, depending on the nature of the fireball. For $\nu_e \leftrightarrow \nu_\mu$ oscillation, the potential is

$$V = V_{\nu_e} - V_{\nu_\mu} = \sqrt{2} G_F N_\gamma \left[ L_e - \left( \frac{7 \xi(4)}{\xi(3)} \right)^2 \frac{T^2}{M_W^2} \right].$$

(4.5)

If the asymmetries $L_{\nu_e} = L_{\nu_\mu}$ is valid in the early universe plasma then it is exactly the same as in the GRB-fireball. So we can say that, GRB can mimic the early universe. One can see that the neutrino potential does not depend on the baryon content of the fireball. The reason is, proton and neutron contributions to the effective potential are the same for $\nu_e$ and $\nu_\mu$ and because of the difference it cancels out. So even if there are baryons in the fireball, they will not affect the $\nu_e \leftrightarrow \nu_\mu$ or the corresponding anti neutrino oscillation processes. Apart from this we can also see that, for the antineutrino process, the potential is negative, hence the resonance condition Eq. (4.3) can never be satisfied. On the other hand for the $\nu_e \leftrightarrow \nu_\mu$ process, depending on the value of $L_e$, there can be a resonance. First of all, for resonance to occur, the condition $L_e > 6.14 \times 10^{-9} T_6^2$ has to satisfy and this is only possible for neutrino processes. Secondly one has to explore the neutrino energy, $\delta m^2$ and mixing angle of the neutrinos to satisfy the resonance condition in Eq. (4.3). For example if we take $T_6 = 3$ then for resonance to take place $L_e > 5.6 \times 10^{-8}$. Apart from this if we further assume a spherical fireball of radius $\sim 100$ Km and $L_e = L_p$ then the fireball is contaminated by a mass $> 1.7 \times 10^{-10} M_5$. In the fireball also, (as in the case of Early Universe) the oscillation of active neutrino to sterile one depends on the baryon and lepton content of the fireball. A detail analysis of the neutrino oscillation in the GRB fireball is in progress.

V. CONCLUSIONS

We have calculated the $1/M^4$ order contribution to the neutrino self energy in Early Universe hot plasma and the GRB fireball, where the higher order contribution to the neutrino effective potential has potential importance. We have looked for the condition of resonant conversion/oscillation of active to active neutrinos, which does not depend on the baryon content of the propagating medium. On the other hand, the active to sterile oscillation/conversion does depend on the baryon content of the medium. It is shown that, under certain circumstances, the GRB fireball can behave as Early Universe hot plasma. The resonant oscillation of neutrino in the GRB fireball depends on its physical parameters (temperature and particle asymmetry) as well as on the neutrino mixing angle and mass square differences. If these conditions are satisfied, the resonant oscillations of active to active neutrinos may have an effect on the energy release of the GRB Fireball. This has to be studied in great detail in future.

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