A model for CMB anisotropies on large angular scales

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Abstract

We investigate the possibility that a low-temperature, low-frequency anomaly in the black-body spectrum, as it emerges when enlarging the Standard Model’s gauge-group factor $U(1)_Y$ to $SU(2)$ (Yang-Mills scale $\sim 10^{-4}$ eV), explains the discrepancy between the Local Group’s velocity as directly observed and as inferred by assuming a purely kinematic origin of the CMB dipole. This discrepancy determines the kinetic term for temperature fluctuations in our model action. Upon subtracting the dipole contribution this model can then be used to predict the low multipoles with $l \geq 2$ in the CMB temperature-temperature correlation and to reinvestigate the issue of statistical isotropy.
1 Introduction

The physics of photon propagation enters an exciting epoch in view of the emergence of a number of experimental and observational results that are unexplained by present theory [1, 2, 3].

The purpose of the present work is to propose a model designed to accommodate the discrepancy between the Local Group’s velocity as directly observed by the motion of galaxies and as inferred by assuming a purely kinematic origin of the CMB dipole. The idea is that in addition to the kinematic contribution to the CMB dipole there exists a dynamic component which (in nonrelativistic approximation) is independent of the velocity of the observer. The ultimate cause for the discrepancy is then attributed to a low-frequency and low-temperature anomaly in black-body spectra as it arises when embedding the U(1)Y-factor of the Standard Model into a new SU(2) gauge symmetry: SU(2)CMB [4, 5, 6, 7, 8]. As a consequence of the nonabelian nature of SU(2)CMB the screening mass of the photon is a function of momentum |p|, temperature T, and Yang-Mills scale ΛCMB. The value of the Yang-Mills scale ΛCMB ∼ 10^{-4} eV follows from the observational fact that today’s photon propagation is not affected by nonabelian fluctuations (thermodynamically decoupled) and completely ignores the thermodynamic ground state of this theory (no preferred rest frame for photon propagation, see [5]). That is, the thermodynamics of SU(2)CMB is at the boundary between the deconfining and preconfining phase (supercooled state [6]) within the present cosmological epoch (no screening of photons).

The only free parameter of the model proposed in the present work is determined by the value of the discrepancy for the Local Group’s velocity. Once this parameter is extracted from the data the model can be used to predict the dipole-subtracted temperature-temperature correlation at small angular resolution. We expect that the model will postdict the observed suppression of and correlation between the low-l multipoles [2] in the temperature-temperature angular power spectrum without the need to invoke early reionization.

The article is organized as follows: In Sec.2 we review the consequences of the thermodynamics of SU(2)CMB in view of an anomaly in black-body spectra at low temperature and frequency as obtained in [7, 8]. We then derive the deviation of the energy density as compared to the conventional case. In Sec.3 we set up our model. Namely, we justify the notion of temperature as a scalar field and investigate the dynamics of temperature fluctuations as driven by the black-body anomaly. In Sec.4 we discuss the discrepancy between observed and inferred Local-Group velocity and explain how this discrepancy is accommodated as a dynamic effect within the realm of SU(2)CMB. Subsequently, we perform a numerical analysis of our evolution equation when assuming spherical symmetry. Finally, we present and interpret our results. In Sec.5 we give a summary and discuss future work.

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1By low temperature we mean a few times $T_{CMB} = 2.73$ K.
2 Anomaly in black-body spectra

The screening effects on photon (γ) propagation of the charged and massive vector excitations \( V^\pm \), as they emerge by virtue of a nontrivial ground state [5], were computed in [7] by evaluating the polarization tensor as a function of temperature \( T \) and (on-shell) momentum \( p \). For the foundations of a nonperturbative approach to SU(2) Yang-Mills thermodynamics in the deconfining phase see [4, 9]. Only this phase is relevant for CMB physics.

Taking screening effects into account, γ’s dispersion law modifies as

\[
\omega^2 = p^2 \quad \longrightarrow \quad \omega^2 = p^2 + G(\omega, p, T, \Lambda),
\]

where \( \omega \) is the energy of the γ-mode and \( p \) its spatial momentum. While the screening function \( G \) is negative (with a small modulus) for large values of \(|p|\) (anti-screening), it is positive and of sizable value for small \(|p|\) (strong screening). For details see [8]. At the critical temperature \( T_c \equiv T_{CMB} = \frac{\Lambda_{CMB}}{2\pi} \lambda_c \approx 13.87 \) K, where the \( V^\pm \) acquire an infinite mass and thus decouple thermodynamically, the propagation of γ is entirely unscreened (\( G \equiv 0 \)) which is in accord with astrophysical observation. For light propagation at temperatures typically prevailing on earth the effect is very weak due to a power suppression of \( G \) for \( T \gg T_c \sim 2.73 \) K.

According to [8] the effect of the function \( G \) on the spectral power of a black body can be expressed as follows:

\[
I_{SU(2)}(\omega) = I_{U(1)}(\omega) \times \left( \frac{\omega - \frac{1}{2} \frac{dG}{d\omega}}{\omega^2} \right) \sqrt{\omega^2 - G} \theta(\omega - \omega^*),
\]

where \( \theta \) is the Heaviside step function, \( \omega^* \) is the root of \( \omega^2 = G \), and \( I_{U(1)} \) denotes the spectral power of the conventional black body. One has

\[
I_{U(1)}(\omega) = \frac{1}{\pi^2} \frac{\omega^3}{\exp[\omega/T] - 1}.
\]

Fig.1 shows the (dimensionless) ratio of the modified spectral power \( I_{SU(2)} \) and \( T^3 \) as a function of (dimensionless) frequency \( Y \equiv \frac{\omega}{T} \) at a temperature of \( T = 10 \) K. In Fig.2 the low-frequency part of the spectrum, where the deviation from the conventional case is best visible, is indicated. The deviation \( \delta \rho \equiv \rho_{SU(2)} - \rho_{U(1)} \) of the energy density then is calculated as

\[
\delta \rho = \int_0^\infty d\omega I_{SU(2)} - \int_0^\infty d\omega I_{U(1)} < 0.
\]

3 Dynamics of temperature evolution

In this section we derive the dynamic equations governing the cosmic evolution of temperature fluctuations. In a first step, we perform a match to the situation of
Figure 1: Dimensionless spectral power $\frac{1}{T^3}$ of a black body as a function of dimensionless frequency $Y \equiv \frac{\omega}{T}$ at $T=10$ K. The black (gray) curve depicts the modified (conventional) spectrum.

Figure 2: Zoom-in of the low-frequency part at $T=10$ K. The black (gray) curve depicts the modified (conventional) spectrum.
Figure 3: The situation relevant for the dynamic contribution to the CMB dipole after the black-body anomaly has evolved an initial inhomogeneity.

a perfect fluid which enables us to interpret temperature as a scalar field subject to an adiabatic approximation. Subsequently, we allow for deviations from the adiabatic limit to obtain a dynamic temperature evolution. Finally, we derive the linearized evolution equation for temperature fluctuations sourced by the anomaly in the black-body spectrum [7,8]. This evolution leads to the situation as sketched in Fig.3

3.1 Temperature as a scalar field

Let us now investigate to what extent it is possible to regard temperature as a scalar field. We start by considering the energy-momentum tensor $T_{\mu\nu}$ of a perfect fluid whose energy density $\rho$ and pressure $p$ are functions of temperature $T$:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu},$$

(5)

where $u_\mu$ is the four-velocity of a fluid segment or the local rest frame of the heat bath, and the signature of the metric tensor $g_{\mu\nu}$ is $(1, -1, -1, -1)$. We now seek an action which produces the right-hand side of Eq. (5) upon use of

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \delta \mathcal{L} \delta g^{\mu\nu},$$

(6)

where $\mathcal{L}$ is a scalar action density and $\det g_{\mu\nu} \equiv g$. We make the ansatz

$$\mathcal{L} = \sqrt{-g} (\alpha u_\mu u_\nu g^{\mu\nu} + \beta)$$

(7)

with parameters $\alpha$ and $\beta$ to be determined such that the perfect-fluid form in Eq. (5) emerges when employing Eq. (6). Notice that in varying the action density in Eq. (7) after $g^{\mu\nu}$ the four velocity $u_\mu$ is kept fixed. The connection between $u_\mu$ and $g^{\mu\nu}$ is made subsequently by virtue of Einstein’s equations.
Comparing the coefficients in front of the two independent tensor structures in Eq. (5) yields:

\[ \alpha = \rho + p, \quad \beta = -\rho - 3p. \]  

(8)

Thus the Lagrangian density in Eq. (7) becomes

\[ \mathcal{L} = \sqrt{-g} \left( (\rho + p) u_\mu u_\nu g^{\mu\nu} - \rho - 3p \right) = -2\sqrt{-g} \rho. \]

(9)

Specializing to a conventional photon gas with equation of state \( \rho = 3p \), we have

\[ \mathcal{L} = -\frac{2}{3} \sqrt{-\tilde{g}} \rho. \]

(10)

Since \( \rho \propto T^4 \) it follows that temperature itself acquires the status of a scalar field as long as the adiabatic approximation underlying the perfect-fluid philosophy remains valid.

### 3.2 Temperature fluctuations

The Lagrangian density in Eq. (10) represents a potential for the scalar field \( T \). We would like to go beyond this adiabatic approximation by allowing for a kinetic term for the temperature fluctuation \( \delta T \) which we also interpret as a scalar field:

\[ T = \bar{T}(t) + \delta T(t, \mathbf{x}). \]

(11)

The deviation \( \delta \rho \) of the energy density due to the black-body anomaly, see Sec. 2, induces the fluctuation \( \delta T \) about the mean temperature \( \bar{T} \). The latter is redshifted by the evolution of the cosmological background. We consider the conventional black-body part \( \bar{\rho}(\bar{T}) \equiv \rho_{\text{U(1)}} = \frac{\pi^2}{45} \bar{T}^4 \) as a fluid which, among other contributions (cold dark matter and dark energy), sources spatially flat Friedmann-Robertson-Walker (background) cosmology:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) dx^2, \]

(12)

where \( a(t) \) denotes the scale factor.

To make the action a scalar the usual factor of \( \sqrt{-g} \) in the action density is expressed in terms of \( \bar{T} \) and \( \bar{T}_0 \equiv T_c \) by virtue of the identity

\[ \frac{a(t)}{a_0} = \frac{\bar{T}_0}{\bar{T}} \quad \Rightarrow \quad \sqrt{-\tilde{g}} = \left( \frac{\bar{T}_0}{\bar{T}} \right)^3, \quad (a_0 \equiv 1). \]

(13)

In Eq. (13) and in the remainder of the article a subscript ‘0’ refers to today’s value of the corresponding quantity. There is a priori no principle fixing the coefficient

\[ \frac{\bar{a}(t)}{\bar{a}_0} = \frac{\bar{T}_0}{\bar{T}} \quad \Rightarrow \quad \sqrt{-\tilde{g}} = \left( \frac{\bar{T}_0}{\bar{T}} \right)^3, \quad (a_0 \equiv 1). \]

(13)

\[ \text{We are only interested in redshifts } z \text{ up to } z = 30 \text{ thus justifying the assumption of a spatially flat Universe driven by } \Lambda \text{CDM.} \]
in front of the kinetic term for $\delta T$. Thus we allow for a dimensionless coefficient $k$ which ultimately needs to be determined observationally. Using Eq. (13) we have:

$$\sqrt{-g} \mathcal{L}_{\text{CMB}} = \left( \frac{T_0}{T} \right)^3 (k \partial_{\mu} \delta T \partial^{\mu} \delta T - \delta \rho(T)).$$

(14)

Let us now define a function $\hat{\rho}(T, T_0)$ as

$$\delta \rho = T_0^2 \hat{\rho}.\quad (15)$$

Varying the action associated with Eq. (14) w.r.t. $\delta T$ then yields the following equation of motion:

$$\partial_{\mu} \partial^{\mu} \delta T - \frac{3}{T} \partial_{\tau} T \partial_{\tau} \delta T + \frac{T_0^2}{k H_0^2} \left[ \frac{1}{2} \left. \frac{d^2 \hat{\rho}}{dT^2} \right|_{T=T_0} \delta T + \frac{1}{2} \left. \frac{d \hat{\rho}}{dT} \right|_{T=T_0} \right] = 0.\quad (16)$$

In Eq. (16) we have performed the coordinate transformation

$$\tilde{x}_0 \equiv \tau = H_0 t, \quad \tilde{x}_i = \frac{da}{dt} x_i, \quad (i = 1, 2, 3).\quad (17)$$

Notice the extremely large factor $(T_0/H_0)^2 \sim 10^{60}$ in front of the square brackets in Eq. (16). This factor arises because we chose to measure time $\tau$ in units of the age of the Universe, distances from the origin $\tilde{x}_i$ in units of the actual horizon size $H^{-1} = a/\frac{da}{dt}$ (as long as $|\tilde{x}_i|$ is sufficiently smaller than unity), and temperature in units of $T_0 = 2.35 \times 10^{-4}$ eV.

Assuming spherical symmetry for the fluctuation $\delta T$, which is relevant for an analysis of the cosmic dipole, see Sec. 4.1 Eq. (16) reads:

$$0 = \partial_{\tau} \partial_{\tau} \delta T - \left( \frac{da}{dt} \right)^2 \left[ \partial_{\tau} \partial_{\tau} \delta T + \frac{2}{\sigma} \partial_{\sigma} \delta T \right] - \frac{3}{T} \partial_{\tau} T \partial_{\tau} \delta T + \frac{T_0^2}{k H_0^2} \left[ \frac{1}{2} \left. \frac{d^2 \hat{\rho}}{dT^2} \right|_{T=T_0} \delta T + \frac{1}{2} \left. \frac{d \hat{\rho}}{dT} \right|_{T=T_0} \right].\quad (18)$$

In Eq. (18) we have introduced $\sigma \equiv \sqrt{\tilde{x}_1^2 + \tilde{x}_2^2 + \tilde{x}_3^2}$.

### 3.3 Background evolution

Here we would like to provide some information about the simple $\Lambda$CDM model for the background cosmology\footnote{The contribution of $\hat{\rho}(T)$ is negligible for $z \leq 30$.} which fits the data best [2, 10, 11, 12]. We assume a spatially flat Universe subject to the following Friedmann equation

$$\left( \frac{\dot{a}}{a} \right)^2 = H_0^2 \left( \frac{\Omega_m}{a^3} + \Omega_\Lambda \right).\quad (19)$$
where $\Omega_m = 0.24$ and $\Omega_\Lambda = 1 - \Omega_m = 0.76$ (fit obtained from WMAP three-year data [13]) are the cold dark-matter and the dark-energy density, respectively, both in units of the critical density. $H_0$ is today’s value of the Hubble parameter, and $\dot{a} \equiv \frac{da}{dt}$. The solution to Eq. (19) is

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \left[\sinh \frac{3\sqrt{\Omega_\Lambda}}{2} H_0 t\right]^{2/3}, \quad (20)$$

where $H_0$ is connected to $t_0$ (present age of the Universe) as

$$H_0 t_0 = \frac{1}{3\sqrt{1 - \Omega_m}} \ln \frac{2 - \Omega_m + 2\sqrt{1 - \Omega_m}}{\Omega_m}, \quad (21)$$

and we use the convention that $a_0 \equiv a(t_0) = 1$. The mean temperature $\bar{T}$ then follows from Eqs. (13) and (20). As a reminder we give the relation between scale factor $a$ and redshift $z$ since we present our results as functions of $z$:

$$a(z) = \frac{1}{1 + z}, \quad (a_0 \equiv a(z = 0) = 1). \quad (22)$$

4 Numerical analysis

4.1 Principle remarks and boundary conditions

To identify a dynamic component in the CMB dipole, which dominates the higher multipoles by two orders of magnitude, the associated solution to Eq. (16) must locally exhibit a singled-out direction. This implies spherical symmetry about the center of an initial inhomogeneity which, by the source term in Eq. (18), induces the built-up of the spatially extended fluctuation $\delta T$. Notice that a superposition of solutions obtained for several such isolated inhomogeneities is not a solution of Eq. (16) due to the presence of the spatially homogeneous source term. Notice also that such an initial situation would evolve to populate higher multipoles of comparable strength as the dipole. This, however, is in contradiction to observation. We conclude that in describing a dynamic component to the CMB dipole spherical symmetry of the fluctuation $\delta T$ is imperative within the horizon of the center of the inducing, initial inhomogeneity. Then almost each observer perceives a modulus of the dynamic CMB-dipole component which is nearly independent of his position. That is, the mean radial gradient approximately serves to define a singled-out direction except at the center of the (as we shall see) bump-like $\delta T$. This exception, however, occurs with vanishing likelihood geometrically, see Fig. 3.

\footnote{The dynamic situation is possibly not unlike the evolution of an initial superposition of static, solitonic configurations in a nonlinear, classical field theory as for example the motion of magnetic (anti)monopoles in an SU(2) adjoint Higgs model [14]. Either there is repulsion pushing participants beyond each other’s horizon or annihilation takes place which destroy the approximate local spherical symmetry about the center of an initial inhomogeneity.}
The modulus of the dynamic component $D_{\text{dyn}}$ as it would be perceived by an observer situated a radial distance $\sigma_0$ away from the center of the bump then is defined as follows:

$$|D_{\text{dyn}}| \equiv \int_{\sigma_0}^{1} d\xi \delta T(z = 0, \xi) - \int_{\sigma_0-1}^{\sigma_0} d\xi \delta T(z = 0, \xi).$$

(23)

The upper limit in the first integral arises from the fact that for $\sigma \geq 1$ the nonexistence of a causal connection to the center of the bump forbids the built-up of a profile. The definition in Eq. (23) states that $|D_{\text{dyn}}|$ is roughly given by the mean gradient of $\delta T$.

Now the coefficient $k$ in Eq. (18) is determined such that the mean gradient in $\delta T(z = 0, \sigma)$ coincides with the dynamic component in the CMB dipole. The latter is attributed to the following discrepancy: On one hand, the velocity of the Local Group $v_{\text{LG,dir}}$ can be determined directly by estimating the gravitational impact on it by all those galaxies contained in successively enlarged concentric, spherical shells and by observing saturation for $z c \geq 6000 \text{ km s}^{-1}$ [15]. Here $c$ is the velocity of light. It is found that $|v_{\text{LG,dir}}| \sim 400 \text{ km s}^{-1}$ [15] with errors typically being $\sim 50 \text{ km s}^{-1}$ [16]. On the other hand, the conventional understanding of the CMB dipole as a purely kinematic effect [17], which implies a velocity of the solar system of $v_{\text{SS}} = (369 \pm 2) \text{ km s}^{-1}$ [18], plus the known relative velocity $v_{\text{LG-SS}}$ between the solar system and the Local Group allows to deduce a velocity of the Local Group of $|v_{\text{LG,dedu}}| \sim 619 \text{ km s}^{-1}$ [20]. In this way the angle $\delta \equiv \angle v_{\text{LG,dedu}}, v_{\text{LG,dir}}$ is extracted as $\delta = (13 \pm 7)^\circ$ [15]. As a consequence, a deficit velocity $v_{\text{dyn}} = v_{\text{LG,dedu}} - v_{\text{LG,dir}}$ is generated which must have a dynamic origin.

Let us explain this in more detail: On one hand, the velocity $v_{\text{LG,dir}}$ generates a kinematic contribution to the CMB dipole, $D_{\text{LG,kin}}$, whose amplitude $\Delta T \equiv \frac{1}{2} |D_{\text{LG,kin}}|$ is calculable according to [17] as

$$\Delta T = \frac{|v|}{c} T_0 + \mathcal{O}\left(\frac{v^2}{c^2}\right).$$

(24)

On the other hand, the CMB dipole $D_{\text{SS}}$, as it is measured in the solar-system rest frame, is supplemented by a purely kinematic contribution $D_{\text{LG-SS,kin}}$ resulting from the relative velocity $v_{\text{LG-SS}}$ between the solar system and the Local Group. This yields the CMB dipole $D_{\text{LG,true}}$ as it is perceived in the rest frame of the Local Group. Knowing $v_{\text{LG,dedu}}$, we can compute $D_{\text{LG,true}}$ by means of Eq. (24). Since

$$D_{\text{LG,true}} = D_{\text{LG,kin}} + D_{\text{dyn}}$$

(25)

5The origin of Eq. (23) is explained as follows: Looking into (opposite to) the direction of the gradient, a surplus (deficit) of photons stemming from the hotter (colder) tail (central region) of the profile $\delta T$ is detected by the observer. This allows to define a mean temperature. The amplitude of the dipole then is half the difference between the temperature into and opposite to the direction of the gradient. These temperatures are obtained by performing a radial average over $\delta T$ within the horizon of the observer.

6In [19] a value of $|v_{\text{LG,dedu}}| = (627 \pm 22) \text{ km s}^{-1}$ was obtained.
Figure 4: Diagram relating observed and deduced quantities concerned with CMB-dipole physics.

the dynamic contribution to the CMB dipole $D_{\text{dyn}}$ follows. In Fig. 4 this situation is sketched.

Eq. (18) is an inhomogeneous partial differential equation which can be solved using the numerical method of lines, see [21]. Four boundary conditions are required, two for the temporal and two for the spatial evolution. We assume the spatial distribution of the initial fluctuation at redshifts $z_i = 5...30$ to be of Gaussian shape with its height chosen such that $\frac{\delta T(z_i, \sigma = 0)}{\bar{T}(z_i)} = 10^{-5}$ as is expected to be provided by primordial causes:

$$\delta T(z_i, \sigma) = 10^{-5} \bar{T}(z_i) e^{-\left(\frac{\sigma}{w}\right)^2},$$

(26)

where the subscript $i$ refers to ‘initial’. The width $w$ of the Gaussian in Eq. (26) will be varied.

Initially, we assume the built-up of the fluctuation to be slow since the source term $\frac{1}{2} \left. \frac{d \delta \rho}{dT} \right|_{T=\bar{T}}$ in Eq. (18) driving this built-up is small for sufficiently large initial redshift $z_i$, see Fig. 5. That is, we prescribe

$$\left. \partial_T \delta T(\tau, \sigma) \right|_{\tau = \tau_i} = 0$$

(27)

and later check for the independence of the result on our choice of $z_i$. For a comparison of orders of magnitude between the two terms in Eq. (18) dependent on the black-body anomaly, the ‘restoring force’ $\frac{1}{2} \left. \frac{d^2 \delta \rho}{dT^2} \right|_{T=\bar{T}}$ is depicted as a function of $z$ in Fig. 6. Since $\delta T < 10^{-2}$ K the source term strongly dominates the restoring term.

As a function of $\sigma$ the fluctuation $\delta T$, being either weakened or enforced during the evolution, remains extremal at $\sigma = 0$ where the initial inhomogeneity (a seed

\footnote{We also set $\frac{\delta T(z_i, \sigma = 0)}{\bar{T}(z_i)} = 0$ at times.}
Figure 5: The source term $\frac{1}{2} \frac{d}{dT} \left. \delta \phi \right|_{T=\bar{T}}$ as a function of redshift $z$.

Figure 6: The ‘restoring force’ $\frac{1}{2} \frac{d^2}{dT^2} \left. \delta \phi \right|_{T=\bar{T}}$ as a function of redshift $z$. 
for $\delta T$) was located. So it should satisfy
\[ \partial_{\sigma} \delta T(\tau, \sigma) \bigg|_{\sigma=0} = 0. \quad (28) \]
Finally, $\delta T$ is zero for $\sigma \geq 1$ (horizon) and for all times. Otherwise, the built-up of $\delta T$ would be noncausal:
\[ \delta T(\tau, \sigma \geq 1) = 0. \quad (29) \]
In order to be consistent with the b.c. in Eq. (26), we approximate the b.c. of Eq. (29) by simply prescribing the value of the profile $\delta T$ at $z_i = 0$ and $\sigma = 1$ for all $z$. This is in good agreement with Eq. (29) if $\omega \ll 1$.

### 4.2 Values for $k$ and higher multipoles

The coefficient $k$ in Eq. (18) is chosen such that half of the mean gradient of the profile $\delta T$ at $z = 0$ equals the inferred amplitude $\Delta T$ for the dynamic contribution $|D_{\text{dyn}}|$. We impose one observationally suggested set (A) and one set with a fictionally large angle $\delta$ and an upper-limit value for $|v_{\text{LG,dir}}|$ (B) (taking $|v_{\text{LG,ded}}| \sim 627 \text{ km s}^{-1}$) as
\[
\begin{align*}
(A) : & \quad |v_{\text{LG,dir}}| = 400 \text{ km s}^{-1}, \quad \delta = 13^o \quad \Rightarrow \\
& \quad |v_{\text{dyn}}| = 253.74 \text{ km s}^{-1}, \quad |D_{\text{dyn}}| = 2.311 \text{ mK} \quad \Rightarrow \quad k = 0.01868 \bar{T}_0^2/H_0^2; \\
(B) : & \quad |v_{\text{LG,dir}}| = 450 \text{ km s}^{-1}, \quad \delta = 30^o \quad \Rightarrow \\
& \quad |v_{\text{dyn}}| = 327.00 \text{ km s}^{-1}, \quad |D_{\text{dyn}}| = 2.978 \text{ mK} \quad \Rightarrow \quad k = 0.01449 \bar{T}_0^2/H_0^2. \\
\end{align*}
\]
(30)
In writing the two values for $k$ in Eq. (30) we have anticipated some results of Sec. 4.3. Namely, our simulations indicate that the mean gradient of the profile $\delta T$ at $z = 0$ does not depend on $z_i$ for $5 \leq z_i \leq 30$, does not depend on the width $w$ in Eq. (26), and does not depend on the height for
\[ 0 \leq \delta T(z_0, \sigma = 0) \leq 10^{-5} \bar{T}(z = 0). \quad (31) \]
The mean gradient does, however, depend roughly linearly on the strength of the source term in Eq. (18), thus generating a definite value$^9$ for $k$.

On one hand, a virtue of the model is to accommodate the possibility for $D_{\text{dyn}}$, the latter serving to fix the value of the coefficient $k$. On the other hand, once $k$ is

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8 This statement is only approximately valid because the employed relation between coordinates $\tilde{x}_i$ and $x_i$ in Eq. (17) actually is only valid for their differentials.

9 The dependence of $|D_{\text{dyn}}|$ on $\sigma_0$, as dictated by Eq. (23), is weak in the vicinity of the maximum at $\sigma_0 \sim 2$. Strictly speaking, the observationally inferred value of $|D_{\text{dyn}}|$ only fixes a curve $\mathcal{C}$ in the $k$-$\sigma_0$ plane, and it must be checked to what extent the postdiction of the dipole-subtracted correlator depends on variations of $k$ and $\sigma_0$ along $\mathcal{C}$. 

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fixed by this observational input a calculation of the dipole-subtracted large-angle correlation function (with a slight abuse of notation)

\[ C(\theta) \equiv \langle \delta T_{\text{dyn},l \geq 2}(\hat{e}_1), \delta T_{\text{dyn},l \geq 2}(\hat{e}_2) \rangle, \quad (\theta \equiv \angle \hat{e}_1, \hat{e}_2), \]  

(32)
is enabled by virtue of Eq. (16). Namely, by subtracting Eq. (18) (dynamic contribution \( \delta T_{\text{dyn},l=1} \) to the dipole) from Eq. (16) (general fluctuation \( \delta T_{\text{dyn}} \), not imposing spherical symmetry) we arrive at

\[
\frac{1}{T} \partial_T \partial_T \delta T_{\text{dyn},l \geq 2} - \frac{3}{T} \partial_T T \partial_T \delta T_{\text{dyn},l \geq 2} + \frac{1}{2} \frac{T_0^2}{k H_0} \left. \frac{d^2 \rho}{dT^2} \right|_{T=T} \delta T_{\text{dyn},l \geq 2} = 0, \]  

(33)

where \( \delta T_{\text{dyn},l \geq 2} \equiv \delta T_{\text{dyn}} - \delta T_{\text{dyn},l=1} \). That is, dipole-subtracted fluctuations obey a wave equation with a cosmological damping term and a ‘restoring force’ (second derivative of the black-body anomaly \( \delta \rho \)). Cartesian two-point correlations of \( \delta T_{\text{dyn},l \geq 2} \) at \( z = 0 \), which are required to postdict \( C(\theta) \), can be computed by an average with primordially provided initial conditions of the product

\[
\delta T_{\text{dyn},l \geq 2}(z = 0, \bar{x}_1) \delta T_{\text{dyn},l \geq 2}(z = 0, \bar{x}_2),
\]  

(34)

where \( \delta T_{\text{dyn},l \geq 2} \) is a solution to Eq. (33) (classical approximation, see [22, 23]). The correlation function \( C(\theta) \) then follows as

\[
C(\theta) = \int_0^1 d|\bar{x}_1| \int_0^1 d|\bar{x}_2| \langle \delta T_{\text{dyn},l \geq 2}(z = 0, |\bar{x}_1|\hat{e}_1) \delta T_{\text{dyn},l \geq 2}(z = 0, |\bar{x}_2|\hat{e}_2) \rangle,
\]  

(35)

and one can check the usual assumption made about statistical isotropy\(^{10}\) by varying \( \hat{e}_1, \hat{e}_2 \) while keeping \( \theta \) fixed. This analysis is reserved for future work.

### 4.3 Results of numerical calculation

Here we present the results of our numerical calculation. Fig. [7] shows the fluctuation \( \delta T \) for \( \sigma = 0.5; 0.05 \) as a function of redshift \( z \) with a width of the initial Gaussian assumed as \( w = 10^{-2} \). Obviously, the major contribution to \( \frac{4T}{T} \) is generated within \( 0 \leq z \leq 1 \) corresponding to a temperature range \( T_0 = 2.73 \text{K} \leq T \leq 8.1 \text{K} \). This is expected from the discussion in [7, 8]. We have checked that, switching off the source term in Eq. (18) and keeping all other conditions the same, the initial profile oscillates in a strongly damped way. This is consistent with our finding that the evolution in \( z \) as described by Eq. (18) possesses an attractor which is determined by this source term, see below.

In Figs. [8] and [9] we show \( \frac{4T}{T} \) as a function of \( \sigma \) at \( z = 0 \) and \( z = 1 \) when varying the shape of the initial profile at \( z_i = 20 \). Our results are practically independent of these initial conditions. Again, it is seen that a major contribution to the profile at \( z = 0 \) is being built up for \( 0 \leq z \leq 1 \). Next, we investigate the dependence of the
Figure 7: $\frac{\delta T}{T}$ at two distances $\sigma = 0.5$ (black curve) and $\sigma = 0.05$ (gray curve) as a function of $z$ setting $z_i = 20$. The left (right) panel corresponds to $k = 0.01868 \frac{T_\theta^2}{H_0^2}$ ($k = 0.01449 \frac{T_\theta^2}{H_0^2}$). A width $w = 10^{-2}$ of the initial Gaussian was assumed.

Figure 8: $\frac{\delta T}{T}$ for two values of $\sigma$: $z = 0$ (black curves: solid line contains the cases $w = 10^{-1}$ and $w = 10^{-4}$, there is practically no difference; dashed curve corresponds to $w = \infty$) and $z = 1$ (gray curves: same as for black curves). The initial redshift is $z_i = 20$, the left (right) panel corresponds to $k = 0.01868 \frac{T_\theta^2}{H_0^2}$ ($k = 0.01449 \frac{T_\theta^2}{H_0^2}$).
Figure 9: $\frac{\delta T}{T}$ for two values of $z$ as a function of $\sigma$: $z = 0$ (two lower curves: initial Gaussian distribution with width $w = 10^{-2}$ (black) and vanishing initial distribution (gray); $z = 1$ (two upper curves: initial Gaussian distribution with width $w = 10^{-2}$ (dark gray) and vanishing initial distribution (light gray)). The initial redshift is $z_i = 20$, the left (right) panel corresponds to $k = 0.01868 \frac{T_0^2}{H_0^2}$ ($k = 0.01449 \frac{T_0^2}{H_0^2}$).

distribution on changes in $z_i$. Fig. 10 shows $\frac{\delta T}{T}$ as a function of $\sigma$ at $z = 0$ and $z = 1$ for $z_i = 5$, $z_i = 20$, and $z_i = 40$. Obviously, there is hardly any dependence on $z_i$.

The plots in Fig. 8 indicate a discontinuity at $\sigma = 0$. As demonstrated by Fig. 11 this is an artefact of the finite lattice constant when solving Eq. (18).

5 Conclusion and Outlook

In the present article we have discussed a model for the temperature-temperature correlation in the cosmic microwave background (CMB) at large angular separation. The key idea is to relate the discrepancy between the observed and the CMB-inferred velocity of the Local Group to a dynamic component in the CMB dipole. The latter arises due to an anomaly in black-body spectra as it is predicted by deconfining $SU(2)$ thermodynamics when postulating that $SU(2)_{CMB}^{\text{today}} = U(1)_Y$, see [4, 5, 6, 7, 8]. As indicated by the WMAP data [2], dipole-subtracted correlations are unexpectedly suppressed at large angles, and the usual assumption of statistical isotropy seems to fail [24]. Our future work thus will focus on the computation of dipole-subtracted large-angle correlations based on the model as proposed here.

\[\text{This assumption is heavily contested in [24] based on a large-angle analysis of the WMAP three-year data.}\]
Figure 10: $\frac{\delta T}{T}$ as a function of $\sigma$ for a width of the initial Gaussian distribution of $w = 10^{-2}$ and for $z = 0$ (lower curves) and $z = 1$ (upper curves). The initial redshifts are chosen as $z_i = 5$, $z_i = 20$, and $z_i = 40$. The left (right) panel corresponds to $k = 0.01868 T_0^2/H_0^2$ ($k = 0.01449 T_0^2/H_0^2$).

Figure 11: $\frac{\delta T}{T}$ for $z = 0$ as a function of $\sigma \leq 0.2$ for varying lattice constants (black curve: grid with 1000 points spatial resolution; gray curve: grid with 100 points). The width of the initial Gaussian distribution is $w = 10^{-2}$, the initial redshift is $z_i = 20$, and the left (right) panel corresponds to $k = 0.01868 T_0^2/H_0^2$ ($k = 0.01449 T_0^2/H_0^2$).
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\frac{1}{2} \left. \frac{d}{dT} \delta \rho \right|_{T=T'_{\pm}} \quad [K^3]
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\[
\frac{1}{2} \left. \frac{d}{dT} \delta \rho \right|_{T=T'_{\pm}} \quad [K^3]
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