Effects of brane-flux transition on black holes in string theory

Oscar Loaiza-Brito\textsuperscript{a} and Kin-ya Oda\textsuperscript{b}

\textsuperscript{a} Centro de Investigaci\'on y de Estudios Avanzados del I.P.N., Unidad Monterrey
Cerro de las Mitras 2565, Col. Obispado, 64060, Monterrey, N.L., Mexico
E-mail: oloaiza@fis.cinvestav.mx

\textsuperscript{b} Theoretical Physics Laboratory, RIKEN, Saitama 351-0198, Japan
E-mail: odakin@riken.jp

Abstract

Extremal $\mathcal{N} = 2$ black holes in four dimensions can be described by an ensemble of D3-branes wrapped on internal supersymmetric three-cycles of Calabi-Yau threefolds on which type IIB superstring theory is compactified. We construct a similar configuration with a NS-NS flux turned on. In this case the D3-brane charge is not conserved since they are classified by torsion classes in twisted K-theory. As a consequence, the D3-branes disappear and the black hole suffers a topological transition, which leaves as a remnant a localized RR three-form flux in the uncompactified four-dimensional space-time.
1 Introduction

Incorporation of extra three-form fluxes in string theory compactification has lead to exciting possibilities such as the stabilization of moduli and a possible explanation for the hierarchy problem, which provide new ways to construct more realistic phenomenological models (for a review see [1] and references therein). Besides the desirable consequences on the effective supergravity, the fluxes impose a stringent non-trivial topological constraint through the Freed-Witten anomaly [2].

This anomaly must be considered in any situation where we have a non-trivial NS-NS three-form flux. Essentially it forbids us to wrap a D-brane on a cycle supporting a cohomologically non-trivial NS-NS flux in type II theories. If one does have such a D-brane, the way to cancel the anomaly is to add magnetic sources for the gauge fields induced by the presence of the flux. Such sources are provided by branes terminating at the anomalous brane, which is especially called instantonic brane when localized in time. The appearance of instantonic branes carrying such an anomaly and its consequent cancellation by the addition of extra branes was firstly described in [3].

In the presence of non-trivial NS-NS flux $H_3$, the instantonic brane triggers a transformation between D-branes and a coupling between NS-NS and RR fluxes. What happens is that $N$ Dp-branes decay into the vacuum by encountering an instantonic D(p+2)-brane which supports $N$ units of NS-NS flux $H_3$. The Dp-brane must be of codimension three in the Euclidean worldvolume of the instantonic brane. The charge of the disappeared Dp-branes is, after the transition, carried by the coupling between the NS-NS flux $H_3$ and the magnetic RR flux $F_{6-p}$ related to the instantonic brane, such that the final flux configuration still carries the same quantum numbers as the disappeared Dp-branes.

Roughly speaking, this topological process is a physical interpretation of the connection between integral cohomology and twisted K-theory [4]. In this connection, some non-trivial cohomology classes are obstructed to be lifted to twisted K-theory. The obstruction follows precisely from the presence of the NS-NS flux $H_3$. Hence, branes wrapped on cycles Poincaré dual to the non-lifted forms are not BPS objects classified by K-theory. The Dp-branes which disappear by encountering the instantonic brane belong to $N$ torsion classes of K-theory. Every configuration of branes wrapped on compact and non-trivial cycles in the presence of a NS-NS flux $H_3$ is potentially anomalous. That implies that what seems to be a stable bunch of Dp-branes can nevertheless be unstable to decay into vacuum by interacting with an instantonic brane.

D-branes wrapped on compact cycles have also been used in literature to describe, in the effective theory, extremal $\mathcal{N} = 2$ black holes in four dimensions [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. They are constructed by wrapping D3-branes on three-cycles of a Calabi-Yau threefold into which type IIB theory has been compactified [5, 7]. All quantum numbers related to this BPS state, as well as the entropy, can be computed in terms of the internal symplectic geometry of the CY moduli space. Recently, there also appeared several considerations on black holes in the presence of fluxes, for instance, the attractor mechanism in [17, 18] and the possible influence of the presence of black holes on the moduli stabilization in [19].

A straightforward question involves the stability of the black hole under the presence of a suitable NS-NS flux $H_3$ in the CY manifold. This is the issue we want to address in the present paper. We construct a configuration where one has a black hole consisting of D-branes, with a NS-NS flux $H_3$ being turned on. We show, under certain assumptions, that the D3-branes-made black hole disappears and turns into a configuration of a remnant RR
three-form flux \( F_3 \) which lives in the uncompactified four spacetime dimensions.

This paper is organized as follows: In section 2 we review the construction of the extremal four-dimensional black hole by wrapping D3-branes on internal cycles. In section 3, we construct a configuration where an extra internal NS-NS flux \( H_3 \) is turned on and study the black hole properties in it. The black hole mass and entropy are estimated. In section 4, we study the transition of D3-branes into R-R fluxes \( F_3 \) due to the appearance of instantonic branes. In section 5, this process is interpreted as a topological transformation acting on the black hole. We estimate the entropy of the fluxes \( F_3 \) into which the D3-branes has been transformed. Finally some comments and open questions are addressed in the last section. In particular a black hole configuration with the remnant three-form flux \( F_3 \) does not exist as a solution for the Einstein-Maxwell equations in four dimensions [20]. We show a possible interpretation that the ensemble of states consisting of the remnant flux \( F_3 \) could still be regarded as a black hole. In the Appendix we briefly review the connection between cohomology and K-theory by the so-called Atiyah-Hirzebruch Spectral Sequence.

2 Extremal black hole from wrapped D3-branes

Consider a compactification of type IIB superstring theory on a Calabi-Yau threefold \( Y \). An extremal charged black hole of the effective \( \mathcal{N} = 2 \) supergravity corresponds to D3-branes wrapped on internal special Lagrangian cycles of \( Y \) from the perspective of the low energy effective action [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. By choosing a symplectic basis for the three-cycles in \( Y \) as \( \{ A_I, B_I \} \) with \( I \) running for \( 0, ..., h^{2,1}(Y) \), let us write the cycle on which \( N \) dyonic D3-branes are wrapped as

\[
\gamma = \sum_I N(e^I A_I - m^I B_I),
\]

(1)

where \( e^I \) and \( m^I \) are the number of times with which a single D3-brane is wrapped on the cycles \( B^I \) and \( A_I \), that is, the electric and magnetic charges for a single D3-brane, respectively. These cycles satisfy \( A_I \cap A_J = B^I \cap B^J = 0 \) and \( A_I \cap B^J = -B^J \cap A_I = \delta^I_J \). By using Poincaré duality, a symplectic basis for 3-forms in \( H^3(Y; \mathbb{Z}) \) is defined as \( \{ \alpha_I, \beta^I \} \) where

\[
\text{PD}(A_I) = \beta^I, \quad \text{PD}(B^I) = \alpha_I,
\]

such that

\[
\int_{A_J} \alpha_I = -\int_{B^I} \beta^J = \delta^I_J.
\]

(3)

Now the self-dual five-form \( G_5 \), from which the electric D3-brane charge is computed, fulfills the following equation of motion [21]

\[
d \ast G_5 = dG_5 = -\mu_3 \rho_5^2 \text{PD}(W_4) = \ast J_4
\]

(4)

and can be decomposed as [7]

\[
G_5 = G_3 \wedge \omega_2,
\]

(5)
where \( \omega_2 \) is the volume unit two-form in four dimensions. The wrapping numbers \((e^I, m^I)\) turn to be as well the electric and magnetic charges for D3-branes. For the case one has \( N \) D3-branes, charge quantization imposes

\[
N m^I = \int_{A_I \times S^2} G_5 = \int_{A_I} G_3, \quad N e^I = \int_{B^I \times S^2} G_5 = \int_{B^I} G_3, \quad (6)
\]

from which

\[
G_3 = N (m^I \alpha^I - e^I \beta^I). \quad (7)
\]

The three-form \( G_3 \) is supported on the 3-cycle \( \Pi_3 \), which reads in terms of the symplectic basis as

\[
\Pi_3 = \sum_I \left( e^I A_I + m^I B^I \right). \quad (8)
\]

Note that

\[
\gamma \cap \Pi_3 = 2 m^I e^I \equiv 1, \quad (9)
\]

according to the Dirac quantization condition and that the cycles are transversal to each other in \( Y \). Therefore, the charge for \( N \) D3-branes is

\[
Q_{D3} = \int_{\Pi_3 \times S^2} G_5 = \int_{\Pi_3} G_3 = N. \quad (10)
\]

The BPS mass of the extremal black hole is computed from the superpotential

\[
\mathcal{W} = \int_{Y \times S^2} G_5 \wedge \Omega = \int_Y G_3 \wedge \Omega, \quad (11)
\]

where \( \Omega \) is the holomorphic three-form in \( Y \) and it reads

\[
M_{\text{BPS}}^2 = e^K |\mathcal{W}|^2, \quad (12)
\]

where the Kähler potential \( K \) is given by

\[
K = -\log i \int_Y \Omega \wedge \overline{\Omega}. \quad (13)
\]

It is convenient to rewrite the above potentials in term of the phases

\[
X^I = \int_{A_I} \Omega \quad \text{and} \quad F_I = \int_{B^I} \Omega, \quad (14)
\]

from which \( \Omega = X^I \alpha^I - F_I \beta^I \). Hence the superpotential reads

\[
\mathcal{W}(X) = N (e^I X^I - m^I F_I) \quad (15)
\]

and the Kähler potential

\[
e^{-K(X, \overline{X})} = 2 (\text{Im} \varphi_{IJ}) X^I \overline{X}^J, \quad (16)
\]
where \( \tau_{IJ} = \partial F_J / \partial X^I \). Now the BPS mass is written as

\[
M_{\text{BPS}}^2 = \frac{N^2}{2 \left( \text{Im} \tau_{IJ} \right) X^I X^J} \left| e_I X^I + m^I F_I \right|^2. 
\]  
(17)

Following the proposal by Ferrara and Kallosh [11], the entropy of the black hole can be computed by extremizing the action

\[
S = -\frac{\pi}{4} \left[ e^{-K(X,\bar{X})} + 2i\mathcal{W}(X) - 2i\bar{\mathcal{W}}(\bar{X}) \right] 
\]  
(18)

with respect to \( X_I \). The extremization condition for \( S \) implies that (following the notation of [7]) at the minimum,

\[
X^I_{e,m} = \left( \frac{-iN}{\text{Im} \tau} \right)^{IJ} (e_J - m^K \tau_{JK}),
\]  
(19)

for which the entropy is written as

\[
S = \frac{\pi}{2} \text{Im} \left[ N^2 \left( \frac{1}{\text{Im} \tau} \right)^{IK} \left( \frac{1}{\text{Im} \tau} \right)^{JL} (e_K - m^P \tau_{PK}) (e_L - m^Q \tau_{QL}) \tau_{IJ} \right] = N^2 S_{\text{unit}}. 
\]  
(20)

Note that the entropy is proportional to \( N^2 \).

There are some comments to be added: The presence of D3-branes in the internal manifold implies the existence of the self-dual five-form \( G_5 \) which can be decomposed, as we have seen above, into an internal three-form \( G_3 \) and the unit volume form \( \omega_2 \) in four dimensions. In particular, the flux \( G_3 \) generates a superpotential \( \mathcal{W} \) which depends on the complex structure moduli through \( \Omega \). Supersymmetry is preserved in the effective theory if \( D_i \mathcal{W} = 0 \). This condition fixes the complex structure moduli and the flux \( G_3 \) to be a real \((3,0) + (0,3)\)-form.

In other words, the complex structure moduli is fixed at the black hole horizon by the attractor mechanism. Since the entropy is computed by extremizing the superpotential (which in turn extremizes the BPS mass), its value correspond to a point in the moduli space in which the complex structure is fixed.

Effectively the flux \( G_5 \) gives rise to an scalar potential \( V_{\text{BH}} \) which is localized [19]. This follows from the fact that upon dimensional reduction, the two-form \( \omega \) has a divergence in the four-dimensional spacetime. The scalar potential \( V_{\text{BH}} \) can be computed by two methods. The first one involves the computation of \( V_{\text{BH}} \) through the superpotential \( \mathcal{W} \),

\[
V_{\text{BH}} = e^K \left( D_i \mathcal{W} D_j \bar{\mathcal{W}} K^{ij} + |\mathcal{W}|^2 \right).
\]  
(21)

The second one is realized upon dimensional reduction from the ten-dimensional IIB supergravity action with a three-form \( G_3 \) turned on. This four-dimensional scalar potential corresponds to the positive definite scalar potential in \( \mathcal{N} = 2 \) gauged supergravity (see for instance [22]). For supersymmetry to be preserved the scalar potential \( V_{\text{BH}} \) has to vanish, leading to the condition that \( e_I = (\text{Re} \mathcal{M})_{IJ} m^J \). We shall see some details of these facts in the next section. Here however we want to point out that SUSY-preserving conditions are not the same for both cases. This would mean that the latter method is merely classical, as was suggested in [6]. A more accurate calculation should invoke the formalism of topological string theory [7] which is beyond the scope of this paper.
3 Turning on extra NS-NS flux

Let us turn on a NS-NS flux $H_3$ in the internal manifold $Y$. An appropriate choice for the flux is (for reasons we shall see) to make it proportional to $G_3$. Our selection is then

$$H_3 = \sum_I M (m^I \alpha_I - e^I \beta^I),$$

for which the Dirac quantization reads

$$\int_{\Pi_3} H_3 = M.$$  \hspace{1cm} (23)

This NS-NS flux also induces the appearence of a scalar potential $V_{\text{eff}}$ in four-dimensions. There are some crucial differences between $V_{\text{eff}}$ and $V_{\text{BH}}$ which we have commented above. Namely, such a potential can be obtained either from the superpotential

$$W_{\text{eff}} = \int H_3 \wedge \Omega,$$

or from the dimensional reduction from the ten-dimensional type IIB supergravity. As we have said, there are differences between them due to quantum effects. Here we stress that the same condition for the complex structure moduli to preserve supersymmetry is kept not only under the presence of D3-branes but also the NS-NS flux. Since we are studying the effective theory, we choose to describe the scalar potentials in the context of dimensional reduction.

Then, when we turn on the flux $H_3$, the effective theory in four dimensions does not correspond to the usual $\mathcal{N} = 2$ supergravity but to the gauged $\mathcal{N} = 2$ supergravity (see for instance \cite{23}). The presence of the NS-NS flux $H_3$ in the internal manifold induces a change in the effective action. The resulting action of the gauged $\mathcal{N} = 2$ supergravity in four dimensions is given in \cite{22}, where the existence of the $\mathcal{N} = 2$ supersymmetry is maintained by the presence of a positive definite scalar potential.

The effective potential $V_{\text{eff}}$ can be deduced by dimensional reduction from 10-dimensional type IIB supergravity. The bosonic part of the 10-dimensional SUGRA action is given by

$$S_{\text{IIB}} = \int e^{-2\phi} \left( -\frac{1}{2} R * 1 + 2d\phi \wedge \ast d\phi - \frac{1}{4} H_3 \wedge \ast H_3 \right) - \frac{1}{2} \int \left( F_1 \wedge \ast F_1 + \bar{F}_3 \wedge \ast \bar{F}_3 + \frac{1}{2} G_5 \wedge \ast G_5 \right) - \frac{1}{2} \int C_4 \wedge H_3 \wedge F_3,$$ \hspace{1cm} (25)

where $H_3 = dB_2$, $\bar{F}_3 = F_3 - C_0 H_3$, and $G_5 = F_5 - \frac{1}{4} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$, with $B_2$ being the NS-NS two-form potential, $F_3 = dC_2$ and $F_5 = dC_4$ the RR field strengths, and $C_0$ and $\phi$ the RR and NS-NS scalars, respectively. From this action, the equation of motion for the field $\bar{F}_3$ is given by

$$d\bar{F}_3 = dF_3 - F_1 \wedge H_3.$$ \hspace{1cm} (26)

When compactifying on $Y$, the total amount of D3-brane charge in $Y$ must be zero, namely $d\bar{F}_3|_Y = 0$ and hence

$$dF_3 = F_1 \wedge H_3.$$ \hspace{1cm} (27)
This result strongly suggests that a current of D3-branes (wrapped in internal 3-cycles of $Y$) are being absorbed by (or emanate from) a D5-brane localized in time that supports a non-trivial NS-NS flux. We shall use this fact later on.

At the moment we are interested in deducing the effective 4-dimensional scalar potential under the presence of the NS-NS flux. From the ten-dimensional action the contribution of the non-trivial flux (22) is

$$S_{4D} = -\frac{1}{2} \int_Y \left( C_0^2 + \frac{e^{-2\phi}}{2} \right) (H_3 \wedge *H_3) = -\int_Y V_{\text{eff}},$$

(28)

from which one gets that

$$V_{\text{eff}} = -\frac{1}{2} \sum_{IJKL} \left( C_0^2 + \frac{e^{-2\phi}}{2} \right) (e_I - \mathcal{M}_{IK}m^K) \left( (\text{Im} \mathcal{M})^{-1} \right)^{IJ} (e_J - \mathcal{M}_{JL}m^L).$$

(29)

The negative definite matrix $\mathcal{M}$ is defined by

$$\left( \int_Y \alpha_I \wedge *\alpha_J = - \left( (\text{Im} \mathcal{M}) + (\text{Re} \mathcal{M})(\text{Im} \mathcal{M})^{-1}(\text{Re} \mathcal{M}) \right)_{IJ} \right),$$

$$\left( \int_Y \alpha_I \wedge *\beta^J = - \left( (\text{Re} \mathcal{M})(\text{Im} \mathcal{M})^{-1} \right)^{J}_I \right),$$

$$\left( \int_Y \beta^I \wedge *\beta^J = - \left( (\text{Im} \mathcal{M})^{-1} \right)^{IJ} \right).$$

(30)

The positive definite scalar potential $V_{\text{eff}}$ is a direct consequence of the presence of the NS-NS flux in the internal manifold, and its existence makes the effective theory to be $\mathcal{N} = 2$ supersymmetric. Of course, the supersymmetries are preserved only if the minima for $V_{\text{eff}}$ is zero. This can be obtained either by turning off the NS-NS flux ($m^I = e_I = 0$) or by setting $e_I = (\text{Re} \mathcal{M})_{IJ}m^J$.

These conditions are exactly the same for the scalar potential $V_{\text{BH}}$ therefore it is allowed to put both D3-branes and NS-NS fluxes in the consistent framework where the complex structure is fixed at the same point in the correspondent moduli space.

### 3.1 Black hole plus NS-NS flux

We are now interested in the study of a configuration of D3-branes wrapped on $\gamma$ in the presence of the flux $H_3$. We wrap $H_3$ on a cycle in $Y$ but put it uniformly distributed in the uncompactified four spacetime dimensions. Effectively this would correspond to putting a 4D black hole in a background given by the scalar potential $V_{\text{eff}}$. The total effective 4D scalar potential (black hole plus flux) would be given as in (19) by

$$S = -\frac{1}{2\kappa_{10}^2} \int d(\text{Vol}_4) \left( \frac{1}{r^4} V_{\text{BH}} + V_{\text{eff}} \right).$$

(31)

In the previous section we have seen that by wrapping D3-branes on internal three-cycles, we get an extremal $\mathcal{N} = 2$ black hole in the four-dimensional effective theory when NS-NS flux is turned off. As is mentioned above, when one turns it on, the effective theory after compactification becomes a $\mathcal{N} = 2$ gauged supergravity (22). In order to compute black hole
properties, we assume that the BPS properties of the Black hole are not altered by the presence of the scalar potential $V_{\text{eff}}$ in this paper. Let us stress the following points implied by our assumption (see also sections 5 and 6 for related discussions to this point):

- In general, having an effective theory preserving $\mathcal{N} = 2$ supergravity does not mean that we have an $\mathcal{N} = 2$ supersymmetric vacuum solution that keeps the BPS condition. To show the latter, further study is required which we leave for future work. In this paper we only point out the following. It is well known that the presence of fluxes in a Calabi-Yau threefold can break half of the supersymmetries even though there is not tadpole contribution. For an $SU(3)$ structure manifold, this happens when one relates the two four-dimensional spinors which appear in the gravitino and dilatino variations. Such conditions have been extensively studied in the past few years (for a detailed exposition, see for instance [22]). In the present paper we are not considering a relation between those two spinors, therefore we expect that the full $\mathcal{N} = 2$ supersymmetry will be preserved even when we turn on the NS-NS fluxes. (Of course there still remains a possibility that the equations of motion do align the two spinors to further break supersymmetry, which we assume is not the case here.)

- In our configuration there is not tadpole contribution from extra flux and hence no back-reaction. This is due to the fact that we have not turned on any RR fluxes transversal to the NS-NS ones. If we had allowed an extra flux contribution to the D3-brane charge from the combination of RR and NS-NS fluxes, the existence of orientifold three-planes with negative tension and charge would be required in order to cancel the tadpole in the internal manifold. However the presence of such orientifolds would spoil the $\mathcal{N} = 2$ supersymmetry in the background by projecting out the graviphotons in the vector multiplet which give rise to the charge of the black hole.

From our assumption, it follows that the black hole properties are not altered by the NS-NS flux $H_3$ since the mass and charge associated to the black hole in four dimensions comes from the mass and charge for the internally wrapped D3-branes. The NS-NS flux $H_3$ does not contribute to the D3-brane charge or mass. By construction, it merely changes the effective potential $V_{\text{eff}}$ uniformly in four dimension. Specifically, since

$$\int A_I H_3 = M m^I$$

is not a two-form in four dimensions, a four dimensional observer does not measure any divergence in the electromagnetic field (the scalar potential $V_{\text{eff}}$ is not localized as $V_{\text{BH}}$). Also, the BPS mass is given in terms of the superpotential $W$ which is null for the NS-NS flux. Note that the flux $H_3$ is supported on $\Pi_3$, which is transversal to $\gamma$ where the D3-branes are wrapped on.

On the other hand, we expect to have a change in the entropy of the whole system, namely the black hole plus flux. Under our assumption, the total entropy of the system is given by

$$S_{\text{total}} = S_{\text{BH}} + S_{\text{NS-NS}},$$

where $S_{\text{NS-NS}}$ is the entropy related to the presence of the NS-NS flux. Note that the reversibility is assumed here (see below). We note again that the presence of the NS-NS flux does not trigger a back-reaction since it does not contribute to the D3-brane charge tadpole.
4 Flux-brane transition

The configuration of $N$ D3-branes wrapped on $\gamma$ and an extra NS-NS flux on $\Pi_3$ suffers from a
topological transition \[3\] \[24\] \[25\]. To see this, consider the D3-brane action in 10 dimensions:

$$S = -\frac{1}{2\kappa_0^2} \int G_5 \wedge *G_5 + \mu_3 \int C_4 \wedge \text{PD}(W_4) + \frac{1}{4\kappa_0^2} \int C_4 \wedge H_3 \wedge F_3,$$

where the last term is the Chern-Simons term from the ten-dimensional type IIB supergravity. Now the equation of motion for the self-dual $G_5$ is modified from Eq. (4) as \[21\]

$$d^*G_5 = dG_5 = -\kappa_0^2 \mu_3 \text{PD}(W_4) + H_3 \wedge F_3.$$  \(35\)

Note that the 3-form fluxes $H_3$ and $F_3$ are supported transversally to the cycle $\gamma$.

The coupling $H_3 \wedge F_3$ contributes to the D3-brane charge and this is usually used in flux compactifications. However if $dF_3 \neq 0$, the presence of a D5-brane is required and there are only two possible ten-dimensional configurations:

1. There is a normal D5-brane on which the D3-brane is attached. The intersection sub-
manifold is of codimension three on $W_6$. Nevertheless, since there are not five-cycles
in $Y$ for a D5-brane to be wrapped, this configuration must be discarded for our model.

2. There is an instantonic D5-brane localized in time (D5 wrapped on a six-dimensional
cycle on $Y$) upon which the D3-brane terminates. A D3-brane disappears for each unit
of NS-NS flux $H_3$ by the topological transition.

Let us study the second option. The equations of motion implies the non-conservation of the
D3-brane current

$$d*J_4 = H_3 \wedge dF_3,$$  \(36\)

where the D3-brane charge is computed through $*J_4$. The coupling $H_3 \wedge F_3$ induces a D3-
brane charge but it does not represent a presence of D3-branes. What it is conserved is the
total charge from D3-branes and fluxes

$$d(*J_4 + F_3 \wedge H_3) = 0.$$  \(37\)

First in the absence of D5-branes, or equivalently for $F_3 = 0$, the only source for D3-brane
charge is the $N$ D3-branes themselves which is measured by $*J_4$. At a certain time, an
instantonic D5-brane appears supporting $M$ units of NS-NS flux. In principle, the brane is
anomalous in the sense of Freed-Witten \[2\]. However the incoming $N$ D3-branes cancel the
anomaly \[3\], by ending at the instantonic brane.

Since the variation of the current of D3-branes is

$$\int d*J_4 = M,$$  \(38\)

there remains only $N - M$ D3-branes after the instantonic disappearance. The $M$ D3-branes
have disappeared by contacting the instantonic brane. However, there is a remnant which

\[2\] We shall use $F_3$ to denote the RR flux which is the magnetic field strength for a D5-brane and $G_3$ as the
internal part of the self-dual five-form $G_5$.\]
corresponds to a magnetic field strength related to the instantonic D5-brane. This flux is a 3-form \( F_3 \) supported on a transversal cycle to the instantonic brane. The lost D3-brane charge is now carried by the coupling

\[
\int_{\Pi_3 \times \mathbb{R}^3} H_3 \wedge F_3 = M \cdot 1 = M, \tag{39}
\]

satisfying the fact that the total charge is conserved.

In [3], formal aspects of this transformation between branes and fluxes have been studied. The authors have shown that in the presence of NS-NS flux \( H_3 \) some branes disappear, as do the D3-branes in our case, due to the fact that the cycles on which the branes are wrapped belong to trivial classes in twisted K-theory. Roughly speaking, non-trivial integral (co)homology classes are not in one-to-one correspondence to twisted K-theory classes. Some of them are lifted to trivial classes while others are obstructed to belong to a K-theory group. The trivial classes correspond physically to the branes which can disappear while obstructed classes are interpreted as the instantonic branes. This connection between cohomology and K-theory is known as the Atiyah-Hirzebruch Spectral Sequence. Hence, the above \( M \) D3-branes belong to an \( M \) torsion K-theory class. This is the process upon which branes are transformed into fluxes (for a more formal description, see Appendix).

Let us be more specific and go back to our model with \( N \) D3-branes wrapped on \( \gamma \) with \( M \) units of \( H_3 \) supported on \( \Pi_3 \). The equation of motion for the flux \( G_5 \) can be reduced from the ten-dimensional spacetime to the compact CY manifold \( Y \) as

\[
d\overline{\tau} G_5 = \overline{\tau} J_4 + H_3 \wedge \overline{\tau} G_7, \tag{40}
\]

where \( \overline{\tau} \) is the Hodge dual acting on the compact manifold \( Y \) and time so that \( \overline{\tau} G_7 \) is a zero-form in \( Y \) and a three-form in the uncompactified three-dimensional space. The flux \( \overline{\tau} G_7 = \tilde{F}_0 \) corresponds to the magnetic field strength for one instantonic D5-brane wrapped on the six-cycle in \( Y \). Hence, before it appears, the flux satisfy \( \overline{\tau} G_7 = 0 \) and the D3-brane charge is measured by

\[
Q_{\text{before}} = \int \overline{\tau} J_4 = \int_{\Pi_3 \times S^2} G_5 = N, \tag{41}
\]

which is equal to the charge in Eq. \( \text{(10)} \). The variation of the current implies that after the appearance of the instantonic brane,

\[
Q_{\text{after}} = \int \overline{\tau} J_4 = \int_{\Pi_3 \times S^2} G_5 = N - M, \tag{42}
\]

from which we conclude that \( M \) D3-branes have disappeared and been transformed into the flux \( H_3 \wedge \tilde{F}_0 \). This topological transformation have an interesting effect for a four-dimensional observer, as we shall see.

\[\text{Footnote:} \text{Such a brane is effectively seen as an instanton in 4D. Although we shall say nothing about it, the appearance of the instantonic brane would be seen by an observer in 4D as the interaction of a black hole with an instanton.}\]
5 Transition of black hole

Let us consider $N$ D3-branes wrapped on $\gamma$ and $M$ units of $H_3$ in $\Pi_3$. A 4D observer measures the charge of the black hole (and an additional scalar field). Via the process described above, $M$ D3 branes disappear by encountering an instantonic D5-brane wrapped on the six-cycle in $Y$. After the disappearance of both D3 and D5-branes, there is a magnetic field strength remnant $F_3$ supported in the three-dimensional space. Hence, the mass of the black hole is reduced since now there are only $(N - M)$ D3-branes wrapped on $\gamma$ and it is given by

$$M_{BPS}^2 = \frac{(N - M)^2}{2(\Im \tau_{IJ}) X^I X^J} |e_I X^I + m I F_I|^2. \quad (43)$$

The situation is catastrophic for the black hole when there are the same number of D3-branes as NS-NS flux ($N = M$). In such a case, the D3-branes are completely transformed into flux. The BPS mass vanishes $M_{BPS}^2 = 0$ and since the RR field strength $G_5$ (for the D3-branes) has also disappeared, the charge cannot be measured by it. Hence, all the D3-brane charge is carried by the NS-NS and RR fluxes,

$$Q_{D3} = \int_{\Pi_3 \times \mathbb{R}^3} H_3 \wedge F_3 = N. \quad (44)$$

The whole process is depicted in Fig. 1.

The fact that the charge is now carried by the internal NS-NS flux $H_3$ and the new RR flux $F_3$ created by the instantonic D5-brane lets us fix another property of $F_3$. Before the topological transition, the D3-brane charge was finite, equal to $N$ in D-brane charge units.

---

\[^4\] The first case for a black hole to become massless was studied in [5] by shrinking to zero the cycle on which the D3-branes are supported.
After the transformation the brane charge is carried by the fluxes and henceforth must also be finite and equal to $N$. The $N$ units are given by the internal NS-NS flux, while

$$\int_{\mathbb{R}^3} F_3 = 1.$$  \hfill (45)

The RR flux $F_3$ can be written as

$$F_3 = F_{123}(\vec{x}) \, dx^1 \wedge dx^2 \wedge dx^3.$$  \hfill (46)

Since the integral of $F_{123}(\vec{x}) = F(\vec{x})$ over the three-dimensional space must be finite, $F(\vec{x}) \to 0$ for $\vec{x} \to \infty$ and the three-flux must be localized in the three-dimensional space. One simple solution would be something roughly of the form $F(\vec{x}) \sim e^{-Ax_i x^i}$, being localized around the center of the former black hole. This is schematically expressed in the upper configuration in Fig. 1.

So far, we have shown how the mass and charge are transformed by the appearance of the instantonic brane. However, as we said before, the entropy also suffers from changes by the presence of external fluxes. We aim at computing the entropy of the configuration after the transformation between branes and fluxes. By assuming that the process is reversible, it is possible to compute the entropy related to the flux $H_3 \wedge F_3$, which lives both in the uncompactified three-dimensional space and in the internal space $Y$. First of all, it is clear that for the remaining $(N - M)$ D3-branes, the black hole entropy reads,

$$S_{\text{BH after}} = (N - M)^2 S_{\text{unit}}.$$  \hfill (47)

Now the $M$ units of D3-brane charge, carried by the fluxes $H_3 \wedge F_3$, should contribute with an entropy $S_{\text{flux}}$ such that

$$S_{\text{flux}} = S_{\text{RR}} + S_{\text{NS-NS}},$$  \hfill (48)

since both fluxes are localized in different spaces. We have assumed that the process is reversible and hence the total entropy is conserved. Then it follows that the entropy of the black hole before the transition should be equal to the entropy of the lowered-mass black hole plus the RR flux

$$S_{\text{BH before}} = S_{\text{BH after}} + S_{\text{RR}}.$$  \hfill (49)

The entropy for the remnant RR flux is given by

$$S_{\text{RR}} = M(2N - M)S_{\text{unit}}.$$  \hfill (50)

Note that for the case $N = M$, all entropy is related now to the RR flux.

One may think that the flux $F_3$ could give rise to a static spherical solution for Einstein-Maxwell (EM) equations that again reproduces a black hole in the four-dimensional effective theory. If this was the case, the difference between the original black hole and the one produced by the flux would only be the type of associated degrees of freedom. However, as was shown in [20] a direct solution is not possible, namely, there cannot be any spherically symmetric charged black hole solution for the EM equations under the presence of the three-form flux $F_3$. In this sense, a three-form flux cannot give rise to a black hole.

If our assumption that the process being reversible should hold, the final flux configuration must somehow describe the same black hole. The following points can be made to support this argument.

\footnote{In [20] a wormhole solution is studied with the same ingredients.}
• Even when the BPS-condition does not hold with non-zero NS-NS flux, the initial D3-branes would still represent a black hole due to the Thorne’s Hoop conjecture. Also the same argument seems to conclude that the final bunch of RR flux in 4D becomes a black hole.

• Regardless of the assumptions, it would be natural that the equivalence of brane and flux holds in general not only for the RR sector but also for the NS-NS sector including gravity.

• The created RR flux is also localized in the three-dimensional space.

• If the three-form remnant flux would have been a NS-NS flux, which directly involves gravity, the conclusion that such a flux describes a black hole would be more naturally expected. A transition between D3-branes and NS-NS fluxes can occur for instantonic NS5-branes in the presence of non-trivial units of RR flux. This process is actually an S-dual picture of the flux-brane transition of our consideration, which has been sucessfully tested for particular cases (see [24] and references therein).

• It might be that the resultant ensemble of states consisting of the flux is still a black hole even though there cannot be a direct black hole solution with flux [27] (see below).

6 Conclusion and Discussions

In this paper we have shown that, under the presence of the non-trivial NS-NS three-form flux \( H_3 \) in the internal space, the four dimensional black hole consisting of D3-branes disappears via the topological process that transforms branes into fluxes. A NS-NS flux induces the Freed-Witten anomaly on an instantonic D5-brane which must be canceled by a D3-brane ending on it. Applying this fact to the system of the D3-branes wrapped on an internal Calabi-Yau cycle under the presence of extra NS-NS flux \( H_3 \), we have shown that the black hole described in four dimensions will suffer from the same topological transition.

Under the four-dimensional perspective, the black hole would disappear leaving as a remnant a RR three-form flux \( F_3 \) localized in the uncompactified four spacetime dimensions. In order to compute the black hole quantities, we have assumed that the black hole represents a BPS state even in the presence of the NS-NS flux \( H_3 \), namely, that there is not interplay between the effective scalar potential induced by the presence of the fluxes and the extremal black hole except for the transition driven by the instantonic brane. In other words, our assumption is that the BPS states in the usual \( \mathcal{N} = 2 \) four-dimensional supergravity are preserved in the \( \mathcal{N} = 2 \) gauged supergravity. The charge (and mass) carried by the black hole before the topological transition driven by the instantonic D5-brane is afterwards carried by the coupling between the NS-NS flux \( H_3 \) and another RR three-form flux \( F_3 \) emanating from the D5-brane.

If our assumption of the BPS-ness of the initial black hole is valid, it implies that even the extremal black hole suffers from the transition that exchanges topologically different configurations which carry the same charge and mass. Of course there remains a possibility that our assumption of the BPS-ness is invalid and the NS-NS flux \( H_3 \) does break supersymmetry.

Regardless of the extremality of the initial black hole, the solution to the Einstein-Maxwell equations in the presence of the remnant RR flux \( F_3 \) cannot be a spherically symmetric charged black hole solution, implying that at the level of gravity the initial black hole does suffer a
change. However, this does not necessarily mean that the final flux configurations cannot make an (extremal) black hole. The wormhole solution is found with which the integration of $F_3$ over a three-cycle can be finite and equal to the charge of the disappeared black hole. Recently, it has been conjectured that an ensemble of the different solutions without horizon corresponds to a black hole \[27\]. According to it, the horizon radius is nothing but the size of the region where the solutions in the ensemble differs each other. This might be the case for the final configurations.

There are many open questions. First of all, the already mentioned topological transformation was explored some years ago in the context of the conifold approximation \[28\]. In this scheme, the moduli space describing the geometry of the conifold on which a D3-brane is wrapped changes after a topological transformation under which the three-cycle of the conifold shrinks to zero and blow up into a two-cycle. Particularly, the complex structure moduli were exchanged into the Kähler moduli. It would be very interesting to go further and study what happens with the Calabi-Yau moduli under the presence of an instantonic brane.

In this paper, we have treated the purely topological aspects of the process and therefore no dynamics is analyzed. Time evolution of the S(pacelike)-brane has already been considered for simpler setup \[29, 30\]. It is of importance to study the dynamics of this transition from black hole into the RR flux.

Another interesting question arises from the computation of the entropy. As was mentioned at the beginning of this paper, the black hole entropy can be computed by extremizing the central charge (which turns out to be also an extremization of the superpotential). This is now a standard procedure to compute it, since we are dealing basically with open strings (with D3-branes present). Now, since we are addressing only a topological transition, we expect to have a configuration keeping the same quantum numbers and more importantly the same entropy. However, due to the fact that for $N = M$ we do not have D3-branes anymore, there are not degrees of freedom related to open strings. The entropy should be computed from a closed string perspective. This strongly suggests a way to compute entropy by other methods as was conjectured in \[14\], where the entropy is gathered from the topological partition function $|Z_{top}|^2$. It would be very interesting to have a detailed description of the entropy for a NS-NS field in terms of the topological partition function, which would provide another support for this conjecture.

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Appendix

A The Atiyah-Hirzebruch Spectral Sequence

Here we briefly review the connection between cohomology and \( \text{K-theory} \) in terms of the Atiyah-Hirzebruch Spectral Sequence (AHSS) (for further details, see [31, 3, 24]). Essentially AHSS is an algorithm which relates integral cohomology with twisted \( \text{K-theory} \).

The relation involves the construction of twisted \( \text{K-theory} \) classes from integral cohomology classes by a finite number of steps. In general, an integral cohomology class \( [\omega_p] \in H^p(X; \mathbb{Z}) \) does not come from a twisted \( \text{K-theory} \) class \( [x] \in K(X) \). Hence, the algorithm begins with cohomology. At this step, cohomology is the first approximation to twisted \( \text{K-theory} \) and it is denoted by \( E_1(X) \). At the \( p \)-th step, the approximate group is denoted by \( E_p(X) \), where

\[
K(X) \sim E^p_m = \frac{\ker d_m|_{E^{p-m}_m}}{\text{Im} d_m|_{E^{p-m}_m}},
\]

such that, the first approximation is

\[
E_1(X) = \bigoplus_p H^p(X; \mathbb{Z}).
\]

The second step is to consider forms which are closed under the differential map

\[
d^3 \equiv \wedge H_3,
\]

with \( d^3: H^p(X; \mathbb{Z}) \to H^{p+3}(X; \mathbb{Z}) \), but discarding those which are exact. This defines the group

\[
E^p_3(X) = \frac{\ker d_3|_{H^p}}{\text{Im} d_3|_{H^{p-3}}}.
\]

After this step, only those forms which are closed will survive and represent stable D-branes in string theory provided the NS-NS field is identified with \( H_3 \). Those forms which are not closed represent the instantonic branes that we discuss in this paper. Finally, \( (p+3) \)-forms which belong to the trivial class satisfy

\[
d^3 \omega_p = \omega \wedge H_3 = \sigma_{p+3},
\]

and represent branes which can be unstable. Note that such forms belong to torsion classes in \( \text{K-theory} \) according to the integral class of \( H_3 \) (upon the isomorphism with the field).

One can go further defining several groups in order to get a closer approximation to \( \text{K-theory} \). However, the algorithm ends after a finite number of steps. At the end, one gets a group which is called “associated graded group” \( \text{Gr}(X) \) given by,

\[
\text{Gr}(K_H(X)) = \bigoplus_p E_p(X).
\]

\(^6\)In this section, “Im” stands for an image rather than an imaginary part.
This group is in some cases the K-theory group\textsuperscript{7} However, in other cases it is necessary to solve an extension problem, since

\[ \text{Gr}(K_H(X)) = \oplus_p K_{H,p}(X)/K_{H,p+1}(X). \]  

(57)

In cases as in this paper, the second step is the final approximation. Hence, by dropping all forms closed under \( d^3 \) one gets a K-theory class. All branes which belong to a non-trivial class in this “cohomology” of \( d^3 \) are stable objects in string theory.

Applying the AHSS to zero and three-forms we can get a particular case in type IIB superstring theory compactified on a Calabi-Yau threefold \( Y \). For instance, for a three-form \( \sigma_3 \) we get

\[ d^3(\sigma_3) = \sigma_3 \wedge H_3. \]  

(58)

Any brane wrapped on the Poincaré dual three-cycle to \( \sigma_3 \) is free of Freed-Witten anomalies provided \( d^3 \sigma_3 = 0 \). (In principle it is possible to have a model in which \( d^3 \sigma_3 \neq 0 \), but here we are interested in D3-branes wrapped around internal three-cycles and are anomaly free.)

However, it is possible that a closed form \( \sigma_3 \) is indeed exact under the action of \( d^3 \). In such a case, \( \sigma_3 \) would belong to the trivial class of \( E_3^3(Y) \). This means that any brane wrapped on the dual three-cycle to \( \sigma_3 \) would be unstable. This is precisely what occurs in the Calabi-Yau compactification with NS-NS fluxes. Consider a zero-form \( \omega_0 \in H_0(Y; \mathbb{Z}) \) which is dual to a six-cycle. By applying the map \( d^3 \), one gets,

\[ d^3 \omega_0 = \omega_0 \wedge H_3 \in H^3(Y; \mathbb{Z}). \]  

(59)

A brane wrapped on the six-cycle (in type IIB) should be anomalous since the Freed-Witten condition is not fulfilled, i.e., \( \omega_0 \) does not belong to \( E_3^3(Y) \). Therefore, an instantonic D5-brane wrapped on this cycle is by itself an anomalous brane. However, as was shown in \[ \text{[?]} \], this anomaly can be cancelled by D3-branes ending at this instantonic D5 brane. Such D3-branes are precisely represented by the form \( \omega_0 \wedge H_3 \). Hence, a D3-brane which is wrapped on a three-cycle in \( Y \) and is represented by \( \sigma_3 \) can nevertheless be unstable to decay into vacua if

\[ \sigma_3 = \omega_0 \wedge H_3. \]  

(60)

For the model studied in this paper, we have

\[ \sigma_3 = PD(-\gamma) = G_3. \]  

(61)

Hence we conclude that \( [\sigma_3] = [0] \in E_3^3(Y) \) which is physically interpreted as the fact that D3-branes disappeared (turned into flux) under the presence of non-trivial amount of NS-NS flux and an instantonic D5-brane. This is in accordance with the description in terms of D3-brane currents given in Section 4.

References


\textsuperscript{7}In the context of string theory, this happens in the absence of orientifold planes.


