Theoretical overview: towards understanding the quark-gluon plasma

Jean-Paul Blaizot
ECT*, Villa Tambosi,
strada delle Tabarelle, 286, I 38050 Villazzano (TN), Italy
E-mail: blaizot@ect.it

Abstract. I give a brief overview of recent theoretical developments concerning the high temperature phase of QCD, and the structure of hadronic wave functions at high energy.

1. From the “ideal gas” to the “perfect liquid”

The study of ultra-relativistic heavy ion collisions offers the possibility to address several fundamental questions, concerning for instance the state of matter at very high temperature and density, or the structure of the wave-function of a nucleus at asymptotically high energy. The reason why these questions refer to extreme situations is of course the fact that simplicity often emerges in asymptotic situations, allowing for a deeper theoretical understanding, that can be eventually extrapolated to the more complex, non asymptotic, situations.

The naive picture of the quark-gluon plasma belongs to such asymptotic idealizations: as a natural consequence of the QCD asymptotic freedom, one expects hadronic matter to turn at high temperature and density into a gas of quarks and gluons whose free motion is only weakly perturbed by their interactions. The beautiful RHIC data that have been collected over the last few years have somehow shaken our hope that such an idealized state of matter can be observed in nuclear collisions: the temperature reached is presumably not high enough, or is attained for too short a period of time to lead to observable consequences. Still, a consensus has been reached that some form of a quark-gluon plasma is produced in RHIC collisions (see [1] [2] [3] [4], and also [5] [6]). This has created a shift of paradigm: we are no longer focusing on the discovery of the quark-gluon plasma, but rather on studying its properties, that RHIC allows us to explore experimentally. Hence the title of this talk, where I have to emphasize, however, that our “understanding” is hampered by the fact that RHIC forces us to look in a regime where, unfortunately, theory is hard.

The impact of the RHIC discoveries on current theoretical investigations has been in fact even deeper, with most of the present discussions emphasizing the strongly coupled
character of the quark-gluon plasma. Three elements have actually conspired to this further shift in paradigm. First, as we have alluded to already, the RHIC data do not provide any evidence for ideal gas behavior. They are better interpreted by assuming that the produced matter behaves as a liquid with low viscosity, the “perfect liquid” \[7, 8\]. Second, perturbation theory is notoriously unable to describe the quark-gluon plasma unless the temperature is extremely high. Third, new techniques have emerged that allow calculations to be done in some strongly coupled gauge theories (that differ however in essential aspects from QCD). The strongly coupled plasma becomes then another idealized picture, of the kind referred to earlier, that could be used as a starting point for the description of the physical quark-gluon plasma.

When discussing the quark-gluon plasma in the context of heavy ion collisions, one has to face the question of how such a state of matter can be produced. This requires understanding the structure of the wave function of a nucleus at very high energy, and the detailed microscopic mechanisms by which its partonic degrees of freedom get liberated and subsequently interact to lead possibly to a thermalized system. These issues will be very briefly discussed in the last part of this talk. It is interesting to observe in this context the remarkable merging of scientific interests in the small x physics and the physics of ultra-relativistic heavy ions, two fields that until recently had little in common: in both domains one is looking into regimes of QCD where the parton densities are large and the non linearities of QCD play a major role.

2. Is the quark-gluon plasma weakly or strongly coupled?

This question does not have a straightforward answer. Indeed, as we shall see, in the quark-gluon plasma coexist seemingly perturbative features, and non perturbative ones. For instance lattice calculations, which provide the most reliable information on the properties of the quark-gluon plasma at high temperature, show that, at very high temperature, the thermodynamical functions go, albeit slowly, towards those of free massless particles (see for instance \[9\]), confirming the expected picture based on asymptotic freedom. At the same time these calculations point to important corrections to naive perturbation theory in all the relevant range of temperatures.

2.1. Breakdown of strict perturbation theory

Much effort has been put into calculating the successive orders of the perturbative expansion for the pressure \[10, 11, 12, 13\] and the series is known now up to order \(g^6 \ln g\)\[13\]. These calculations have revealed that perturbation theory makes sense only for very small values of the coupling constant, corresponding to extremely large values of \(T\). For not too small values of the coupling, the successive terms in the expansion oscillate wildly and the dependence of the results on the renormalization scale keeps increasing order after order (see e.g. \[14\]). Clearly, the corrections to the ideal quark gluon plasma cannot be calculated in strict perturbation theory.
This situation is to be contrasted with what happens at zero temperature, where perturbative calculations achieve a reasonable accuracy already at the GeV scale. The point is that the validity of a weak coupling expansion depends not only on the strength of the coupling, but also on the number of active degrees of freedom. At zero temperature, one deals most of the time with a very limited number of degrees of freedom (the colliding particles and the reaction products), while at finite temperatures, as we shall see shortly, the thermal fluctuations alter the infrared behavior in a profound way. There are of course situations at zero temperature where one needs to take into account many degrees of freedom through appropriate resummations; the small $x$ resummations provide an example. Similarly, in the study of cold dense matter, the BCS instability towards color-superconductivity provides another example of a weak coupling situation made non-perturbative by the presence of many degenerate degrees of freedom. Having recognized this, we shall see that a lot can be learned from weak coupling calculations, in particular how to capture the right physics that can allow for smooth extrapolations towards the strong coupling regime.

2.2. The role of thermal fluctuations

A simple characterization of the strength of the interactions in classical plasmas is provided by the dimensionless parameter $g^2 \equiv e^2 n^{1/3}/T$, which is essentially the ratio between the average potential energy per particle ($\sim e^2 n^{1/3}$) and the mean kinetic energy per particle ($\sim T$), with $T$ the temperature, $n$ the number density and $e$ the electric charge. The condition that the plasma be ideal, or weakly interacting, is then $T \gg e^2 n^{1/3}$, or simply $g \ll 1$. The Debye screening length is $\sim \sqrt{T/n e^2} \sim n^{-1/3}/g$ which, when $g$ is small, is large compared to the interparticle distance ($\sim n^{-1/3}$), a criterion for collective behaviour.

In ultrarelativistic plasmas, the temperature and the density are no longer independent control parameters since $n \sim T^3$, and in QCD, the parameter $g$ reduces to the gauge coupling (at a scale $\sim T$). Note that, just above $T_c$, $g$ is not small, but not huge either, $g \simeq 2$ (see e.g. [15]). However, as already mentioned, to decide whether the quark-gluon plasma is strongly or weakly coupled it is not enough to consider only the strength of the coupling: the effects of the interactions depend also on the magnitude of the relevant thermal fluctuations, which depends on their wavelengths. Predicting the effect of the interactions amounts to comparing the kinetic energy $\sim \langle (\partial A)^2 \rangle \sim k^2 \langle A^2 \rangle$ with the interaction energy $g^2 \langle A^4 \rangle \sim g^2 \langle A^2 \rangle^2$, or equivalently $k^2$ with $g^2 \langle A^2 \rangle$ ($A$ is the gauge field). At weak coupling a hierarchy of scales of thermal fluctuations emerges, each scale being associated with well identified physics (see e.g. [16] for a more detailed discussion). At one end we have the thermal fluctuations corresponding to the plasma particles $\langle A^2 \rangle_T \sim T^2$. These constitute the dominant contribution to the energy density at weak coupling. The plasma particles have typical momenta of the order of the temperature, and mostly kinetic energies, $k^2 \sim T^2 \gg g^2 T^2$ if $g$ is small. For them, perturbation theory works as well as at zero temperature. At the other end, we have long
wavelength ($\sim 1/g^2T$), unscreened, magnetic fluctuations which remain strongly coupled
$\langle A^2 \rangle g^2T \sim g^2T^2$, so that $k^2 \sim g^4T^2 \sim g^2 \langle A^2 \rangle g^2T$, however small the coupling, hence however high the temperature. These magnetic fluctuations prevent the perturbative calculation of the pressure at order $g^6$ and beyond.

Thus in non abelian ultra-relativistic plasmas, some degrees of freedom can be weakly coupled while others remain strongly coupled for any values of the coupling. The contribution of the magnetic fluctuations to the pressure is not known quantitatively, although there are indications from dimensional reduction (see below) that it is small. Then the main difficulty with thermal perturbation theory, as far as the calculation of the pressure is concerned, occurs already in scalar field theories [14]. It is not so much related to the fact that the coupling is not small enough (for the relevant temperatures the coupling is not huge), but rather to the interplay of degrees of freedom with various wavelengths, possibly involving collective modes. To deal with this aspect of the problem, weak coupling techniques are useful: they allow us to identify and perform the appropriate reorganizations and resummations of the perturbative expansion. Once this is done, the extrapolation to strong coupling is much smoother than what strict perturbation theory could lead us to expect.

2.3. Effective theories and resummations

A powerful technique to handle situations where modes at different scales couple is that of effective theories. Among those, one focuses on the modes carrying zero Matsubara frequencies. The construction of the effective theory for these modes is known as dimensional reduction [17, 18, 19]. The coefficients of the resulting effective lagrangian are usually determined perturbatively as a function of the gauge coupling $g$ by matching. Calculations based on this scheme have been pushed to high order [20], but the determination of the order $g^6$ contribution to the pressure depends on an as yet undetermined 4-loop matching coefficient. By adding a parameter to account for this uncalculated contribution, one can match the four-dimensional lattice results. The required value of the coefficient is not very large, suggesting that the contribution of the magnetic sector to the pressure is numerically not important at high temperature, as mentioned before.

Other ways to reorganize the perturbative expansion have been tried. One proposal, called “screened perturbation theory” [21, 22], has been generalized to the resummation of the full QCD hard thermal loops [23, 24]. A different approach, borrowed from early studies of another strongly interacting liquid, namely liquid helium, is based on an expression for the entropy density that can be obtained from a $\Phi$-derivable two-loop approximation [25, 26] (sometimes also called 2PI formalism). This approach has also the virtue of providing a clear physical picture: the dominant effect of the interactions is to turn the original degrees of freedom, quarks and gluons, into massive quasiparticles, with weak residual interactions. It was shown in Refs. [25, 26] that the lattice results for the entropy of the gluonic plasma [27, 28] were quite well reproduced for $T \geq 3T_c$. 

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The formalism used in this calculation of the entropy has been tested \cite{29} in the limit of a large number of quark flavors, which can be solved exactly. One then found that the 2PI approximation scheme allows for a smooth extrapolation that is accurate up to quite large values of the coupling constant.

The weak coupling approximation schemes can be fully justified only when the coupling is truly small, in which cases the various relevant degrees of freedom can be arranged in a clean hierarchy of scales \((T, gT, g^2T)\). The non-perturbative renormalization group \cite{30, 31, 32, 33, 34} sheds some light on what happens as the coupling grows. There is some analogy between the effective field theory approach and the non-perturbative renormalization group: in effective field theory one integrates out degrees of freedom above some cut-off; in the renormalization group this integration is done smoothly. In a way, the renormalization group builds up a continuous tower of effective theories that lie infinitesimally close to each other, and are related by a renormalization group flow equation. This picture is independent of the value of the coupling, so that the renormalization group provides a smooth extrapolation from the regime of weak coupling, characterized by a clean separation of scales, towards the strong coupling regime where all scales get mixed. In a recent study \cite{37}, it has been shown in the case of a scalar \(\phi^4\) theory with \(O(N)\) symmetry, that such a technique can provide a smooth extrapolation to strong coupling, which turns out to be similar to that of a simple 2PI approximation.

The physical picture that emerges from these various weak coupling calculations, with proper resummations, is that at high \(T\), the dominant degrees of freedom are quark and gluon quasiparticles. The main effect of the interactions is to affect the propagation of these quasiparticles that suffer otherwise very little residual interactions. This picture is consistent with lattice calculations for temperatures above \(3T_c\). The physics in the temperature range \(T_c \lesssim T \lesssim 3T_c\) remains poorly understood, with many unanswered questions, for instance: What are the degrees of freedom relevant for an effective theory? Are remnants of confinement important, such as those captured by effective theories for the Polyakov loop (see for instance \cite{38, 39} or, on a more phenomenological side, \cite{40})? What is the role, if any, of bound states \cite{41}? What is their fate as the temperature grows, do they survive at large temperature, as recent lattice data suggest \cite{42}? What are the charge carriers \cite{43, 44}? Etc.

2.4. The strong coupling regime from the AdS/CFT correspondence

New techniques for doing calculations in strongly coupled gauge theories have emerged recently. These techniques are based on a duality between some supersymmetric Yang-Mills theories and gravitation theories. The duality involves an interchange of the regimes of weak and strong coupling: weak coupling in the gravity theory corresponds to strong coupling in the gauge theory. This duality offers the possibility to study strongly coupled gauge theories by performing essentially classical calculations in their gravity duals. This possibility has been exploited in a number of recent publications (see
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the talk by Hong Liu at this conference). One prediction of such calculations concerns the entropy $S$ which behaves, in strong coupling, as

$$S/S_0 = \frac{3}{4} + \frac{45}{32} \zeta(3) \frac{1}{\lambda^{3/2}}.$$  \hspace{1cm} (1)

Here $\lambda \equiv 2g^2N_c$ and $S_0$ is the entropy of the non interacting system. Thus, in the limit of strong coupling, $\lambda \to \infty$, the entropy is bounded from below by the value $S/S_0 = 3/4$. The fact that this value is close to that obtained in lattice calculations at temperatures above $T_c$ has led to the suggestion that the quark-gluon plasma above $T_c$ could be in a strongly coupled regime analogous to the strong coupling regime of SYM theories. Comparing the two theories is, however, delicate. Note first that SYM theories, the most discussed of such theories, and the only ones that I consider in this short discussion, have symmetries that make them rather different from QCD: in particular the coupling constant does not run, and there is no phase transition. The running of the coupling constant is however an essential property of QCD; in particular, above the transition region it is accompanied by a breaking of conformal symmetry that is very well observed in lattice calculations [27, 46, 47]. So one cannot compare $\mathcal{N} = 4$ SYM theories with QCD in the temperature range where the QCD coupling is the largest (the minimal value of 3/4 is obviously not compatible with lattice data near $T_c$ where the entropy nearly vanishes, and the coupling is the strongest). Thus one should compare $\mathcal{N} = 4$ SYM theories with QCD only for temperatures large enough for conformal symmetry to be approximately satisfied, that is for $T \gtrsim 3T_c$. But in this temperature range, the entropy density is about half-way between its weak coupling value and its strong coupling value, so there is no compelling reason to favor at that point an interpretation based on strong coupling (see [48] for a recent discussion).

3. Hadronic wave-functions at high energy and the early stages of nucleus-nucleus collisions

I turn now to the second fundamental issue mentioned in the introduction, that related to the structure of the wave functions of nuclei at very high energies. This is a domain where significant progress has been achieved recently. I should emphasize that because of lack of space and time, I shall not be able to do justice to these exciting developments. I wish to refer to the talks by F. Gelis and M. Strickland at this conference for more information. As we shall see, much of the discussion relies on weak coupling arguments, although strong interactions are generated by classical color fields.

3.1. The physics of saturation and the color glass condensate

The degrees of freedom involved in the early stages of a nucleus-nucleus collision at sufficiently high energy are partons, mostly gluons, whose density grows as the energy increases (i.e., when $x$, their momentum fraction, decreases). This phenomenon has been well established at HERA [52]. One expects however that the growth of the gluon density should eventually “saturate” when non linear QCD effects start to play a role.
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The existence of such a saturation regime has been predicted long ago \[53, 54\], together with estimates for the typical transverse momenta where it could set in in heavy ion collisions \[55\]. But it is only during the last decade that equations providing a dynamical description of the saturated regime have been obtained \[56, 57\] and \[58, 59, 60, 61, 62\]. A remarkable feature which emerges from the solution of these equations is that the dense, saturated systems of partons that make hadronic wave functions at high energy have universal properties (see below), the same for all hadrons or nuclei.

The momentum scale \(Q_s\) that characterizes the onset of saturation is called the saturation momentum and is given by \(Q_s^2 \sim \alpha_s(Q_s^2) x G(x, Q_s^2) / \pi R^2 \[?\]. Partons in the wave function have different transverse momenta \(k_T\). Those with \(k_T > Q_s\) are in a dilute regime; those with \(k_T < Q_s\) are in the saturated regime. Note that at saturation, naive perturbation theory breaks down, even though \(\alpha_s(Q_s)\) may be small if \(Q_s\) is large: the saturation regime is a regime of weak coupling, but large density. In fact one can easily estimate the number of partons occupying a small disk of radius \(1/Q_s\) in the transverse plane. At saturation, this number is proportional to \(1/\alpha_s\), a large number is \(\alpha_s\) is small. In such conditions of large numbers of quanta, classical field approximations may become relevant to describe the nuclear wave-functions. This observation is at the basis of the McLerran-Venugopalan model \[63\]. The color glass formalism provides a more complete physical picture. It relies on the separation of the degrees of freedom into Lorentz contracted frozen color sources \(\rho\) flying along the light-cone, and low \(x\) partons which are described by classical gauge fields \(A^\mu(x)\) determined by solving the Yang-Mills equations with the source \(\rho\). An average over all acceptable configurations must be performed in order to calculate observables. In this average, the weight of a given configuration is a functional \(W_{x_0}[\rho]\) of the density \(\rho\) of color sources, which depends on the separation scale \(x_0\) between the modes which are treated as sources, and those which are treated as fields. As one lowers this separation scale, more and more modes are included among the frozen sources, and therefore the functional \(W_{x_0}\) evolves with \(x_0\) according to a renormalization group equation. This leads to the non linear evolution equations alluded to earlier, namely the Balitsky-Kovchegov equation \[56, 57\] and the JIMLWK equation \[58, 59, 60, 61, 62\].

The saturation momentum increases as the gluon density increases. This may come from an increase of the gluon structure function as \(x\) decreases \((Q_s^2 \sim x^{-0.3})\). This may also come from the additive contributions of several nucleons in a nucleus. In large nuclei, one expects \(x G_A(x, Q_s^2) \propto A\), and hence \(Q_s^2 \propto \alpha_s A^{1/3}\), where \(A\) is the number of nucleons in the nucleus. Thus, the saturation regime sets in earlier (i.e., at lower energy) in collisions involving large nuclei than in those involving protons. In fact, the parton densities in the central rapidity region of a Au-Au collision at RHIC are not too different from those measured in deep inelastic scattering at HERA. The study of \(dA\) collisions at RHIC, in the fragmentation region of the deuteron, gives access to a regime of smaller \(x\) values, where quantum evolution could be significant. Indeed, very exciting results have been obtained in this regime \[49\], which have been interpreted as evidence.
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3.2. Fluctuations

The non linear JIMWLK or BK evolution equations have solutions that exhibits universal asymptotic properties (when the evolution is carried to high rapidity) [64]. One remarkable property is that of geometrical scaling, observed at HERA [65]. It has been recognized recently that those asymptotic properties are those of a large universality class of non linear equations that appear in various areas of statistical physics, and describe typically reaction-diffusion processes [66].

This connection with problems in statistical mechanics has been deepened. In particular, the role of fluctuations in the dilute part of the partonic systems has been recognized [67]. Such fluctuations are ignored in the usual non linear evolution equations, which emphasize the saturation regime where the large densities validate a mean field approximation. New evolution equations that incorporate the physics of fluctuations have been constructed [68, 69]. A remarkable outcome is that the fluctuations in the dilute regime influence the whole approach to saturation. As a result, the saturation scale becomes a fluctuating quantity, which, in a given event, may reach different values for different impact parameters. This could have phenomenological consequences at the very high energies of the LHC (although perhaps no so much in A-A collisions, since the large size of the system will, a priori, damp then the possible fluctuations). A preliminary study of such possible phenomenological consequences is given in [70].

3.3. Initial conditions and the approach to thermalization

The knowledge of the initial parton distribution is obviously essential in order to determine the early stages of heavy ion collisions. Typically the partons which are set free during the collision are those which carry a transverse momentum of order $Q_s$, and they are freed on a time scale of order $1/Q_s$. In the “bottom-up” scenario [71], these hard gluons radiate a lot of soft gluons and thermalization proceeds through these soft gluonic modes that eventually collect all the energy initially stored in the hard gluons. Such a scenario has been revised recently [73, 74] in order to take into account the plasma instabilities generated by the anisotropy of the initial momentum distributions [72]. These instabilities show up in many numerical simulations of the initial stages of nucleus-nucleus collisions [75, 76, 77]. Whether they play an essential role in the thermalization, or lead perhaps to turbulent behaviour [78, 79, 80], or anomalous viscosity [81], are issues, among others, very much discussed at the moment.

References

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