Comparing P-stars with Observations

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Abstract. P-stars are compact stars made of up and down quarks in \( \beta \)-equilibrium with electrons in a chromomagnetic condensate. P-stars are able to account for compact stars as well as stars with radius comparable with canonical neutron stars. We compare p-stars with different available observations. Our results indicate that p-stars are able to reproduce in a natural manner several observations from isolated and binary pulsars. On the other hand, we argue that the standard model based on neutron stars is getting in troubles leading to the need for a drastic revision of the standard paradigm.

Key words. pulsars: general

1. Introduction

In few years since their discovery (Hewish et al. 1968), pulsars have been identified with rotating neutron stars, first predicted theoretically by Baade & Zwicky (1934a,b,c), endowed with a strong magnetic field (Pacini 1968; Gold 1968).

It is widely believed that there are no alternative models able to provide as satisfactory an explanation for the wide variety of pulsar phenomena as those built around the rotating neutron star model. Nevertheless, recently we have proposed (Cea 2004a,b) a new class of compact stars, p-stars, which is challenging the standard paradigm.

P-stars are compact stars made of up and down quarks in \( \beta \)-equilibrium with electrons in a chromomagnetic condensate. In our previous studies (Cea 2004a,b), we find that p-stars are able to account for compact stars, stars with radius comparable with canonical neutron stars, as well as super massive compact objects. Moreover, we show that p-stars once formed are absolutely stable. The cooling curves of p-stars compare rather well with observations. We also suggested that p-matter produced at the cosmological deconfinement phase transition could be a viable can-
In addition, in Cea (2006) we discuss p-stars endowed with super strong dipolar magnetic field. We find that soft gamma-ray repeaters and anomalous X-ray pulsars can be understood within our theory. In particular, we point out that within our p-star model there is a quite natural mechanism to account for the generation of dipolar surface magnetic fields up to $10^{16}$ Gauss. We succeed in obtaining a well defined criterion to distinguish rotation powered pulsars from magnetic powered pulsars. We show that glitches, that in our theory are triggered by magnetic dissipative effects in the inner core, explain both the quiescent emission and burst activity in soft gamma-ray repeaters and anomalous X-ray pulsars. We are able to account for braking and normal glitches observed in soft gamma-ray repeaters and anomalous X-ray pulsars. We discuss and explain the observed anti correlation between hardness ratio and burst intensity. Within our p-star theory we are able to account quantitatively for light curves from both gamma-ray repeaters and anomalous X-ray pulsars.

In the present paper we focus on few selected topics with the aim to furnish further compelling evidences in support of our model. In section 2 we compare recent determination of masses and radii of isolated and binary pulsars with our p-star model. In section 3 after a brief review of cooling in p-stars, we compare our theoretical cooling curves with several observational data. Finally, we draw our conclusions in Section 4.

2. Masses and Radii

In a recent analysis of the low-mass X-ray binary pulsar EXO 0748-676 observational data Özel (2006) concluded that the determination of the mass and radius of this pulsar appears to rule out all the soft equations of state of neutron-star matter. Indeed, multiple phenomena have been observed from this low-mass X-ray binary that can be used to determine uniquely the mass and radius.

The three quantities used in Özel (2006) are the Eddington limit $F_{\text{Edd}}$, the gravitational redshift $z$, and the ratio $F_{\text{cool}}/\sigma T^4_c$. Following Özel (2006), we have:

\begin{equation}
F_{\text{Edd}} = \frac{1}{4\pi D} \frac{4\pi GM}{\kappa_e} \left[ 1 - \frac{2GM}{R} \right]^\frac{1}{2},
\end{equation}

\begin{equation}
z = \left[ 1 - \frac{2GM}{R} \right]^{-\frac{1}{2}},
\end{equation}

\begin{equation}
\frac{F_{\text{cool}}}{\sigma T^4_c} = f_\infty^4 \frac{R^2}{D^2} \left[ 1 - \frac{2GM}{R} \right]^{-1}.
\end{equation}

Here and in the following we shall adopt natural units where $\hbar = c = k_B = 1$.

In Eqs. (1), (2), and (3) $G$ is the gravitational constant, $M$ and $R$ are the stellar mass and radius, $D$ the distance to the source, and $\kappa_e$ is the electron scattering opacity:

\begin{equation}
\kappa_e \approx 0.2 (1 + X) \text{ cm}^2 \text{ g}^{-1},
\end{equation}
\( X \) being the hydrogen mass fraction of the accreted material. Finally, \( f_\infty \) is the colour correction factor which relates the colour temperature \( T_c \) to the effective temperature \( T_{\text{eff}} \) of the star.

Özel pointed out that the following parameterization of the colour correction factor:

\[
\frac{f_\infty}{\kappa} \equiv 1.34 + \left( \frac{1 + X}{1.7} \right)^{2.2} \left[ \frac{(T_{\text{eff}}/10^7 \, \text{K})^4}{g/(10^{13} \, \text{cm} \, \text{s}^{-2})} \right]^{2.2}, \tag{5}
\]

leads to an accurate description of the results from model atmosphere calculations (Madej et al. 2004).

Remarkably, Eqs. (1), (2), and (3) can be solved to uniquely determine the stellar mass, radius, and distance (Özel 2006):

\[
M = \frac{f_\infty^4}{4\kappa G} \frac{[1 - (1 + z)^{-2}]^2}{(1 + z)^3} \frac{F_{\text{cool}}}{\sigma T_c^4} \frac{1}{F_{\text{Edd}}}, \tag{6}
\]

\[
R = \frac{f_\infty^2}{2\kappa} \frac{1 - (1 + z)^{-2}}{(1 + z)^3} \frac{F_{\text{cool}}}{\sigma T_c^4} \frac{1}{F_{\text{Edd}}}, \tag{7}
\]

\[
D = \frac{f_\infty^2}{2\kappa} \frac{1 - (1 + z)^{-2}}{(1 + z)^3} \frac{[F_{\text{cool}}/\sigma T_c^4]^b}{F_{\text{Edd}}}, \tag{8}
\]

The value of the redshift has been reported in Cottam et al. (2002):

\[
z = 0.35. \tag{9}
\]

Moreover, the Eddington limit luminosity \( F_{\text{Edd}} \) has been obtained in Özel (2006) by averaging the values determined with RXTE (Wolff et al. 2005) and EXO SAT (Gottwald et al. 1986):

\[
F_{\text{Edd}} = 2.25 \pm 0.23 \times 10^{-8} \, \text{erg cm}^{-2} \, \text{s}^{-1}. \tag{10}
\]

Finally, the ratio \( F_{\text{cool}}/\sigma T_c^4 \) has been inferred from the RXTE data (Wolff et al. 2005):

\[
\frac{F_{\text{cool}}}{\sigma T_c^4} = 1.14 \pm 0.10 \, (\text{km/kpc})^2. \tag{11}
\]

Using the values in Eqs. (9), (10), and (11) we easily obtain from Eqs. (6), (7), and (8):

\[
\frac{M}{M_\odot} = [0.2009 \pm 0.0271] \frac{f_\infty^4}{\kappa}, \tag{12}
\]

\[
R = [1.315 \pm 0.177] \, \text{km} \frac{f_\infty^4}{\kappa}, \tag{13}
\]

\[
D = [1.232 \pm 0.137] \, \text{kpc} \frac{f_\infty^2}{\kappa}. \tag{14}
\]

The main uncertainty that affects the mass, radius, and distance of the binary pulsar resides in the hydrogen mass fraction \( X \) of the accreting material and the colour correction factor \( f_\infty \).

Following Özel (2006), if we assume for the colour correction factor:

\[
f_\infty \approx 1.37, \tag{15}
\]

and using the extreme value of hydrogen mass fraction \( X \approx 0.70 \), we get:

\[
\frac{M}{M_\odot} = 2.10 \pm 0.28. \tag{16}
\]
Fig. 1. Full lines are the gravitational mass $M$ versus the stellar radius $R$ for p-stars for different values of $\sqrt{gH}$ (in GeV). The thin dashed line (labelled $MS0$) is the mass-radius curve for neutron stars by assuming the stiffest equation of state. Full triangles correspond to the EXO 0748-676 mass-radius constraints assuming $f_{\infty} \approx 1.37$ for two different values of hydrogen mass fraction $X$. Full square is the the minimum allowed mass-radius values for EXO 0748-676. Full circles are the best-fitting mass-radius values of the model 2 in Table 1 of Poutanen & Gierlinski (2003) for the accreting X-ray millisecond pulsar J1808. The thick dashed line corresponds to the mass-radius curve for RXJ 1856 obtained by solving Eq. (25) with $R_{\infty} \approx 6.6$ km.

$$R = 13.62 \pm 1.84 \text{ km} \quad \text{(17)}$$

On the other hand, for binary systems like EXO 0748-676 a helium-rich companion should be expected (see, for instance Levin et al. 1993). In this case the value $X \approx 0.30$ seems to be more appropriate. If this is the case, we get:

$$\frac{M}{M_\odot} = 2.72 \pm 0.37 \quad \text{(18)}$$

$$R = 17.82 \pm 2.40 \text{ km} \quad \text{(19)}$$

In Fig. 1 we compare Eqs. (16), (17), and Eqs. (18), (19) with the mass-radius relation obtained within our p-star theory (Cea 2004a). We can see that for both hydrogen abundances our model is able to account for the inferred values, Eqs. (16), (17) and Eqs. (18), (19), together with the constraint Eq. (9). On the contrary, the mass-radius curve for neutron stars is consistent with the determination Eq. (16) and Eq. (17) only by assuming the stiffest equation of state (labelled $MS0$ in Fig. 1) see Lattimer & Prakash 2001), which corresponds to high-density neutron matter without non-linear vector and isovector interactions (Muller & Serot 1996). Note, however, that even the stiffest high-density neutron matter equation of state is ruled out if we consider the more
realistic hydrogen mass fraction $X \approx 0.30$.

Even more, Eqs. (12) and (13) show that the mass and radius of the star depend on the combination $f_{\infty}^4/\kappa_e$. By using the parameterization Eq. (5) for the colour correction factor and Eq. (4) for the electron opacity, we find that the ratio $f_{\infty}^4/\kappa_e$ is a smooth function of the hydrogen mass fraction. Indeed, in Fig. 2 we display the ratio $f_{\infty}^4/\kappa_e$ as a function of $X$ assuming that:

$$\left(\frac{T_{\text{eff}}}{10^7 \, \text{K}}\right)^4 \frac{g}{(10^{13} \, \text{cm s}^{-2})} \approx 1 .$$

(20)

We see that, indeed, $f_{\infty}^4/\kappa_e$ does display a minimum at $X^* \approx 0.43$, and:

$$\frac{f_{\infty}^4}{\kappa_e}(X^*) \approx 18.22 .$$

(21)

In this case, from Eqs. (12) and (13) we get (see Fig. 1):

$$\frac{M}{M_\odot} = 3.66 \pm 0.49 , \quad R = 23.97 \pm 3.23 \, \text{km} .$$

(22)

Interestingly enough, we find that the distance of the binary system is quite tightly constrained:

$$(8.87 \pm 0.99) \, \text{kpc} \lesssim D \lesssim (12.38 \pm 1.39) \, \text{kpc} .$$

(23)

It is worthwhile to stress that $M$ and $R$ in Eq. (22) are the minimum allowed values for EXO 0748-676. From Fig. 1 we infer that these minimum values are not compatible with any equation of state for neutron-star matter.

Our previous results strongly indicate that the compact star in the binary system EXO 0748-670 cannot be a neutron star. On the other hand, Fig. 1 shows that EXO 0748-670 can be accounted for by our p-star model irrespective on the assumed value of the hydrogen mass fraction.
Even assuming that the stiffest equation of state for neutron-star matter is marginally compatible with observations from EXO 0748-670, we argue in a moment that this is incompatible with several pulsar data.

It has been proposed long time ago that the compact accreting object in the famous X-ray binary Hercules X-1 is a strange star (Li et al. 1995). This proposal was based on the comparison of a phenomenological mass-radius relation for Hercules X-1 (see for instance Shapiro & Teukolsky 1983) with theoretical M-R curves for neutron and strange stars. The analysis in Li et al. (1995) has, however, been criticized by Reynolds et al. (1997). These authors, using a new mass estimate together with a revised distance, which leads to a somewhat higher X-ray luminosity, argued that the hypothesis that Hercules X-1 is a neutron star is not disproved. As a matter of fact, Reynolds et al. (1997) found that there is marginal consistency with observations if one adopts for neutron stars a very soft equation of state. At the same time, these authors pointed out that the hypothesis of a strange star can be ruled out since the theoretical curves no longer intercept the observational relations within the permitted mass range. However, it is evident from Fig. 1 in Reynolds et al. (1997) that the mass-radius constraints for Her X-1 are incompatible with the stiffest equation of state displayed in our Fig. 1.

The millisecond pulsar SAX J1808-3658 (henceforth J1808) is one of the most studied accreting pulsar (for instance, see van der Klis 1995). Recently, Poutanen & Gierlinski (2003) studied the pulse profile of J1808 at different energies. In particular, they derived simple analytical formulae for the light curves. By fitting the observed pulse profiles in the energy band 3 – 4 keV and 12 – 18 keV Poutanen & Gierlinski (2003) constrain the compact star mass and radius. The best-fitting parameters for two different models are presented in Table 1 of Poutanen & Gierlinski (2003). It turns out that the results from the two adopted models are quite consistent (Poutanen & Gierlinski 2003). In Fig. 1 we report the results form model 2 in Poutanen & Gierlinski (2003). Again, we see that the mass-radius constraints for J1808 are incompatible with the stiffest equation of state for high density neutron matter, while our p-star model can easily account for the observed mass-radius values for the accreting X-ray millisecond pulsar J1808.

Remarkably, our previous conclusion is confirmed by the recent analysis of the light curves of J1808 during its 1998 and 2002 outbursts (Leahy et al. 2007). Indeed, Leahy et al (2007) obtain that at the 3 $\sigma$ level the radius must satisfy $R < 11.9$ km and the mass $M < 1.56 M_\odot$ which rule out very stiff neutron matter equations of state.

RXJ 1856.5-3754 (in the following RXJ 1856) is the nearest and brightest of a class of isolated radio-quiet compact stars. RXJ 1856 has been observed with Chandra and XMM-Newton (Burwitz et al. 2003), showing that the X-ray spectrum is accurately fitted by a black-body law. Assuming that the X-ray thermal emission is due to the surface of the star, Burwitz et al. (2003) found for the effective radius and surface temperature:

$$R^\infty \approx 4.4 \frac{d}{120 \text{ pc}} \text{Km}, \quad T^\infty \approx 63 \text{ eV},$$

(24)
where
\[ R^\infty = \frac{R}{\sqrt{1 - \frac{2GM}{R}}} \quad \text{and} \quad T^\infty = T \sqrt{1 - \frac{2GM}{R}}. \] (25)

It should be stressed, however, that in the observed spectrum there is also an optical emission in excess over the extrapolated X-ray blackbody. By interpreting the optical emission as a Rayleigh-Jeans tail of a thermal blackbody emission, one finds that the optical data can be also fitted by the blackbody model yielding an effective radius (Burwitz et al. 2003):
\[ R^\infty \gtrsim 16 \text{ Km} \frac{d}{120 \text{ pc}}. \] (26)

Interestingly enough, quite recently the distance measurement of RXJ 1856 has been reassessed and it is now estimated to be at about 180 pc (Kaplan 2003). Moreover, from recent parallax measurements (Kaplan 2003, van Kerkwijk & Kaplan 2006) we infer that there is a lower limit to the distance of RXJ 1856:
\[ d \gtrsim 160 \text{ pc}, \] (27)

which excludes the two-blackbody interpretation. Indeed, in this case from Eq. (26) we obtain:
\[ R^\infty > 21 \text{ Km}, \] (28)

which is too large for a neutron star. On the other hand, assuming \( d \approx 180 \text{ pc} \), from Eq. (24) we get \( R^\infty \approx 6.6 \text{ Km} \). From this value of \( R^\infty \) we can solve Eq. (25) to constrain the mass and radius of RXJ 1856 (Cea 2004b). The result of this analysis, displayed in Fig. 1, indicates that there are stable p-star configurations which agree with observational data, while the allowed mass-radius values cannot be accounted for within the neutron star models.

3. Cooling

The thermal evolution of compact stars is an important tool to investigate the state of dense matter at supra-nuclear densities. In fact, observations of the thermal photon flux emitted from the surface of the stars provide valuable information about the physical processes operating in the interior of these objects.

The cooling in p-stars has been discussed for the first time in Cea (2004a). As for neutron stars, the predominant cooling mechanism of newly formed p-stars is neutrino emission. In p-stars neutrino cooling dominates for about \( 10^2 - 10^3 \) years. Subsequently, after the internal temperature has sufficiently dropped, the photon emission overtakes neutrinos.

Let us briefly review the cooling in p-stars (Cea 2004a). Assuming stars of uniform density and isothermal, the cooling equation is:
\[ C_V \frac{dT}{dt} = - (L_\nu + L_\gamma), \] (29)

where \( L_\nu \) is the neutrino luminosity, \( L_\gamma \) is the photon luminosity and \( C_V \) is the specific heat. Assuming blackbody photon emission from the surface at an effective surface temperature \( T_s \) we get:
\[ L_\gamma = 4\pi R^2 \sigma_{SB} T_s^4, \] (30)
where $\sigma_{SB}$ is the Stefan–Boltzmann constant. Following Shapiro & Teukolsky (1983), in Cea (2004a) we assumed that the surface and interior temperature were related by:

$$\frac{T_s}{T} = 10^{-2} a \, , \, \, 0.1 \leq a \leq 1.0 \, .$$  \hspace{1cm} (31)

Eq. (31) is relevant for a p-star which is not bare, namely for p-stars which are endowed with a thin crust. It results (Cea 2004b) that p-stars have a sharp edge of thickness of the order of about 1 fermi. On the other hand, electrons which are bound by the coulomb attraction, extend several hundred fermis beyond the edge. It follows, then, that on the surface of the star there is a positively charged layer which is able to support a thin crust of ordinary matter. The vacuum gap between the core and the crust of order of several hundred fermis leads to a strong suppression of the surface temperature with respect to the core temperature. In principle, the actual relation between $T_S$ and $T$ can be obtained by studying the crust and core thermal interactions. In any case, our phenomenological relation Eq. (31) allows a wide variation of $T_S$, which encompasses the neutron star relation (see, for instance, Gundmundsson et al. 1983). Moreover, as we discuss below, our cooling curves display a rather weak dependence on the parameter $a$ in Eq. (31).

In Cea (2004a) we showed that the dominant cooling processes by neutrino emission are the direct $\beta$-decay quark reactions (Iwamoto 1980, Burrows 1980):

$$d \rightarrow u + e + \bar{\nu}_e \, , \, \, u + e \rightarrow d + \nu_e \, .$$  \hspace{1cm} (32)

We find the following neutrino luminosity (Cea 2004a):

$$L_\nu \simeq 3.18 \times 10^{36} \frac{\text{erg}}{s} \frac{T_9^8}{M_{\odot}} \frac{\epsilon_0}{e} \frac{\sqrt{gH}}{1 \text{ GeV}} \, ,$$  \hspace{1cm} (33)

where $T_9$ is the temperature in units of $10^9 \degree K$, and $\epsilon_0 \approx 2.51 \times 10^{14} \text{g cm}^{-3}$ is the nuclear density. So that, for typical values of parameters we have:

$$L_\nu \sim 10^{36} \frac{\text{erg}}{s} T_9^8 \, .$$  \hspace{1cm} (34)

Note that the neutrino luminosity $L_\nu$ has the same temperature dependence as the neutrino luminosity by modified URCA reactions in neutron stars (see, for instance Shapiro & Teukolsky 1983), but it is more than two order of magnitude smaller. This peculiar dependence on the core temperature is due to the presence of the strong chromomagnetic condensate which strongly constraints the quark transverse motion.

It is worthwhile to stress that, since our neutrino luminosity is reduced by more than two order of magnitude with respect to neutron stars, the maximum allowed quiescent luminosity of isolated pulsars is about two order of magnitude greater than the maximum allowed surface luminosity in neutron stars (Van Riper 1991). Thus, while our theory allows to account for luminosities up to $10^{36} \text{ erg s}^{-1}$ as observed in magnetars (Cea 2006), the standard model based on neutron stars is in striking contradiction with observations.

A further support on our neutrino luminosity Eq. (34) comes from superbursts, namely rare, extremely energetic, and long duration type I X-ray bursts, from low mass X-ray binaries (for instance, see Kuulkers 2004). Indeed, it has been pointed out that to account for the...
Fig. 3. Comparison of p-star cooling curves with several data for the effective surface temperature. Different lines correspond to compact star mass values indicated in the legend and different values of the parameter $a$ in Eq. (31). Data points with error bars are taken from literature.

observed ignition depths in superbursts one needs slow neutrino emission processes with emissivity [Page & Cumming 2005, Stejner & Madsen 2006]:

$$\epsilon_\nu = \frac{Q_\nu}{T_9^8} \frac{\text{erg}}{\text{cm}^3 \text{s}}, \quad Q_\nu \sim 10^{18} - 10^{22}.$$  (35)

It is evident that the phenomenological neutrino emissivity Eq. (35) needed to explain observed superbursts from low mass X-ray binaries cannot be accounted for within the standard neutron star model. On the contrary, from our Eq. (34) we infer for the neutrino emissivity in p-stars:

$$\epsilon_\nu \sim 10^{18} \frac{T_9^8}{\text{erg/cm}^3 \text{s}},$$  (36)

which naturally fits in the phenomenological allowed range of values.

Finally, to determine the thermal evolution of p-stars we need the specific heat [Cea 2004a]:

$$C_V \approx 0.92 \times 10^{55} T_9 \frac{M}{M_\odot} \frac{\epsilon_0}{\epsilon} \left(\frac{\sqrt{gH}}{1 \text{GeV}}\right)^2,$$  (37)

which, for typical parameter values, in physical units reads:

$$C_V \sim 10^{39} \frac{\text{erg}}{K} T_9.$$  (38)

Note that, from Eq. (38) it follows that the p-star specific heat is of the same order of the neutron star specific heat [Shapiro & Teukolsky 1983]. To obtain the thermal history of a p-star we integrate Eq. (29) by assuming the initial temperature $T_9^{(i)} = 1.4$ [Cea 2004a].

In Fig. 3 we report our cooling curves for three different values of the parameter $a$ in Eq. (31).

It is worthwhile to note that the cooling curves are almost independent on the p-star mass.
Moreover, there is a weak dependence on $a$ up to age $\tau \sim 10^3 \text{years}$. We compare our theoretical cooling curves with several pulsar data taken from the literature.

In any case we see that the agreement between theoretical cooling curves and observational data is quite satisfying. In particular, we see that, at variance with neutron stars, our peculiar neutrino luminosity allows sizeable surface effective temperature, up to $10^5 \text{K}$, for compact stars with age $\tau > 10^6 \text{years}$. Indeed, Fig. 3 suggests that this distinguishable feature of the p-star model is corroborated by observations.

4. Conclusions

The proposal for p-stars originates from our non-perturbative investigations of QCD (Cea & Cosmai 2003, 2005) which suggested that quarks and gluons get deconfined in strong enough chromomagnetic fields. This, in turns, leads us to argue that the deconfined QCD vacuum is characterised by long-range chromomagnetic correlations and that p-matter, namely almost massless up and down quarks immersed in a chromomagnetic condensate, is formed in the collapse of the core of an evolved massive star (Cea 2004a). In addition, we already argued that p-stars are more stable than neutron stars whatever the value of the chromomagnetic condensate. As a consequence, the true ground state of QCD in strong gravitational fields is not hadronic matter, but p-matter. In other words, the final collapse of massive stars leads inevitably to the formation of p-stars.

The results discussed in the present paper, indeed, indicate that p-stars are able to reproduce in a natural manner several observations from pulsars. On the other hand, it should be evident that the standard model based on neutron stars is unable to account for recent observational data from isolated and binary pulsars.

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