Constrained Electroweak Chiral Lagrangian

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We update the uncertainty analysis on $S$ parameter of the electroweak chiral Lagrangian (EWCL) by including the LEP-II W pair production data. We find that experimental data still allow a positive $S^{\text{EXP}}(1\text{TeV})$.

1. Introduction

Technicolor models are one of the natural candidates beyond the standard model.\textsuperscript{1} But it is widely said that the Technicolor models are ruled out by electroweak precision data. The claim\textsuperscript{2} is established in two logical steps: 1) by extrapolating the electroweak precision data from $\mu = m_Z$ to 1 TeV (1 TeV is argued as the scale of compositeness) by perturbation calculation with only 6 quadratic operators of the EWCL, it was found that the value of the parameter $S$ is negative. In our fit, when triple gauge coupling (TGC) effects are not included, $S$ is determined as

$$S^{\text{EXP}}(1\text{TeV}) = -0.17 \pm 0.10,$$

(1)

2) by using the unsubtracted dispersion relations with the assumption of custodial symmetry and vector meson dominance and the low energy hadronic QCD data of $\rho$ and $a_1$ mesons as input, and by scaling up the value of $S$ to 1 TeV, it was found that the value of $S$ parameter is positive,\textsuperscript{2} which is given as

$$S^{\text{TH}}(1\text{TeV}) = 0.3 \frac{N_{\text{TFF}} N_{\text{TC}}}{6}.$$

(2)

The discrepancy between $S^{\text{EXP}}$ and $S^{\text{TH}}$ is at least 3$\sigma$, which is interpreted as an evidence that Technicolor models are ruled out by precision data.

In this article we summarize our study on the uncertainty analysis in $S^{\text{EXP}}(1\text{TeV})$\textsuperscript{3} by extending the 6-operator analysis to the 14-operator one. Part of results was reported in.\textsuperscript{4} Our result show that the uncertainty induced by TGC measurement dominates the error bar of $S^{\text{EXP}}(1\text{TeV})$ and
experimental data still allows positive $S^{\text{EXP}}(1\text{TeV})$ in the parameter space. Therefore we argue that it is premature to claim that the electroweak precision data has ruled out Technicolor models.

2. Our knowledge on the chiral coefficients

We follow the standard analysis of chiral Lagrangian method\textsuperscript{5,6} and include 14 operators up to mass dimension four in the EWCL.\textsuperscript{7} Our study extends the RGE analysis of Bagger \textit{et. al.},\textsuperscript{8} who have considered the effects of 6 out of the 14 operators.

The 14 gauge invariant operators constructed in the EWCL are supposed to describe EWSB models defined at 1TeV in a model independent way, either strong or weak interaction models. Below we describe how to determine 14 chiral coefficients of the EWCL in our analysis at $\mu = m_Z$. Three of six two-point chiral coefficients $g$, $g'$, and $v$, are determined by the following inputs $1/\alpha_{\text{em}}(m_Z) = 128.74$, $m_Z = 91.18$ GeV, and $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$. They are assumed to be free from error bars. The next three two-point-function chiral coefficients $\alpha_1$, $\alpha_0$, and $\alpha_8$ are extracted from Z-pole data, precise W mass measurement, and top quark mass measurement (Here we use data of PDG2004). We perform the analysis with three best measured quantities $m_W = 80.425 \pm 0.038$ GeV, $\sin^2 \theta_W = 0.23147 \pm 0.00017$ and the leptonic decay width of $Z$, $\Gamma_\ell = 83.984 \pm 0.086$ MeV for the $S$, $T$ and $U$ fit. We take $m_t = 175$ GeV in our $S$, $T$, and $U$ fit.

The central values with $1\sigma$ errors of the $S$, $T$, and $U$ parameters are found as

$$
S(m_Z) = (-0.06 \pm 0.11) \\
T(m_Z) = (-0.08 \pm 0.14) \rho_{\text{co.}} = \begin{pmatrix} 1 \\ .9 \\ -1.4 \\ -1.6 \end{pmatrix} \\
U(m_Z) = (+0.17 \pm 0.15)
$$

which roughly agrees with.\textsuperscript{8}

The relations among two-point chiral coefficients $\alpha_1$, $\alpha_0$, and $\alpha_8$ of EWCL with the $STU$ parameters are found to be

$$
\alpha_1(\mu) = -\frac{1}{16\pi} S(\mu), \\
\alpha_0(\mu) = \frac{1}{2} \alpha(\mu)_{\text{EM}} T(\mu), \\
\alpha_8(\mu) = -\frac{1}{16\pi} U(\mu).
$$

From Eqs. (3-4), $\alpha_1(m_Z)$, $\alpha_0(m_Z)$, and $\alpha_8(m_Z)$ are determined as

$$
\alpha_1(m_Z) = (+0.13 \pm 0.21) \times 10^{-2} \\
\alpha_0(m_Z) = (-0.03 \pm 0.05) \times 10^{-2} \\
\alpha_8(m_Z) = (-0.35 \pm 0.29) \times 10^{-2}
$$
Three three-point chiral coefficients $\alpha_2$, $\alpha_3$ and $\alpha_9$ are extracted from the LEP-II measurements via the process $\gamma_+ \gamma_- \rightarrow W^+ W^-$. The experimental observables of anomalous TGC\cite{10} between $\delta k_7$, $\delta k_Z$, $\delta g^Z_1$, and three-point chiral coefficients, $\alpha_2$, $\alpha_3$, $\alpha_9$, can be simplified as

\begin{align}
\delta k_7 &= (\alpha_2 + \alpha_3 + \alpha_9)g^2, \\
\delta k_Z &= (\alpha_3 + \alpha_9)g^2 - \alpha_2 g^2, \\
\delta g^Z_1 &= \alpha_3(g^2 + g'^2).
\end{align}

There is no experimental data relaxing the custodial symmetry except L3 collaboration\cite{11} from where we take $\delta k_Z = -0.076 \pm 0.064$ as one of the inputs. Other inputs $\delta k_7 = -0.027 \pm 0.045$ and $\delta g^Z_1 = -0.016 \pm 0.022$ are taken from LEP Electroweak working group.\cite{12,13} We found TGC errors are quite large as reported in D0 collaboration\cite{14} at Tevatron. Because of this fact, we use LEP data in our analysis.

We also relax the custodial $SU(2)$ gauge symmetry as it is natural in the framework of the EWCL to have a non-vanishing $\alpha_9$ if the underlying dynamics break this symmetry explicitly.\cite{15} By assuming these data are extracted independently, we can obtain three-point chiral coefficients as

\begin{align}
\alpha_2(m_Z) &= (+0.09 \pm 0.14) \\
\alpha_3(m_Z) &= (-0.03 \pm 0.04) \rho_\gamma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
\alpha_9(m_Z) &= (-0.12 \pm 0.12)
\end{align}

We observe that $\alpha_3(m_Z)$ is more tightly constrained than $\alpha_2(m_Z)$ and $\alpha_9(m_Z)$. In our numerical analysis, we consider the two-parameter fit data from L3 collaboration and this combined data as two scenarios to show the effects of TGC to $S^{\text{EXP}}$.

There is no experimental data to bound 5 four-point chiral coefficients, which usually are assumed to be of order one. Partial wave unitary bounds of longitudinal vector boson scattering processes can be used to put bounds on the magnitude of those chiral coefficients. We use the following five $J = 0$ channels to bound 5 chiral coefficients ($\Lambda$ is arbitrary, which should correspond to the UV cutoff of the EWCL), $\alpha_4$, $\alpha_5$, $\alpha_6$, $\alpha_7$, and $\alpha_{10}$:

\begin{align}
|4 \alpha_4 + 2 \alpha_5| &< 3 \pi \frac{\Lambda^4}{m^4}, \\
|3 \alpha_4 + 4 \alpha_5| &< 3 \pi \frac{\Lambda^4}{m^4}, \\
|\alpha_4 + \alpha_6 + 3(\alpha_5 + \alpha_7)| &< 3 \pi \frac{\Lambda^4}{m^4}, \\
|2(\alpha_4 + \alpha_6) + \alpha_5 + \alpha_7| &< 3 \pi \frac{\Lambda^4}{m^4}, \\
|\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})| &< 3 \pi \frac{\Lambda^4}{m^4},
\end{align}

where bounds are obtained from $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$, $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, $W_L^+ Z_L \rightarrow Z_L Z_L$, $W_L^+ Z_L \rightarrow W_L^+ Z_L$, and $Z_L Z_L \rightarrow Z_L Z_L$, respectively.
Quartic gauge couplings will not contribute to $S$ parameter, but do affect $T$ parameter. For completeness, we list these theoretical bounds here.

3. Uncertainty in $S^{\text{EXP}}(\Lambda)$ parameter

In the framework of effective field theory method, all chiral coefficients of the EWCL depend on renormalization scale $\mu$. $S(\Lambda)$, $T(\Lambda)$, and $U(\Lambda)$ are values of parameters $S$, $T$, and $U$ at the matching scale $\Lambda$, where the EWCL matches with fundamental theories, Technicolor models, extra dimension models, Higgsless models, etc. In the perturbation method, theoretically, the $S(m_Z)$, $T(m_Z)$, and $U(m_Z)$ can be connected with the $S(\Lambda)$, $T(\Lambda)$, and $U(\Lambda)$ by improved renormalization group equations.

The effect of TGC measurement is depicted on $S(\Lambda) - T(\Lambda)$ plane as shown in Fig. 1(a) and Fig. 1(b). We highlight some features of these two figures. (1) In Fig. 1(a), in absence of TGC contribution (dashed-line contours), $S(\Lambda)$ becomes more negative as $\Lambda$ increases with reference to the reference LEP-I fit contour at $\Lambda = m_Z$. This is roughly in agreement with the observation of Ref., Ref., and PDG. Two-parameter fit data with custodial symmetry condition from L3 collaboration shows that $S(\Lambda)$ is driven to the positive value region (the solid line). The central value of $S^{\text{EXP}}(1\text{TeV})$ is 0.8. Furthermore, the error bars of $S - T$ are greatly enlarged by the uncertainty of TGC measurements.

(2) In Fig. 1(b), without the custodial symmetry assumption on the experimental data, as given in Eq. (7), inclusion of TGC contribution makes the central value of $S(\Lambda)$ almost unchanged. Meanwhile, the error bars of $S - T$ are enlarged at least by a factor of 2.

Fig. 1 clearly demonstrate a fact that the TGC measurement affect the value of $S^{\text{EXP}}(\Lambda)$ significantly.

4. Discussions and Conclusions

In the unsubtracted dispersion relation, the fact that $S^{\text{EXP}}$ is a running parameter might not be transparent. In the RGE analysis, the quantum fluctuations of active degree of freedoms $S$ parameter to run:

$$ S^{\text{EXP}}(\Lambda) = S^{\text{EXP}}(m_Z) + \beta_S \ln \frac{\Lambda}{m_Z}. $$

Active degree of freedoms and new resonances can contribute to $\beta_S$ function and affect the value of $S^{\text{EXP}}(\Lambda)$. We propose a naive subtracted dispersion relation without assuming custodial symmetry and vector meson.
Fig. 1. $S(\Lambda) - T(\Lambda)$ contours at $\Lambda = m_Z$, 300 GeV, 1 TeV, and 3 TeV, respectively. TGC uncertainty is included in solid line contours while not included in dashed line. Fig1(a) corresponds to the L3 two-parameter fit data. The parameter $U(m_Z)$ is taken as its best fit value, $U(m_Z) = +0.17$. Fig1(b) corresponds to the combined data. The parameter $U(m_Z)$ is taken as its best fit value, $U(m_Z) = 0.00$.

dominance, which can read as

$$S(q^2) = S(q^2 = 0) - \frac{q^2}{3\pi} \int_{s > m^2_Z}^{\infty} ds \frac{R_{3Y}(s)}{s(s - q^2)}.$$  \(10\)

Where $R_{3Y} = -12\pi Im \Pi_{3Y}$, which is to count the degree of freedoms belonging to the representations of both $U_Y(1)$ and $SU_L(2)$. The $S(q^2 = 0)$ corresponds to the value determined from LEP-I Z-pole data. The second term include the contributions of active degree of freedoms and new resonances, either fermionic or bosonic. With Eqs. (9-10), the equivalence between the description of RGE and dispersion relation becomes obvious.

Our most conservative numerical result from Fig. 1(b) can be put as

$$S^{Exp}(1 \text{TeV}) = -0.08 \pm 0.20.$$  \(11\)

To obtain these numerical values, we have assumed the perturbation method is valid from the energy scale $m_Z$ to 1 TeV. When data in PDG2006 is used, the central value of $S^{Exp}(1 \text{TeV})$ shifts to $-0.02$, which is in agreement with the observation in.\(^20\) If there exist new resonances below or near 1 TeV, the perturbation method might be invalid before $\Lambda = 1 \text{TeV}$ and effects of threshold and tail of new resonances might modified the value.

\(^{20}\)We thank Han-Qing Zheng and Kenneth Lane for comments on this point.
of $S(1\text{TeV})$ drastically. This might occur, for example, in the low energy Technicolor model.\textsuperscript{21} Hence, we stress that both the center values and error bars of $S(1\text{TeV})$ can only be interpreted as reference values obtained in perturbation method.

Whether $S$ and $T$ should run in a logarithmic or power way or whether the decoupling theorem should hold in the process of extrapolating the data from $m_Z$ scale to the matching scale is still a debatable issue. In the analysis of the minimal standard model with a Higgs,\textsuperscript{18} the model is renormalizable and Higgs boson plays the role of regulator. Therefore only logarithmic terms are taken in the standard global fit. In the EWCL (a non-renormalizable theory), when operators beyond the $O(p^2)$ order are included, terms proportional to the power of $\Lambda_{\text{UV}}/v$ enter into the radiative corrections. As the most conservative calculation, we adopted the logarithmic running. However, if power running is used, error bars of $S$ parameters would be much larger than those shown Fig. 1.

One-loop contribution of TGC in our analysis can be attributed as part of $O(p^6)$ order effects in the standard chiral derivative power counting rule. There are analysis by including $O(p^6)$ operators in order to accommodate data of $e^+e^-\rightarrow f\bar{f}$ above $Z$ pole, as done in.\textsuperscript{19} Even when these tree level effects of $O(p^6)$ operators are included, near $T\text{eV}$ region, the sign of $S^{\text{EXP}}(1\text{TeV})$ can not change from negative to positive. Another remarkable fact is that when more operators are introduced the error bar of $S^{\text{EXP}}(1\text{TeV})$ becomes a few larger. But effects of these operators are smaller than those of TGC operators.

One may worry about the two-loop contributions of $O(p^2)$, which are also part of $O(p^6)$ order effects. However, due to the two loop suppression factor, they must be tiny. Therefore, uncertainty induced by the TGC dominates the error bar of $S^{\text{EXP}}(1\text{TeV})$. Our results show that the sign of $S(1\text{TeV})$ can be changed from negative to positive by TGC.

It is an open question to construct a realistic model which can have a large deviation from the prediction of the SM in TGC sector. In the Higgsless model with ideally delocalized fermions and the gauge-Higgs unification model in the warped space-time, it was found that the deviation is small.\textsuperscript{22} The Higgsless model in warped 5D space-time might provide a solution.\textsuperscript{12}

We show here that electroweak precision data have constrained both the oblique parameters STU significantly and the anomalous TGC considerably. But, current precision of electroweak data is not sufficient to rule

\textsuperscript{19} We thank Kinya Oda mention this to us.
out Technicolor models, due to the large uncertainty in $\alpha_2$ and $\alpha_9$. Technicolor models can provide dark matter candidates.\textsuperscript{23} Therefore, in our opinion, Technicolor models are still quite competitive and promising as a candidate of EWSB.\textsuperscript{24}

**ACKNOWLEDGMENTS**

This work is partially supported by the JSPS fellowship program and by NCTS (Hsinchu, Taiwan). We would like to thank Ulrich Parzefall for communication on the TGC measurements, Masaharu Tanabashi and Masayasu Harada for stimulating discussions.

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