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1. INTRODUCTION

Cooling of proton and antiproton beams¹⁻⁵⁾ has recently led to spectacular results. Amongst these are the following:

- the increase of the lifetime of stored beams by a factor of 50⁶⁻⁷⁾ due to the compensation of blow-up by multiple scattering on the residual gas;
- the non-destructive observation of as few as 50 circulating particles⁸⁾ (corresponding to a beam current of 3×10^{-11} A), by reducing their frequency spread to less than 10^{-5} ;
- the accumulation of 2×10^{11} antiprotons from batches of about 3×10^6 ⁹⁾;
- the preparation of sharply collimated beams with a final beam size of less than a millimetre and a momentum spread of 10^{-4} at intensities of 10^8 - 10^9 circulating protons¹⁰⁾.

At CERN, these new techniques have made an ambitious programme of antiproton physics¹¹⁾ possible, covering a large range of energies from almost 0 to 600 GeV/c in the centre of mass and with beams of very high quality.

In the light of these achievements and prospects it is natural to consider the application of similar techniques to other rare particles like heavy-ion and/or polarized-particle beams. The use of electron cooling in this context was in fact suggested already some time ago by the Novosibirsk group^{3,4)}. In the present paper we try to work out in some detail the features of both stochastic and electron cooling of heavy-ion beams. The problems considered are cooling times, beam lifetime, particle loss due to electron capture, equilibrium states, and possible experimental techniques and applications.

2. ELECTRON COOLING

2.1 Principle

The principle is simple (see Fig. 1). Electrons produced in a gun travel together with the ions over part of the ion storage ring and absorb transverse and longitudinal energy deviation of the ions by Coulomb interaction. It is instructive to consider the process in the "electron rest frame", moving at the average electron

Fig. 1 The principle of an electron cooling system in a storage ring. The electrons travel together with the ions over part of the storage ring. They absorb transverse and longitudinal energy deviation of the ions via Coulomb interactions.

velocity which is equal to the desired ion equilibrium velocity $v_0 = \beta c$. In this frame, ideally, ions are "stopped" because they lose their energy deviation by friction with the electrons, similarly to the slowing down of particles in matter by virtue of their energy loss to the atomic electrons.

2.2 Cooling time and equilibrium

The cooling time can be worked out in the electron rest frame^{1,2)}, taking only binary collisions into account. This picture permits us to "scale" the expressions obtained by the more general theory to the case of heavy ions of mass $A_i M_p$ and charge $Z_i e$ being cooled by particles of mass $M_c m_e$ (i.e. M_c times the electron mass) and charge $Z_c e$. One finds for the cooling rate

$$\frac{1}{\tau} = \frac{1}{k} \frac{Z_i^2}{A_i} \frac{Z_c^2}{M_c} \frac{(j_c/Z_c e) r_p r_e}{\beta^4 \gamma^5 \theta^3} \eta_c \ln \left[\frac{\hat{\rho}_e M_c \sqrt{M_c}}{\tilde{\rho}_p Z_i Z_c \sqrt{Z_c}} \right]. \quad (1)$$

Here k depends on the ion-electron velocity distribution in the electron rest system: $k = 3/(2\sqrt{2}\pi) \approx 0.6$ for a uniform Maxwellian distribution, $k = 1/2\pi = 0.16$ for a flattened distribution (where the longitudinal velocity spread of the electrons is negligible), $r_p = 1.54 \times 10^{-18}$ m, $r_e = 2.82 \times 10^{-15}$ m, $\eta_c = \ell_i/2\pi R$ is the fraction of the cooling ring circumference ($2\pi R$) over which cooling takes place.

$$\hat{\rho} = \left(\frac{v}{c} \right) \sqrt{4\pi \frac{j_c}{Z_c e} \frac{Z_c^2 e^2}{M_c m_e}} = \hat{\rho}_e \sqrt{\frac{Z_c}{M_c}} \quad (2)$$

is the maximum impact parameter due to Debye screening, expressed here in terms of the Debye length in the plasma of cooling particles with the current density

$j_c = n_c Z_c e \beta c$. In a similar way

$$\check{\rho} = r_e \left(\frac{c}{V} \right)^2 \frac{Z_i Z_c}{M_c} = \check{\rho}_0 \frac{Z_i Z_c}{M_c} \quad (3)$$

is the minimum impact parameter entering into the Coulomb logarithm. Finally, V is the r.m.s. of the ion-electron relative velocity and $\theta = V/\beta c = v/v_0$ is the normalized velocity spread. For transverse components of v it can be interpreted as angular spread.

For $Z_i = Z_c = 1$, $A_i = M_c = 1$ we recover the familiar results for electron cooling of protons. When all other parameters are equal (especially j_c , θ , β) then the cooling speed for heavy ions becomes slower with the mass of the "c" and "i" particles and faster with their charge:

$$\frac{1}{\tau} \propto \frac{Z_i^2 Z_c}{A_i M_c} \ln r_0 \frac{M_c^{3/2}}{Z_i Z_c^{3/2}}, \quad (4)$$

or neglecting the logarithm dependence

$$\frac{1}{\tau} \propto \frac{Z_i^2 Z_c}{A_i M_c}. \quad (5)$$

Equations (4) or (5) permit us to scale the cooling time; we recall that M_c is the mass of the cooling particles in units of the *electron* mass.

We note that the linear dependence $1/\tau \propto Z_c$ comes about because we assume a current limited cooling plasma and hence a constant current j_c of cooling particles. If, instead, one assumes a constant number density the cooling rate is proportional to Z_c^2 , and thus symmetric in its dependence on Z_i and Z_c as one might expect from elementary considerations.

Another quantity of interest is the equilibrium beam velocity spread. From equalization of temperatures in the rest system $[(A_i m_p/2)v_i^2 = (M_c m_e/2)v_c^2]$, the velocity deviations of the ions and the cooling particles are related. Ideally,

$$v_i = \sqrt{\frac{M_c m_e}{A_i m_p}} v_c = \sqrt{\frac{M_c}{A_i}} \frac{v_c}{42.8}$$

or in terms of the divergence

$$\theta_i = \frac{v_i}{\beta c} = \theta_{(ep)} \sqrt{\frac{M_c}{A_i}} . \quad (6)$$

Here $\theta_{(ep)}$ denotes the equilibrium *proton* beam divergence with *electron* cooling. Equation (6) indicates stronger collimation for heavier ions and lighter cooling particles.

These equilibrium considerations are only true in the absence of any other heating mechanism. For intense beams the equilibrium can be considerably affected by scattering amongst the particles of the same beam ("intra beam scattering"). This leads to an intensity-dependent equilibrium spread $\Delta p/p$, as was measured at Novosibirsk³⁾ and CERN⁸⁾ for proton beams. As the interaction is via the Coulomb force of the particles, we expect that for ions of charge Z_i and mass number A , the interaction and hence the equilibrium spread are Z^2/A_i times the spread for the same number of protons. For a more detailed analysis we have to balance the cooling strength $1/\tau$ with the total heating rate $d(\Delta p)/dt$ or dE/dt (for momentum spread or emittance respectively), and this leads to equilibrium values for momentum spread and emittance of the order of

$$\begin{aligned} \Delta p_{eq} &\approx \frac{d(\Delta p)}{dt} \tau \\ E_{eq} &\approx \frac{dE}{dt} \tau . \end{aligned} \quad (7)$$

Apart from intra-beam scattering, heating can be due to various other processes, for example: collisions with the residual gas, ripple of the guide fields, etc. (see Section 4.2).

2.3 Electron capture

Ions can pick up cooling electrons and get lost from the storage ring owing to the change of their charge state. At the electron densities in question the dominant mechanism is radiative capture:

This process has been well known since the late 1920's and exact calculations of the cross-section have been performed¹³⁾ for hydrogen-like ions, i.e. bare nuclei or partially stripped ions if the captured electron can be assumed to move in the Coulomb field of a (partially screened) point charge.

In connection with electron cooling of protons, recombination has been studied by Budker and Skrinsky³⁾ and more recently by M. and J.S. Bell¹⁴⁾. The Novosibirsk Group has successfully used the neutral hydrogen formed in the cooling section to observe and optimize the cooling process.

We will now try to scale these results to heavier ion beams. Using the cross-section for radiative capture given by Seaton¹⁵⁾

$$\sigma = Ax \left\{ \frac{1}{n} (\ln x) + \gamma_1 + \gamma_2 x^{-1/3} \right\} \quad (9)$$

where

$$x = \frac{E_b}{E} = \frac{\text{ground state binding energy}}{\text{electron energy in ion rest system}} \quad (10)$$

and

$$\gamma_1 = 0.140, \quad \gamma_2 = 0.525, \quad A = 2.11 \times 10^{-22} \text{ cm}^2.$$

Bell and Bell¹⁴⁾ obtain the recombination rate $\alpha = \langle v_e \sigma(v_e) \rangle$ by averaging over a Maxwellian or a flattened velocity distribution for v_e . This distribution may be written as

$$f(\vec{v})_{\text{Maxwell}} = \left(\frac{m}{2\pi E_T} \right)^{3/2} \exp \left(- \frac{1}{2} m v^2 / E_T \right)$$

$$f(\vec{v})_{\text{flatt.}} = \frac{m}{2\pi E_T} \exp \left(- \frac{1}{2} m v^2 / E_T \right) \delta(v_{\parallel}),$$

where E_T is the typical energy spread between ions and electrons. Defining now $x = E_b/E_T$ in terms of this "temperature" E_T , one gets

$$\alpha(\text{Maxwell}) = A c^2 \sqrt{\frac{2}{\pi}} \sqrt{x} \sqrt{\frac{E_b}{m_0 c^2}} \left\{ \gamma_3 + \gamma_4 x^{-1/3} + \frac{1}{2} \ln x \right\}$$

$$\alpha(\text{flatt.}) = A c^2 \sqrt{\frac{2}{\pi}} \sqrt{x} \sqrt{\frac{E_b}{m_0 c^2}} \left\{ \gamma_3 + \gamma_5 x^{-1/3} + \frac{1}{2} \ln 4x \right\}$$

with $\gamma_3 = 0.429$, $\gamma_4 = 0.469$, $\gamma_5 = 0.334$.

The results for protons can be generalized to fully stripped ions by noting that the ground-state binding energy is

$$E_b \approx Z^2 E_b(\text{hydrogen}) = Z^2 \cdot 13.6 \text{ eV}.$$

For very heavy fully stripped nuclei E_b is even larger than Z^2 times the Rydberg constant. We shall neglect this for the rest of this paper and replace E_b by $Z^2 E_b$, $x \rightarrow Z^2 x$ in Eq. (10). Numerically with E_T in eV we have from Bell and Bell

$$\alpha(\text{Maxwell}) = \frac{1.92Z^2}{\sqrt{E_T}} \left\{ \ln \left(\frac{5.66Z}{\sqrt{E_T}} \right) + 0.196 \left(\frac{E_T}{Z^2} \right)^{1/3} \right\} \times 10^{-13} \text{ cm}^3/\text{s}$$

$$\alpha(\text{flatt.}) = \frac{3.02Z^2}{\sqrt{E_T}} \left\{ \ln \left(\frac{11.32Z}{\sqrt{E_T}} \right) + 0.14 \left(\frac{E_T}{Z^2} \right)^{1/3} \right\} \times 10^{-13} \text{ cm}^3/\text{s}.$$

These are Eqs. (27) and (28) of Bell and Bell¹⁴⁾ generalized for $Z \neq 1$. Usually, $E_T \approx 0.1-1$ eV so that one can neglect the $1/3$ power term especially for large Z . Hence, we find the scaling of α with charge state

$$\alpha \propto Z^2 \ln \left(aZ/\sqrt{E_T} \right), \quad a = \begin{cases} 5.66 & \text{Maxwell} \\ 11.32 & \text{flattened} \end{cases} \quad (11)$$

or, neglecting the logarithmic dependence,

$$\alpha \propto Z^2. \quad (12)$$

The recombination rate $1/\tau_R = \alpha n_e n_c$ is obtained by multiplying α with the volume density n_e of the electron beam. For the cooling as for the recombination problem one has to include the ratio n_c (cooling length/storage ring circumference), which gives the fraction of time the ion stays inside the electron cloud; τ_R may then be interpreted as the lifetime of the ion beam against recombination. Note that an equation similar to this was given by Budker and Skrinsky³⁾ who write:

$$\alpha = 19.4Z^2 r_e^2 c^2 \frac{\ln[Zc/(137v)]}{137 v_e}.$$

Our main result is that the recombination rate scales like $Z^2 \ln a Z/\sqrt{E_T}$ or approximately $1/\tau \propto Z^2$. This is true for hydrogen-like systems, i.e. for bare nuclei. For partially stripped ions Seaton's formula (10) is no longer true as the captured electron will find itself in a shielded potential into which its orbit may partially plunge.

The difference is most pronounced for singly stripped ions. However, from the work of Bates¹³⁾ one can conclude that the recombination coefficient even for singly charged ions can be estimated to within about 40% by using formula (10) with $Z = 1$. This is explained by the fact that for the higher state orbits the ion looks more and more hydrogen like and the unexcited state of the valence orbit contributes only a fraction of 25-50% to the total capture probability (see Tables 1 and 2 taken from

the work of Bates). With these reservations we will use Eqs. (10) to (12) for arbitrary ions of charge state Z_i .

Table 1

Total radiative recombination coefficients at 250 K (0.02 eV)

Ion	H ⁺	He ⁺	Li ⁺	C ⁺	N ⁺	O ⁺	Ne ⁺	Na ⁺	K ⁺
$\alpha(10^{-12} \text{ cm}^3/\text{s})$	4.8	4.8	3.7	4.2	3.6	3.7	3.4	3.2	3.0

Table 2

Radiative recombination of H⁺ ions (from Ref. 14)

Principle quantum number n	Rate coefficient α (in cm^3/s) for an electron temperature of:		
	1000 K (0.08 eV)	4000 K (0.35 eV)	16000 K (1.4 eV)
1	5.07×10^{-13}	2.56×10^{-13}	1.20×10^{-13}
2	2.79×10^{-13}	1.32×10^{-13}	5.63×10^{-14}
3	1.88×10^{-13}	8.44×10^{-14}	3.19×10^{-14}
.	.	.	.
.	.	.	.
.	.	.	.
Sum α_Σ	1.99×10^{-12}	7.85×10^{-13}	2.93×10^{-13}

2.4 Comparison of cooling and recombination rate

In the approximation outlined above, the beam lifetime in the presence of recombination now scales as

$$\tau_r \propto \left[Z_i^2 \ln (aZ_i/\sqrt{E_T}) \right]^{-1}, \quad (13)$$

or approximately as

$$\tau_r \propto Z_i^{-2},$$

whereas the electron cooling time from expression (4) varies as

$$\tau_c \propto \frac{Z^{-2} A_i}{\ln(r_0/Z_i)}.$$

The ratio τ_r/τ_c thus becomes more unfavourable (roughly as $1/A_i$) as the ion mass increases. This is illustrated in Table 3, where cooling and recombination time are scaled from protons to heavier ions.

We also see from Table 3 and Eqs. (4) and (11) that electron cooling times tend to become long for singly charged heavy ions, whereas recombination times become short for highly stripped heavy nuclei.

Table 3

Scaling of cooling times, recombination time, and equilibrium beam divergence for three different ion species singly or fully stripped. Normalization such that cooling time $\tau = 1$ s, recombination time $\tau_r = 10^5$ s, and equilibrium proton divergence $\theta_{eq} = 0.1$ mrad for electron cooling of a proton beam. Note the different dependence $\tau \propto V^3$, $\tau_r \propto V$.

Particle to be cooled			Cooling plasma	Cooling time τ (s)	Ideal equilibrium divergence (mrad)	Recombination time τ_r (s)	τ_r/τ
Type	Mass number A	Charge state z_i					
p	1	1	e	1	0.1	10^5	10^5
			p	1836	-	-	-
Ne	20	1	e	20	0.02	10^5	5×10^3
			p	3.7×10^4	-	-	-
		10	e	0.20	0.02	580	2.9×10^3
			p	37	-	-	-
U	238	1	e	238	0.006	10^5	4.2×10^2
			p	4.4×10^5	-	-	-
		98	e	0.025	0.006	4	1.6×10^2
			p	46	-	-	-

The alternative of cooling heavy ions by protons, which in turn are cooled by electrons, is also considered in Table 3. It has already been suggested by the Novosibirsk group. "Proton cooling" avoids the recombination problem but it is slower even if the high density obtainable in electron beams could be achieved in the proton beam. Certainly the need for an additional proton storage ring is a serious drawback.

One concludes that electron cooling of heavy-ion beams seems feasible. However if one wants to obtain long storage with low recombination losses one will have to reduce the electron current, which of course also reduces the cooling strength.

3. STOCHASTIC COOLING OF HEAVY ION BEAMS

3.1 Principle

This second cooling method indicated in fig. 2 uses an electronic feedback system to correct the momentum deviation and the betatron oscillations of each individual particle^{2,4)}. A set of pick-up electrodes senses the error of a particle; the signal is amplified in a high-gain wide-band amplifier and applied on a corrector downstream in the storage ring. Stochastic cooling may be viewed as the competition between three effects: the action of a test particle upon itself via the cooling system ("coherent cooling effect"), the perturbing action of the other particles on the test particle ("incoherent heating effect"), and the perturbing influence of the thermal noise of the amplifier (heating by amplifier noise). Owing to the finite bandwidth (W) of the cooling system, the correction signal of a particle will be present during a time $T_s = 1/2W$, where T_s is the response (i.e. decay) time of the system. All particles passing during this time interval will influence the test particle. To minimize their heating effect it is important to have large bandwidth and a small number of particles.

Another requirement is referred to as mixing: stochastic cooling only works if the test particle continuously changes its neighbours -- if it stays in the same company the perturbing effect is no longer random. After a passage through the system cooling would stop. Fortunately, owing to momentum spread, faster particles continuously overtake the slower ones and under those conditions the incoherent effect keeps the required random character.

Fig. 2 The principle of (horizontal) stochastic cooling. The pick-up measures horizontal deviation; the kicker corrects angular deviation. They are spaced by a multiple plus one quarter of the betatron wavelength. A position error at the pick-up transforms into an error of angle at the kicker. This angular error is corrected.

3.2 Basic limitations

The above qualitative considerations may help to understand the cooling-rate expression, which can be written as⁴⁾

$$\frac{1}{\tau} = \frac{W}{N} \left[2g - g^2(M+U) \right], \quad (14)$$

where $g \leq 1$ is related to the amplifier gain, W is the system bandwidth, N the total number of particles in the coasting beam. $M \geq 1$ is the number of turns for mixing. Finally, $U \geq 0$ is the ratio of noise to signal power. In Eq. (14) the $2g$ term represents the coherent effect, the g^2M term the incoherent effect (including partial mixing $M > 1$), and the g^2U term the heating by amplifier noise. Note that the cooling rate (14) is fastest $1/\tau = (W/N)[1/(M+U)]$ if the optimum gain $g_0 = [1/(M+U)]$ is chosen.

We can now scale from protons to ions of charge $Z_i > 1$. For the same particle number N both the coherent and the incoherent signals will increase in the same way with Z_i ; hence their balance is not affected. Mixing is unchanged, but the signal power increases with Z_i^2 . Hence, for given amplifier noise

$$U = \text{noise/signal power} \rightarrow U_0/Z_i^2.$$

We can thus use

$$\frac{1}{\tau} = \frac{W}{N} \left[2g - g^2 \left(M + \frac{U}{Z_i^2} \right) \right],$$

which suggests faster cooling

$$\tau_0 = \frac{N}{W} \left(M + \frac{U}{Z_i^2} \right) \quad (15)$$

for the optimum choice $g_0 = [M + (U/Z_i^2)]^{-1}$ with the noise term reduced by Z_i^2 . We thus find the same limitation with particle number and the same mixing limitations, but a more favourable noise limit than for protons.

Equation (15) is shown in Fig. 3 (from Ref. 4). The linear increase of τ with N is in the range of "large intensity" where $U \ll M$. For small intensity, where $U \gg M$, the curves level off as the noise-to-signal ratio U itself is proportional to $1/N$. Together with the N outside the bracket in Eq. (15) this leads to constant cooling time independently of intensity. For multiple charged ions, the conclusion is that this noise-limited regime is "pushed downwards", thus permitting (much) shorter cooling times for (very) small particle number. Stochastic cooling obviously avoids recombination losses. There are no basic differences in the performance limits for stochastic ion and stochastic proton cooling. Figure 3 thus can be used for our scaling purposes; it also includes the working points of some CERN machines and one can see that these points fall on lines with $M \approx 5-10$.

Present-day systems work with cooling times of 1 s for 10^7 protons or antiprotons or 1 day for 10^{12} . A factor of 10 can probably be gained in the near future by pushing the bandwidth into the gigahertz region.

Fig. 3 Normalized cooling time versus intensity. The inclined lines represent the mixing limit. For low intensity the cooling time levels off because of noise. The points represent initial cooling in various proton and antiproton machines. During cooling noise and/or mixing become more important and the cooling time increases, i.e. the working point moves upwards. Note that the vertical scale is normalized for 100 MHz bandwidth.

Another limitation of stochastic cooling should be understandable from the preceding discussion: as the cooling of a beam proceeds, stochastic damping tends to become increasingly slower. This is because it becomes more difficult to detect and feed back the decreasing error signals from the shrinking beam and also because mixing decreases with $\Delta p/p$. This "slowing down" tendency is quite opposite to the behaviour of electron cooling, which tends to work faster for cooler beams -- at least as long as the ion-electron velocity spread is dominated by the ion beam properties.

These differences have led the scepticists to the statement that electron cooling works well when one does not need it and that stochastic cooling gets worse and worse as it proceeds. But more positively: stochastic cooling is very efficient for hot beams like antiprotons from a conversion target or ions from a large emittance source, and electron cooling is well suited to (post)-freezing and/or conditioning beams which are already cool. The combination of both techniques¹⁶⁾ will be used in the low-energy antiproton ring (LEAR), thus profiting from the complementarity of the two cooling techniques.

4. APPLICATION OF PHASE-SPACE COOLING

4.1 Accumulation of rare particles

In principle, all long-lived charged reaction products can be injected into a storage ring, where they can be cooled and accumulated.

One might consider collecting the rare fractions of highly stripped heavy ions which emerge from a stripper, or accumulating polarized reaction products, or rare isotopes, or stable isomers, which emerge from a reaction target. Cooling then allows preparing perfect-quality beams of these particles for new experimental applications. The particles can be used inside the storage ring on internal targets (Section 4.3), in beam-beam interactions (Section 4.4), or on external targets after fast or slow extraction. For a good duty cycle one needs extraction times longer than the accumulation time. Stochastic extraction¹⁷⁾ will be a means of obtaining extremely long spill-out times.

In the CERN Antiproton Accumulator^{9,11)} (AA) the preparation of a high-density beam of rare particles by cooling is realized on a large scale. Stochastic cooling is used to achieve stacks of several 10^{11} antiprotons from batches of 10^7 particles. These batches emerge from a conversion target where antiprotons are produced from 26 GeV protons impinging on it. Schematically the aperture of the AA vacuum chamber can be subdivided into the precooling region, which houses the fresh antiproton batch, and the stacking region, where antiprotons are accumulated.

A fast precooling system reduces the momentum spread of the injected pulse from 1.5 to 0.25% in 2.5 s before a new pulse arrives. Stochastic cooling is well adapted because the antiproton production rate is relatively small and the beam as taken from the target has extremely large emittances.

The cooled pulse is transferred into the stack region, where additional cooling systems continuously make phase space available. Momentum ($\Delta p/p$) phase space is used for stacking. At the same time, horizontal and vertical cooling (with time constants of the order of 30 min) slowly improve the transverse density as required to obtain high luminosity beams. For proton-antiproton operation of the CERN SPS, accumulation takes ~ 24 hours, and during this time a factor of about 10^4 in antiproton intensity and a factor of about 10^8 in over-all phase-space density is gained in the AA by cooling. This large improvement makes antiproton physics possible with beams which have only one order of magnitude less *circulating* beam intensity (5×10^{11} particles circulating) than SPS proton beams but similar density. The repetition rate is of course much lower than for proton beams but this can be partly made up for by doing experiments with two circulating colliding beams or a circulating beam and an internal target.

4.2 Compensation of emittance increase

In LEAR¹⁶⁾, now in its running-in phase, at CERN, stochastic cooling will be used to compensate for the adiabatic increase of emittances. This increase occurs when a particle beam is decelerated. As is well known, the product of momentum with horizontal (E_H), vertical (E_V), and longitudinal ($E_L = \Delta p/p$ bunch length) emittance

is an invariant of (ideal) particle beams as long as no cooling can be applied:

$$E_H p = \text{const} , \quad E_V p = \text{const} , \quad E_L p = \text{const} .$$

Cooling of the three emittances prior to and/or during deceleration makes it possible to reduce these invariants, and this permits slowing down the beam to small momentum without loss. Slowing down for experiments at low energy is used in LEAR because in our world antiprotons are produced in sufficient number only at relatively high energy (~ 3 GeV if 26 GeV primary protons are converted). By collecting antiprotons at 3 GeV and decelerating them rather than taking low-energy particles from the target, 4 to 7 orders of magnitude are gained in antiproton intensity.

Apart from the adiabatic emittance increase, other heating mechanisms which are governed by their proper time constants tend to spoil the quality of stored beams. Amongst them are:

- *Diffusion* due to betatron resonances driven by small errors of the electromagnetic guide fields of the ring (high-order resonances) together with various sorts of field ripple.
- *Scattering* on the residual gas. Pile-up effects of small-angle Coulomb scattering increase the beam emittance with a time constant τ_{ms} . Single scattering with angles larger than the acceptance causes losses with a typical time τ_{ss} . Both τ_{ms} and τ_{ss} are inversely proportional to the residual vacuum pressure P , and they depend strongly on the particle velocity β and the Lorentz factor γ :

$$\tau_{ms}, \tau_{ss} \propto \frac{\beta^3 \gamma^2}{P} . \quad (16)$$

- *Beam-beam interaction* via electromagnetic fields of a counter-rotating colliding beam.
- *Beam instabilities* due to the wake field of the particles in the beam environment.

Such mechanisms can be compensated if the cooling rate exceeds the blow-up rate. A cooling system with a strong damping rate $1/\tau_c = (1/E)(dE/dt)$ will lead to equilibrium values of emittance E_{eq} and momentum spread $(\Delta p/p)_{eq}$

$$E_{eq} = \tau_c \left. \frac{dE}{dt} \right|_{\text{blow-up}} , \quad \left(\frac{\Delta p}{p} \right)_{eq} = \tau_c \left. \frac{d(\Delta p/p)}{dt} \right|_{\text{blow-up}} . \quad (17)$$

They are small compared to the acceptance if τ_c is short enough. The remaining losses are then due to single scattering with angles or momentum deviations larger than the acceptance. Since large deviations occur with very small probability, cooling in all three directions is therefore expected to increase dramatically the lifetime of a circulating beam up to $\lesssim \tau_{ss}$. This effect was clearly observed in the cooling experiments at CERN.

4.3 Internal targets

The advantages of an internal target in conjunction with phase-space cooling have been recognized by Skrinsky et al.^{3,7)}. For LEAR, internal targetry has been promoted since the very beginning and the (machine-dependent) balance between cooling and single as well as multiple scattering on the target has been worked out¹⁸⁾. LEAR has been designed to include three possible locations for the addition of gas targets. Recently, this idea has been taken up by Pollock et al.¹⁹⁾, who are working on a project based on the use of gaseous targets in an ion storage ring with electron cooling for nuclear physics.

The preferred operation of an internal target will be a cyclic mode with (fast) filling and (slow) depletion of the stack by interactions on the target. The interaction time constant is given by

$$\tau_i = (v t \sigma)^{-1}, \quad (18)$$

where v is the revolution frequency of the stack, t the target thickness (particles per cm^2), and σ the total effective cross-section. The time τ_i should be larger than the cooling time constant τ_c if one wants to ensure that cooling makes a strong improvement in emittance and resolution. On the other hand a large reaction rate r requires a short interaction time constant τ_i , since r depends on τ_i and on the number n of stored particles:

$$r = n \frac{1}{\tau_i} = n t v \sigma. \quad (19)$$

So one has to balance reaction rate (statistical precision) against angular and energy resolution. Both can be improved simultaneously with increasing cooling speed.

Maximal count rate with optimal duty cycle (~ 1) is reached when r equals the available average particle flux. Equation (19) shows that this defines an upper limit for $\{nt\}$. At LEAR, where about 10^6 $\bar{p}$ /s are available on the average, we get at ~ 800 MeV/c where $\sigma \approx 150$ mb with $n = 10^9$ $\bar{p}$ stored: $\{nt\} \approx \{10^9 \times 5 \times 10^{-9} \text{ g/cm}^2\}$. This tells us that it does not make sense to have internal proton targets thicker than $5 \times 10^{-9} \text{ g/cm}^2$ in the case of LEAR.

Important properties and advantages of internal targets are:

a) Recirculation of all particles occurs with the high revolution frequency v ($\geq 10^6 \text{ s}^{-1}$) except for those which undergo the wanted reactions or scattering by an angle larger than θ_{ap} , the acceptance limit of the machine (see Fig. 4).

The fast recirculation leads to high luminosity and allows economic use of rare particles. The efficiency for hadronic interaction is

$$\epsilon \approx \frac{\sigma_h}{\sigma_h + \Delta\sigma_{Cb}} = \frac{\sigma_h}{\sigma}. \quad (20)$$

Fig. 4 Schematic view of components relevant for internal gas target operation. Particles which do not interact in the target can be recycled if they are scattered by less than the acceptance angle θ_{ap} . The energy loss ΔE in the target and single scattering below θ_{ap} (and therefore also multiple scattering) are compensated by a phase-space cooling system.

Here $\sigma = \sigma_h + \Delta\sigma_{Cb}$ is the above-mentioned effective total cross-section; σ_h is the total hadronic cross-section and $\Delta\sigma_{Cb}$ is the Coulomb cross-section for scattering with angles larger than the acceptance angle θ_{ap}

$$\Delta\sigma_{Cb} = \int_{\theta_{ap}}^{180^\circ} \frac{d\sigma_{Cb}}{d\Omega} d\Omega \approx 261 \text{ mb} \left(\frac{Z_{beam} Z_{target}}{p^* \beta^* \theta^*} \right)^2. \quad (21)$$

The star indicates c.m. quantities. In Fig. 5 we compare efficiencies for internal and external target operation for $p+p$. If high resolution is needed then external targets need to be thin and they become very inefficient as shown in Fig. 5.

b) Very good angular and energy definition is possible, limited only by the equilibrium between cooling strength and blow-up effects (mostly) on the target.

With electron cooling at LEAR one hopes to reach a momentum spread $\Delta p/p < 10^{-4}$. The corresponding energy resolution in the c.m. system $\delta\sqrt{s} = \delta p_{lab} m_{target} / \sqrt{s}$ is shown for $p+p$ in Fig. 6 and compared with external target operation.

In Fig. 7 we show as an example the equilibrium emittance expected for antiprotons in LEAR with a proton gas jet target which leads to 10^6 interactions per second with 10^9 stored antiprotons. Figure 7 gives an indication of the possible angular resolution $\delta\theta \approx E/\Delta x$, where Δx is the dimension of the beam-target overlap (typical < 1 cm).

Very good energy resolution is needed if one wants to find narrow structures in excitation functions, e.g. in intermediate nuclear compound systems. It may also be desirable in studies of spontaneous positron emission in heavy-ion interactions. Energy and angular resolution are crucial if one plans to do precise measurements close to thresholds²⁰).

Fig. 5 Target efficiencies for proton proton interactions: a) on an internal hydrogen gas target with perfect phase-space cooling. The resolution here is defined by the cooling. b) On an external hydrogen transmission target with a thickness chosen to give a certain resolution $\Delta\sqrt{s}$.

Fig. 7 Equilibrium beam size (horizontal).

Fig. 6 The c.m. energy resolution $\Delta\sqrt{s}$ as a function of the beam momentum in pp interactions for constant momentum spread $\delta p = 0.6$ MeV/c and $\delta p = 0.06$ MeV/c and for liquid hydrogen targets of 1 mm and 10 mm thickness.

It is worth noting that the quality of the circulating beam can easily be improved at the expense of "beam particle economy" [Eq. (20)] if one reduces artificially the machine acceptance angle θ_{ap} . This increases $\Delta\sigma_{Cb}$ and reduces the interaction time constant τ_i . This poses no problems if one has a fast enough cycling source (like a cyclotron). The advantage lies in the fact that large deviations in momentum and angle are cut out so that the beam temperature [θ in Eq. (1)] stays smaller and electron cooling becomes *faster*.

c) There are no multiple interactions in the extremely thin target. Hence all kinematical correlations, even for very heavy and/or slow recoil particles, are undisturbed. This is, of course, attractive for coincidence measurements, especially in heavy-ion interactions, where energy loss and angular straggling impose very stringent limits. It might also have applications for atomic physics because, for example, δ -electrons or Auger electrons can be seen undisturbed.

d) Full advantage can be taken of the high density of cooled circulating beams. This is because extraction and beam transport to an external target -- processes both liable to mis-steering and jitter -- are not needed.

e) A polarized hydrogen or deuterium atomic beam can be used²¹⁾ as a target with a high degree of polarization and virtually free from carbon and other contaminants. It still gives excellent luminosity in the recirculated beam if it reaches a (realistic) thickness of $\sim 10^{-12}$ g/cm².

f) Thin filaments ($\emptyset \lesssim 1 \mu\text{m}$) can be employed as internal targets²²⁾. Their apparent thickness is reduced to reasonable values owing to the small overlap with the beam. Their advantage is simplicity. Furthermore, they have extremely high brilliance for the reaction products, a very attractive feature for a subsequent magnetic spectrometer.

g) There is virtually no beam dump. Therefore the background conditions can be very clean. One has to worry only if $\Delta\sigma_{Cb}$ is large compared to σ_h . But also under these conditions the (uninteresting) Coulomb forward scattered particles can be absorbed on an aperture limiter, which is installed in the ring far away from the detectors.

4.4 Colliding beams

Hadronic interactions in colliding beams are under study so far only at CERN in the Intersecting Storage Rings and in the SPS $p\bar{p}$ collider. There the beam energies are so high (> 10 GeV) that Coulomb scattering is not important. In the low-energy range blow-up effects become crucial [Eq. (16)]. Here collider operation with protons and heavier ions will be feasible only if sufficient cooling can be applied.

The luminosity L in a collider with coasting co-rotating or counter-rotating (0° crossing angle) beams is

$$L = \frac{1}{4} \frac{N_1 N_2}{U_1 U_2} \frac{A_{\min}}{A_1 A_2} \ell c |\beta_1 \mp \beta_2| \quad (22)$$

Here ℓ is the length of the interaction region. $U_{1,2}$ are the circumferences, $N_{1,2}$ the number of particles stored, and $A_{1,2}$ the beam cross-sections for the storage rings 1 and 2, respectively. $A_{\min}$ is the overlap area of both beams. Ideally $A_1 \approx A_2 \approx A_{\min}$. These beam areas and therefore the luminosity are constrained by disruptive effects of the space-charge field of one beam on the other²³⁾. Phase-space cooling will counteract this effect to some (so far not tested) extent. Furthermore $c\beta_{1,2}$ are the velocities of the stored particles. The + and - signs in Eq. (22) apply for counter-rotating and co-rotating beams, respectively.

With heavy ions losses will occur also owing to stripping or pick-up of electrons. This problem can be reduced when using fully stripped ions and/or machines with very high momentum acceptance Δp . Since $\Delta p/p \approx \Delta Z/Z$ one can store several ionization states which differ by $\sim \Delta Z$. Machines for this purpose have been frequently discussed²⁴⁾.

If one foresees a storage ring with non-dispersive straight sections one can install there electron cooling, which equalizes the velocities of all particles, independently of their charge states. In the dispersive regions, where different charge states have separated orbits, stochastic cooling could be applied on these orbits individually. Clearly when working with partially stripped ions one has to foresee also dispersion-free interaction regions. One may consider the AA ring at CERN⁹⁾ as an example where these criteria (e.g. $\Delta p/p \approx 6\%$) are realized.

Finally, we wish to give some further examples of possible future applications of cooling, which are however quite exotic.

4.4.1 Experiments with ultimate resolution

A double-ring system with counter-rotating or co-rotating beams allows the study of a large variety of ion-ion interactions in a wide energy range. If one works with fully stripped ions, and only then, one can obtain optimal energy resolution. Only under these conditions can one avoid the degradation of resolution caused otherwise by ejection of δ -electrons (Landau tails) or by inelastic excitations of electronic states. The resolution of the invariant mass $\sqrt{s}$ in interactions of fully stripped beams depends on the momentum spread of the cooled beams

$$\begin{aligned} \partial \sqrt{s} &= \partial p_1 \frac{(E_2/E_1)p_1 - \cos \Theta_{12} p_2}{\sqrt{s}} \\ &= \partial p_1 \frac{E_2(\beta_1 - \cos \Theta_{12} \beta_2)}{\sqrt{s}} \end{aligned} \quad (23)$$

We see from Eqs. (23) that for ultimate resolution the interaction angle θ_{12} also has to be well defined by cooling.

4.4.2 Studies of charge exchange in collisions of two ions

A double-ring system as in Fig. 8 with two partially stripped ion beams (different in charge and/or mass) may be tuned to have nearly the same ion velocities $\beta_1 \approx \beta_2$. The excitation energy of the compound system can be made so small that it comes into the range of typical electronic excitation energies. Equation (23) shows that the energy resolution also gets better with $(\beta_1 - \beta_2)$. One can study excitation functions of atomic charge-exchange processes. The new charge states appear in well-defined directions, where they can be easily measured even in coincidence. In order to illustrate the energy resolution, consider a hypothetical case where, for example, Ca^{20+} and Ca^{10+} beams are stored with $\beta = 0.1$ (5 MeV/A) and $\beta = 0.1 (1 + 3 \times 10^{-3})$. This corresponds to an excitation energy of about 1 keV. Assuming a momentum resolution of $\Delta p/p \approx 10^{-5}$ one would obtain in this case a centre-of-mass resolution of 60 eV.

Fig. 8 Schematic view of the double ring system (for $\bar{d}$ production, see Ref. 25). The rings 1 and 2 overlap in a common straight section. Here $\bar{d}$ are produced and stored in ring 1.

4.4.3 Accumulation of reaction products

In a double-ring collider (Fig. 8) a given collision energy can be realized with different momentum combinations p_1 and p_2 of the stored beams. What changes is the centre-of-mass velocity of the compound system. For certain conditions one can arrange for two-body reactions of the type $m_1 + m_2 \rightarrow m_3 + m_4$ that the momentum vectors in the laboratory system are equal: $\vec{p}_1 \approx \vec{p}_3$ and $\vec{p}_2 \approx \vec{p}_4$ (25). In the case of heavy ions one can arrange that $\vec{p}_1/Z_1 = \vec{p}_3/Z_3$ or $\vec{p}_2/Z_2 = \vec{p}_4/Z_4$, where Z denotes ionization

states. The consequence is that for such settings a fraction of the reaction products 3 (and) or 4 is permanently accepted in ring 1 (and) or 2. The new particles have the correct magnetic rigidity but they can be distinguished by different revolution frequencies from their parent particles 1 and 2.

One might think of "breeding" and accumulating rare isotopes, etc. This trick has been discussed in Ref. 25 for the production of antideuterons in the reaction $\bar{p}p \rightarrow \bar{d}\pi^-$. The limits of this method come with losses induced by elastic forward Coulomb scattering.

4.4.4 Production of fully or highly stripped heavy ions

For phase-space cooling, a stored beam of (partially) stripped heavy ions overlaps with an intense electron beam of nearly the same velocity. One can easily increase the relative electron-ion energy. Instead of pick-up we expect then knockout of electrons from the ions, as long as enough energy is available in the collision. What counts for the maximal obtainable ionization state is the velocity difference between the ion and the stripper electron.

In order to get fully stripped Xe^{54+} (U^{92+}) on a fixed stripper an ion beam with $> 80 \text{ MeV/A}$ ($> 250 \text{ MeV/A}$) would be needed. An electron beam with an energy of only $> 40 \text{ keV}$ ($> 125 \text{ keV}$) overlapping with a stored slow ion beam can, in principle, do the same.

The heavy ions could be kept at a convenient energy in a storage ring. In a non-dispersive section the electron beam can overlap with all charge states. Large $\Delta p/p \approx \Delta Z/Z$ would allow simultaneous storage of all charge states which are produced. Injection and extraction is on separate orbits in dispersive regions. It could be done in a pulsed or continuous way. Electron cooling could be performed in a second non-dispersive straight section or stochastic cooling could be applied to the separated charge states in the dispersive regions. Cooling is needed to avoid blow-up and to provide sufficient storage time to reach large ionization. As in electron beam ionization sources (EBIS)²⁶⁾ also in our example we have to worry about the time constants with which a given charge state is populated. For the production of an ion with charge $i+1$ from ionization state i the time $\tau_{i \rightarrow i+1}$ is determined by the ionization cross-section $\sigma_{i \rightarrow i+1}$, the current density of the electron beam j , and the fraction η of the storage ring which is filled with the ionizing electron beam

$$\tau_{i \rightarrow i+1} = (\sigma_{i \rightarrow i+1} j \eta)^{-1}.$$

Table 4 gives order-of-magnitude information for time constants for the increase of ionization of Xe and U ions. In Table 4 we assume $\eta = 3\%$, $j = 16 \text{ A/cm}^2 \approx 10^{20} \text{ electrons/cm}^2\text{s}$ and for $\sigma_{i \rightarrow i+1}$ we use a conservative approximation, $\sigma_{i \rightarrow i+1} \gtrsim 2 \times 10^{-14} \text{ cm}^2$

Table 4

Order of magnitude of the time constant for ionization of inner-shell electrons in Xe and U ions by an electron current density of 16 A/cm^2

Ion	$i \rightarrow i+1$	$\tau_{i \rightarrow i+1}$ (s)	E_i (keV)	$\sigma_{i \rightarrow i+1}$ (cm^2)
Xe	50 $\rightarrow$ 51	250	5.5	1.35×10^{-21}
	51 $\rightarrow$ 52	480	5.5	6.94×10^{-22}
	52 $\rightarrow$ 53	10^4	34.6	3.4×10^{-23}
	53 $\rightarrow$ 54	2×10^4	34.6	1.7×10^{-23}
U	88 $\rightarrow$ 89	3.8×10^3	21.8	8.7×10^{-23}
	89 $\rightarrow$ 90	7.7×10^3	21.8	4.5×10^{-23}
	90 $\rightarrow$ 91	1.1×10^5	115.6	3×10^{-24}
	91 $\rightarrow$ 92	2.2×10^5	115.6	1.5×10^{-24}

$1/(E_i/\text{eV})^2$, which holds when the electron energy $E_e \approx 2E_i$ to $12E_i$ (see Ref. 27). The separation energies E_i are taken from Ref. 28.

If we have n heavy ions of a charge state i stored, then we can get a continuous flux of charge state $i+1$ of the order $I_{i+1} \sim n/\tau_{i \rightarrow i+1}$. For example, with 10^6 Xe^{53+} (U^{91+}) stored we can get 50 Xe^{54+} (4.6 U^{92+}) per second. These higher charge states can either be cooled and then extracted or they can be accumulated for internal use in the machine or for further ionization. In order to reach fully stripped ion states one may consider a cascade of stripper rings each for a certain range of ionization ΔZ .

High-quality beams of highly or fully stripped heavy ions can be efficiently handled in a given accelerator structure; such beams could have many applications, for example:

- in atomic physics studies, where the high-quality beams could be decelerated without losses to very low energy.
- in studies of spontaneous positron emission from scattering of very heavy ions, where well-defined kinematical parameters and K-shell vacancies already in the beam particles would be an advantage.

5. CONCLUSIONS

We have discussed some properties and limitations of the new techniques of phase-space cooling relevant in nuclear and heavy-ion physics. Many applications seem to

be possible in the future. We have tried to indicate some of them. But in order to take full advantage of cooled beams, detectors and experimental techniques will have to be adapted. Phase-space cooling is already applied on a large scale in the anti-proton projects at CERN.

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