78-5-51

Ref.TH.2485-CERN

POINT-SPLITTING DERIVATION OF ANOMALOUS DIVERGENCE

FOR THE SPINOR CURRENT

H. Inagaki
CERN -- Geneva

A B S T R A C T

The anomaly for the divergence of the spinor current in the supersymmetric Yang-Mills theory is obtained by the point-splitting regularization method. The results agree with those of the lowest order of perturbation theory. The anomaly is removed by redefinition of the gauge invariant conserved current.

Ref.TH.2485-CERN

5 April 1978

12

12

12

•

•

•

•

Recently the anomaly of spinor current in a global supersymmetric Yang-Mills theory has been studied in the lowest order of perturbation theory ^{1),2)}. The action of the Lagrangian

$$\mathcal{L} = -\frac{1}{4} \text{tr} F_{\mu\nu} F_{\mu\nu} + \frac{i}{2} \bar{\psi} \gamma_\mu D_\mu \psi \quad (1)$$

is invariant under the global supersymmetric transformation ³⁾ that is generated by the Noether current

$$J_\mu(x) = \text{tr} F_{\rho\nu}(x) \tilde{\sigma}_{\rho\nu} \gamma_\mu \psi(x) \quad (2)$$

This spinor current is conserved in the classical level but in fact has been shown to have the anomaly in the one-loop level of the perturbation theory,

$$\partial_\mu J_\mu = -\frac{3g^2}{8\pi^2} \text{tr} F_{\rho\nu} \gamma_\rho D_\nu \psi \quad (3)$$

for the case of the SU(2) gauge group. It has also been emphasized ²⁾ that the gauge invariance of the whole calculation is crucial to get the anomaly ³⁾. We shall show in this note that the result (3) can be regained by the point-splitting regularization method ⁴⁾. That method would be useful in future to investigate the topological origin of the anomaly in the supersymmetric gauge theories.

We note first of all that $F_{\rho\nu}(x)$ in Eq. (2) includes the product of the fields $A_\rho(x)$ at the same point. It is also the case even if we split $F_{\rho\nu}(x)$ and $\psi(y)$ into two points for which they stand. This gives rise to the meaningless divergence, and we shall consider in the background approximation to get rid of this difficulty. Let us write the gauge field into two parts,

$$A_\mu = A_\mu^{cl} + A_\mu' \quad , \quad (4)$$

where A_μ^{cl} satisfies the classical equation of motion. We find the equation for the quantum fluctuations A_μ' and ψ ⁵⁾,

$$\begin{cases} \hat{D}^2 A'_\mu - \hat{D}_\mu \hat{D}_\nu A'_\nu + 2ig[A'_\nu, F_{\nu\mu}^a] = ig[\bar{\psi}, \gamma_\nu \psi] \\ i\gamma^\mu (\partial_\mu \psi + ig[A_\mu, \psi]) = 0 \end{cases} \quad (5)$$

(6)

with $\hat{D}_\mu = \partial_\mu + ig[A_\mu^{cl}, \cdot]$. Here the ψ is treated as the quantum fluctuation and the terms more than cube in the bosonic order are neglected. In this case the current (2) is linearized, in which $F_{\rho\nu}$ is given by

$$F_{\rho\nu} = \hat{D}_\rho A'_\nu - \hat{D}_\nu A'_\rho \quad (7)$$

The current (2) is again conserved in this order of approximation by using Eqs. (5) and (6).

For the latter convenience, we shall obtain the Green function for the bosonic quantum fluctuation. From Eq. (5) it should satisfy

$$\hat{D}^2 G_{\mu\rho}(x, y) + 2ig[G_{\nu\rho}(x, y), F_{\nu\mu}^a(x)] = g_{\mu\rho} \delta(x-y), \quad (8)$$

where we have fixed the gauge for the quantum field A_μ^a to be

$$\hat{D}_\nu A'_\nu = 0 \quad (9)$$

From (8), in the order of g , we obtain the Green function which has the following short-distance singularities,

$$\begin{aligned} G_{\mu\nu}^{ab}(x, y) = & g_{\mu\nu} \delta^{ab} D(x-y) + \frac{g}{8\pi^2} \epsilon^{abd} F_{\mu\nu}^{d\ell} \left(\frac{x+y}{2}\right) \ln(x-y)^2 \\ & + \frac{g}{4\pi^2} g_{\mu\nu} \epsilon^{abd} A_\rho^{d\ell} \left(\frac{x+y}{2}\right) \frac{(x-y)_\rho}{(x-y)^2} + g_{\mu\nu} f^{ab}(x, y) \end{aligned} \quad (10)$$

with f being an arbitrary function satisfying

$$\square^x f^{ab}(x, y) = 0 \quad (11)$$

Here $D(x-y)$ is the propagator of massless particles and ϵ^{abc} is the structure function for the $SU(2)$ gauge group. Now we require the manifest gauge covariance. For that purpose we note that the third term of the Green function (10) can be cancelled by the last term. This is always possible from the degree of freedom of gauge transformation for A_μ^{cl} . Thus we get

$$G_{\mu\nu}^{ab}(x, y) = g_{\mu\nu} \delta^{ab} D(x-y) + \frac{g}{8\pi^2} \epsilon^{abd} F_{\mu\nu}^{d, cl}(\frac{x+y}{2}) \ln(x-y)^2 \quad (12)$$

We are in a position to derive the anomaly. Let us define the split gauge invariant current ^{*)}

$$J_\mu(x, y) = T \left\{ \text{tr} F_{\mu\nu}(x) \sigma_{\nu\rho} \gamma_\mu K(x, y) \psi(y) K^\dagger(x, y) \right\} \quad (13)$$

with

$$x_\mu = z_\mu + a \epsilon_\mu, \quad y_\mu = z_\mu - b \epsilon_\mu, \quad a, b > 0, \quad \epsilon_0 > 0,$$

and T being the time-ordered product. Here $K(x, y)$ transforms such as

$$K(x, y) \longrightarrow e^{-i\chi(x)} K(x, y) e^{i\chi(y)} \quad (14)$$

with a gauge function χ , and reduces rapidly to the unit matrix for $x \rightarrow y$. Taking the derivative, we obtain

^{*)} This form is inferred from the supercurrent ⁶⁾, which has the split gauge invariant form

$$V_{\alpha\beta}(x, y, \theta) = \text{tr} \left\{ e^{V(x, \theta)} W_\alpha(x, \theta) K(x, y) e^{-V(y, \theta)} \bar{W}_\beta(y, \theta) K^\dagger(x, y) + (x \leftrightarrow y) \right\}$$

The symmetrization for x and y is needed to get the correct form for the axial current. In the Wess-Zumino gauge, the component for the spinor current gives (13).

$$\begin{aligned} \partial_\mu \bar{J}_\mu(x, y) = T \operatorname{tr} \{ & \partial_\mu \bar{F}_{\rho\nu}(x) \hat{O}_{\rho\nu} \delta_\mu K(x, y) \psi(y) K^\dagger(x, y) \\ & + \bar{F}_{\rho\nu}(x) \hat{O}_{\rho\nu} \delta_\mu K(x, y) \partial_\mu \psi(y) K^\dagger(x, y) + \bar{F}_{\rho\nu}(x) \hat{O}_{\rho\nu} \delta_\mu \partial_\mu K(x, y) \psi(y) K^\dagger(x, y) \\ & + \bar{F}_{\rho\nu}(x) \hat{O}_{\rho\nu} \delta_\mu K(x, y) \psi(y) \partial_\mu K^\dagger(x, y) \} \end{aligned} \quad (15)$$

The third and the last terms drop out when we take the matrix element between the vacuum and one fermion, because $\bar{F}_{\rho\nu}(x)$ includes quantum field $A'_\rho(x)$ linearly as is seen from Eq. (7). The first and the second terms give, with making use of Eq. (6),

$$\langle \partial_\mu \bar{J}_\mu(x, y) \rangle = T \{ \langle \operatorname{tr} D_\mu \bar{F}_{\rho\nu}(x) \hat{O}_{\rho\nu} \delta_\mu \psi(y) \rangle + \langle \operatorname{tr} \bar{F}_{\rho\nu}(x) \hat{O}_{\rho\nu} \delta_\mu (-A'_\mu(x) + A'_\mu(y)) \psi(y) \rangle \} \quad (16)$$

The first term vanishes by using equation of motion and the Fierz re-arrangement. Then we are left with

$$\begin{aligned} \langle \partial_\mu \bar{J}_\mu(x, y) \rangle = & -2g \epsilon^{abc} \hat{O}_{\rho\nu} \delta_\mu \{ \theta(x_0 - y_0) (x - y)_\alpha \partial_\rho^\alpha \partial_\nu^\beta \langle A'_\nu{}^a(x) A'_\mu{}^c(y) \rangle_0 \psi(y) \\ & + (x \leftrightarrow y) \} + O(z^2) \end{aligned} \quad (17)$$

where the Green function of the quantum fluctuation has appeared. Note that the only contributed part of the Green function is antisymmetric for the $SU(2)$ index and has the form

$$\partial_\rho^\alpha \partial_\nu^\beta G_{\nu\rho}{}^{ac}(x, y) = \frac{2}{8\pi^2} \epsilon^{acd} F_{\nu\rho}{}^{d\alpha} \left(z, \frac{1}{(a+b)z} \right) \left\{ -\frac{2g_\mu}{\epsilon^2} + \frac{4g_\mu \epsilon_\mu}{\epsilon^4} \right\} \quad (18)$$

Inserting it in Eq. (17) and averaging, we end up with

$$\lim_{\epsilon \rightarrow 0} \langle \partial_\mu \bar{J}_\mu(z) \rangle = -\frac{3g^2}{8\pi^2} \operatorname{tr} \bar{F}_{\rho\nu}(z) \delta_\rho \partial_\nu \psi(z) + O(z^2) \quad (19)$$

By covariantizing it, we reach the result (3) in this order of approximation.

A few comments are in order. In this regularization method, it is easy to see from Eq. (13) that

$$\delta_\mu \bar{J}_\mu = 0, \quad (20)$$

which is against the result of Ref. 2). However, since the anomaly is written in the form of total divergence, we can construct the gauge invariant conserved current. From Eq. (3) we obtain

$$\partial_\mu \bar{J}_\mu = - \frac{3g^2}{8\pi^2} \partial_\mu (\text{tr } \bar{F}_{\rho\mu} \gamma^\rho \psi) \quad (21)$$

Then we may define the new current

$$\bar{J}_\mu' = \bar{J}_\mu + \frac{3g^2}{8\pi^2} \text{tr } \bar{F}_{\rho\mu} \gamma^\rho \psi \quad (22)$$

which is conserved

$$\partial_\mu \bar{J}_\mu' = 0 \quad (23)$$

and has the value

$$\delta_\mu \bar{J}_\mu' = - \frac{3g^2}{4\pi^2} \bar{F}_{\rho\mu} \sigma_{\rho\mu} \psi \quad (24)$$

Equations (23) and (24) state that the conformal spinor current is anomalous instead of the Poincaré spinor current and, furthermore, the coefficient of this anomaly agrees with that of Ref. 7) ^{*)}. To summarize, we have calculated the anomaly of the current (2) and our results are Eqs. (3) and (20) or, equivalently, are Eqs. (23) and (24).

The method is easily generalized to the curved space, as far as the gravitational field is treated as the background. In this case Dirac matrices in Eq. (2) depend on x . Since the current is a local Lorentz spinor, we must take the covariant derivative

^{*)} The author would like to thank Professor B. Zumino for clarifying this point.

$$D_\mu J^\mu \equiv (\partial_\mu J^\mu + \Gamma_{\mu\nu}^\mu J^\nu + \frac{1}{2} \omega_{\mu ab} \sigma^{ab} J^\mu) ,$$

and it again vanishes in the classical level using covariantized equations of motion. Here $\Gamma_{\mu\nu}^\rho$ is the affine connection and $\omega_{\mu ab}$ is the vierbein connection. However, the similar calculation shown above again leads to the anomaly (3), in which the covariant derivative includes vierbein connection. When the background is replaced by the supergravity ⁸⁾ the covariantized current for (2) in the matter system by itself is no longer conserved, and so the discussion of this kind of anomaly is not valid in this case.

Acknowledgements

The author wants to thank H. Römer, M. Grisaru and S. Ferrara for several discussions.

REFERENCES

- 1) L. Abbott, M. Grisaru and H. Schitzer - Brandeis University Preprint (1977).
- 2) H. Inagaki and Y. Shimiza - Trieste Preprint IC/77/1129 (1977).
- 3) S. Ferrara and B. Zumino - Nuclear Phys. B79, 413 (1974) ;
A. Salam and J. Strathdee - Phys.Letters 51B, 353 (1974) ;
B. de Wit and D. Freedman - Phys.Rev. D12, 2286 (1975).
- 4) In the case of the axial current anomaly, this method was extensively studied by many authors ; for instance, see :
J. Schwinger - Phys.Rev. 82, 664 (1951) ;
N.K. Nielsen, H. Römer and B. Schroer - CERN Preprint TH. 2402 (1977).
- 5) H. Inagaki - Phys.Letters 71B, 403 (1977).
- 6) S. Ferrara and B. Zumino - Nuclear Phys. B87, 207 (1975).
- 7) T. Curtright - Phys.Letters 71B, 185 (1977).
- 8) D.Z. Freedman, P. van Nieuwenhuizen and S. Ferrara - Phys.Rev. D13, 3214 (1976) ;
S. Deser and B. Zumino - Phys.Letters 62B, 335 (1976) ;
S. Ferrara, F. Gliozzi, J. Scherk and P. van Nieuwenhuizen - Nuclear Phys. B117, 333 (1976).