Sweet spot supersymmetry

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Abstract: We find that there is no supersymmetric flavor/CP problem, $\mu$-problem, cosmological moduli/gravitino problem or dimension four/five proton decay problem in a class of supersymmetric theories with $O(1)$ GeV gravitino mass. The cosmic abundance of the non-thermally produced gravitinos naturally explains the dark matter component of the universe. A mild hierarchy between the mass scale of supersymmetric particles and electroweak scale is predicted, consistent with the null result of a search for the Higgs boson at the LEP-II experiments. A relation to the strong CP problem is addressed. We propose a parametrization of the model for the purpose of collider studies. The scalar tau lepton is the next to lightest supersymmetric particle in a theoretically favored region of the parameter space. The lifetime of the scalar tau is of $O(1000)$ seconds with which it is regarded as a charged stable particle in collider experiments. We discuss characteristic signatures and a strategy for confirmation of this class of theories at the LHC experiments.

Keywords: Supersymmetric Standard Model, Supersymmetry Phenomenology, Cosmology of Theories beyond the SM.
1. Introduction

In spontaneously broken supersymmetric theories, there is a spin-half Goldstino fermion which is eaten by the gravitino as its longitudinal components. By supersymmetry, the Goldstino must be accompanied with its superpartner whose spin is zero if supersymmetry is broken by a vacuum expectation value of the $F$-component of a chiral superfield. A chiral supermultiplet is formed by the Goldstino, its scalar superpartner, and the non-vanishing $F$-term, which we call the chiral superfield $S$. The low energy physics is then described by matter superfields, gauge superfields and the chiral superfield $S$.

There are variety of possibilities for couplings between matter/gauge superfields in the supersymmetric standard model and the superfield $S$. These possibilities have been classified as follows. If we assume that the couplings are suppressed by the Planck scale ($M_{Pl}$),
such as \( \mathcal{L} \ni [(S/M_{Pl})W^\alpha W_\alpha]_\theta^2 \) with \( W^\alpha \) being gauge fields, the model is called the “gravity mediation” \(^1\).\(^2\). Another possibility that the gauge kinetic function is of the form, \( \mathcal{L} \ni [(\log S/(4\pi)^2)W^\alpha W_\alpha]_\theta^2 \), is called the “gauge mediation” \(^3\).\(^4\). This is the form we obtain after integrating out vector-like fields which obtain masses proportional to \( \langle S \rangle \).\(^5\)

If the coupling is more suppressed than the Planck scale, effects of the “anomaly mediation” \(^6\) give the largest contribution to supersymmetry breaking terms in the Lagrangian. Among those scenarios, gauge mediation assumes the strongest interaction between the matter/gauge fields and \( S \) while the anomaly mediation effects are the weakest. The size of supersymmetry breaking, \( F_S \), therefore has a relation, \( F_{\text{gauge}} \ll F_{\text{gravity}} \ll F_{\text{anomaly}} \), when we fix the scale of gaugino/sfermion masses. The gravitino masses are \( m_{3/2}^{\text{gauge}} \ll m_{3/2}^{\text{gravity}} \ll m_{3/2}^{\text{anomaly}} \) as \( m_{3/2}^{\text{grav}} \propto F_S \).

The question is what size of the gravitino mass (i.e., the supersymmetry breaking scale) is preferred by phenomenological and cosmological requirements. This is an interesting question since each scenario predicts a quite different pattern in the spectrum of the supersymmetric particles, which we will search for at the LHC experiments. Strategies for finding supersymmetric particles and measurements of model parameters will be also different for different scales of \( m_{3/2} \).

There have been many model-building efforts in making supersymmetric models realistic in each category: gravity, gauge or anomaly mediation. If one of them had been completely successful, we could have believed in the scenario and used the model as the standard supersymmetric model. However, unfortunately, there is no such a standard model so far because of the fact that neither of these scenarios are fully realistic by different reasons. In the gauge and anomaly mediation scenarios, there is a problem with the electroweak symmetry breaking, i.e., the \( \theta \)-problem. The pure anomaly mediation, in addition, predicts tachyonic scalar leptons which are not acceptable. Although the gravity mediation scenario does not suffer from those problems, it has been known that sizes of flavor and CP violation are expected to be too large. There are also cosmological constraints. In particular, in gravity mediation models, a moduli problem caused by fields in the supersymmetry breaking sector destroys cosmological successes of the (supersymmetric) standard model \(^6\), such as the big-bang nucleosynthesis (BBN) and also cold dark matter by thermal-relic neutralinos.

In this paper, we reconsider problems in supersymmetric models by using an effective field theory described by the field \( S \) and the matter/gauge fields. By doing so, we can discuss each of these scenarios as a different choice of functions of \( S \) which define an effective theory. The labeling can be done by projecting the function space onto a one-dimensional axis of the gravitino mass. In this formulation, we find that there is a sweet spot in between the gauge and gravity mediation \( (m_{3/2} \sim O(1) \text{ GeV}) \) where the theory is perfectly consistent with various requirements. All the classic problems, such as the flavor/CP problem and the \( \mu \)-problem are absent. The theory also avoids a cosmological moduli problem caused by the scalar component of \( S \). Non-thermally produced gravitinos through the decay of the \( S \)-condensation naturally account for dark matter of the universe. A simple ultraviolet (UV) completion of the theory exists, which is actually a model of grand unification without neither the doublet-triplet splitting problem nor the proton decay.
problem. Relations to the strong CP problem and the supersymmetric fine-tuning problem are also addressed. We discuss a characteristic spectrum of supersymmetric particles, and demonstrate how we can confirm this scenario.

In the next section, we rewrite the various supersymmetric models in terms of the effective Lagrangian described by the Goldstino multiplet $S$ and particles in the minimal supersymmetric standard model (MSSM). The section includes review of the supersymmetry breaking and its transmission. A concrete set-up of our proposal, Sweet Spot Supersymmetry, is defined in subsection 2.4 and discuss its successes there. We then discuss low energy predictions of the framework in section 3. A parametrization of the model and a way of calculating the spectrum of supersymmetric particles are presented. Collider signatures are discussed in section 4. We demonstrate a method of extracting model parameters in the case where the scalar tau (stau) is the next to lightest supersymmetric particle (NLSP).

2. Theoretical set-up

We construct a phenomenological Lagrangian of the supersymmetric standard model and consider various requirements from particle physics and cosmology. We will arrive at a scenario with $m_{3/2} \sim 1$ GeV.

2.1 $S$ sector

We derive here a description of a supersymmetry breaking sector by the Goldstino chiral superfield $S$. This corresponds to the construction of the Higgs sector in the standard model. As any models of the electroweak symmetry breaking flow into the standard model with various mass ranges of the Higgs boson at low energy, the model below provides a standard low energy description of a variety of supersymmetry breaking models.

We concentrate on $F$-term supersymmetry breaking scenarios as most of the supersymmetry breaking models are of this type. To ensure a non-vanishing vacuum expectation value of the $F$-component of a chiral superfield $S$, we add a source term in the Lagrangian:

$$L \ni m^2 F_S + h.c.$$  \hspace{1cm} (2.1)

This term can be expressed in terms of the superfield as follows:

$$W \ni m^2 S.$$  \hspace{1cm} (2.2)

We can also write down an arbitrary Kähler potential, $K_S$, for the kinetic and interaction terms of $S$. As long as $\partial^2 K_S/(\partial S \partial S^\dagger)$ is a non-singular function, $F_S \neq 0$ is obtained by the equation of motion. For example, the low energy effective theory of the O’Raifeartaigh model [10] has a Kähler potential:

$$K_S = S^\dagger S - \frac{(S^\dagger S)^2}{\Lambda^2},$$  \hspace{1cm} (2.3)

where $\Lambda$ is the mass scale of the massive fields which have been integrated out. In general, if $S$ carries some approximately conserving charge, the Kähler potential is restricted to the
form in eq. (2.3) (up to the sign of the second term).\(^1\) The second term gives a mass to the scalar component of \(S\), \(m_S\):

\[
m_S = \frac{2FS}{\Lambda} = \frac{2m^2}{\Lambda} = 2\sqrt{3}m_{3/2}\left(\frac{M_{Pl}}{\Lambda}\right),
\]

(2.4)

and stabilizes the value of \(S\) at

\[
S = 0.
\]

(2.5)

Here we ignored supergravity effects. The fermionic component of \(S\) remains massless. This is the Goldstino fermion associated with the spontaneous supersymmetry breaking.

Note that the existence of the chiral superfield \(S\) in the above effective theory does not necessarily mean that the supersymmetry breaking sector contains a gauge singlet chiral superfield in the UV theory. The \(S\) field can originate from a component of some multiplets or can be a composite operator in physics above a ‘cut-off’ scale \(\Lambda\). It is totally a general argument that there is a gauge singlet chiral superfield \(S\) in the effective theory below the scale of supersymmetry breaking dynamics, \(\Lambda\), as long as \(\Lambda^2 \gtrsim m^2\).

The Lagrangian discussed above is analogous to the Higgs sector in the standard model. The two parameters \(m^2\) and \(m^2/\Lambda^2(\sim m_{3/2}^2/m^2)\) correspond to the parameters \(v^2\) and \(\lambda_H(\sim m_{3/2}^2/v^2)\) in the Higgs potential, \(V = (\lambda_H/4)(|H|^2 - v^2)^2\). We should not trust this effective theory if \(m^2/\Lambda^2 \gtrsim \sqrt{4\pi}\) as it violates the unitarity of scattering amplitudes of the gravitinos at high energy just like the standard model with \(\lambda_H \gtrsim 4\pi\).

\[\text{2.2 Matter/gauge sector}\]

The superpotential of the MSSM is

\[
W_{\text{MSSM}} = QH_uU + QH_dD + LH_dE + \mu H_uH_d,
\]

(2.6)

where we suppressed the Yukawa coupling constants and flavor indices. The last term, the \(\mu\)-term, is needed to give a mass to the Higgsino, but it should not be too large. For supersymmetry to be a solution to the hierarchy problem, i.e., \(\langle H_u,d\rangle \ll M_{Pl}\), the \(\mu\)-term is necessary to be of the order of the electroweak scale (or scale of the soft supersymmetry breaking terms).

This is called the \(\mu\)-problem. The fact that \(\mu\) is much smaller than the Planck scale suggests that the combination of \(H_uH_d\) carries some approximately conserving charge.

There are many gauge invariant operators we can write down in addition to the above superpotential such as

\[
W_R = UDD + LLE + QLD,
\]

(2.7)

and

\[
W_{\text{dim.5}} = QQQL + UDE.
\]

(2.8)

\(^1\)In fact, the cubic term in the Kähler potential, \(K \ni S^1S^2 + \text{h.c.}\) can be eliminated by the shift of \(S\) in general. However, once we take into account interaction terms between \(S\) and the MSSM fields, the origin of \(S\) has a definite meaning and we cannot shift away the cubic term.
These are unwanted operators as they cause too rapid proton decays.

The $\mu$-problem and the proton decay problem above are actually related, and there is a simple solution to both problems. The Peccei-Quinn (PQ) symmetry with the following charge assignment avoids too large $\mu$-term and the proton decay operators.

\[
PQ(Q) = PQ(U) = PQ(D) = PQ(L) = PQ(E) = -\frac{1}{2},
\]
\[
PQ(H_u) = PQ(H_d) = 1.
\] (2.9)

This symmetry is broken explicitly by the $\mu$-term, $PQ(\mu) = -2$. Since it is a small breaking of the PQ symmetry, the coefficients of the dimension five operators are sufficiently suppressed. The unbroken $Z_4$ symmetry, which includes the $R$-parity as a subgroup, still forbids the superpotential terms in eq. (2.7) and ensures the stability of the lightest supersymmetric particle (LSP), leaving us to have a candidate for dark matter of the universe.

The Majorana neutrino mass terms, $W \ni LLH_uH_u$, are forbidden by the PQ symmetry, but large enough coefficients can be obtained by introducing another explicit breaking of the PQ symmetry. For example, we can write down $LLH_uH_u/M_N$ with $PQ(M_N) = 1$ without introducing proton decay operators or too large $\mu$-term. The $Z_4$ symmetry above is broken down to the $R$-parity with this term.

In fact, there is another symmetry which can play the same role as the PQ symmetry, called $R$-symmetry. The charge assignment is

\[
R(Q) = R(U) = R(D) = R(L) = R(E) = 1,
\]
\[
R(H_u) = R(H_d) = 0.
\] (2.10)

Again, $R(\mu) = 2$ explicitly breaks the $R$-symmetry down to the $R$-parity. In this case, the $LLH_uH_u$ term is allowed by the symmetry.

In summary, there are approximate symmetries, $U(1)_{PQ}$ and $U(1)_R$, in the Lagrangian of the MSSM. If one of them is a good (approximate) symmetry of the whole system, it provides us with a solution to the $\mu$ and the proton decay problems.

### 2.3 Interaction to mediate the supersymmetry breaking

Now we discuss interaction terms between the $S$-sector and the MSSM sector. These interactions determine the pattern of supersymmetry breaking parameters which are relevant for low energy physics. We review here three famous mechanisms; gravity, gauge, and anomaly mediation models, as choices of the form of the interactions. Each of these scenarios suffers from different problems. Understanding nature of those problems guides us to a phenomenologically consistent model.

#### 2.3.1 Gravity mediation

The simplest scenario is to assume general interaction terms suppressed by the Planck
scale. This is called the gravity mediation. The Kähler potential is

\[ K^{(\text{matter})}_{\text{gravity}} = -\frac{S^\dagger S}{M_{\text{Pl}}^2} + \left( \frac{S^\dagger \Phi \Phi}{M_{\text{Pl}}} + \text{h.c.} \right) + \cdots \]

(2.11)

and

\[ K^{(\text{Higgs})}_{\text{gravity}} = (H_u H_d + \text{h.c.}) + \left( \frac{S^\dagger H_u H_d}{M_{\text{Pl}}} + \text{h.c.} \right) + \left( \frac{S^\dagger S H_u H_d}{M_{\text{Pl}}^2} + \text{h.c.} \right) - \frac{S^\dagger S (H_u^\dagger H_u + H_d^\dagger H_d)}{M_{\text{Pl}}^2} + \cdots \]

(2.12)

where \( \Phi \) represents the quark and lepton superfields in the MSSM, and we omit \( O(1) \) coefficients.

Planck suppressed operators in the gauge kinetic function generate gaugino masses:

\[ f_{\text{gravity}} = \left( \frac{1}{g^2} + \frac{S}{M_{\text{Pl}}} \right) W^\alpha W_\alpha, \]

(2.13)

where \( g \) is the gauge coupling constant.

The first term in eq. (2.11) and the second term in the bracket in eq. (2.13) generate sfermion masses and gaugino masses, respectively. Both of them are of \( O(F_S/M_{\text{Pl}}) \sim m_{3/2} \). Therefore, the gravitino mass, \( m_{3/2} \), is \( O(100) \) GeV in this scenario. The \( \mu \)-problem is completely solved in a quite natural way \[1\]. The first and second terms in eq. (2.12) generates \( \theta \sim O(m_{3/2}) \).

This mechanism for the \( \mu \)-term generation is consistent with the discussion in the previous subsection. The \( R \)-symmetry introduced before can be preserved once we assign \( R(S) = 0 \). The term in eq. (2.2) breaks the \( R \)-symmetry by \( R(m^2) = 2 \) at the intermediate scale, \( m^2 = \sqrt{3} m_{3/2} M_{\text{Pl}} \). This is still small enough for the proton decay operators.

On the other hand, the natural solution to the \( \mu \)-problem is not compatible with the PQ symmetry. The term in eq. (2.13) restricts the PQ charge of \( S \) to be vanishing, whereas the term responsible for the \( \mu \)-term generation, \( K \ni S^\dagger H_u H_d \), determines that \( PQ(S) = 2 \). The PQ symmetry must, therefore, be maximally violated. This implies that none of the terms in eq. (2.12) can be forbidden by approximate symmetries of the theory. This fact becomes important in the discussion of the supersymmetric CP problem.

Even though the \( \mu \)-problem is solved perfectly, there are several serious problems in this scenario. Since there is no reason for the alignment of the flavor structure in the first term in eq. (2.11), too large rates for flavor changing processes are predicted. We expect flavor mixings of \( O(1) \) from this form of Lagrangian. Such large mixings are unacceptable unless the sfermion masses are of \( O(10) \) TeV or heavier \[12\]. The CP violating phases in the supersymmetry breaking terms are also expected to be \( O(1) \). In particular, a phase of the combination, \( m_{1/2} \mu (B \mu)^* \), with \( m_{1/2} \) the gaugino mass and \( B \mu \) defined by \( \mathcal{L} \ni B \mu H_u H_d + \text{h.c.} \), cannot be eliminated by field redefinitions. The \( \mu \) and \( B \mu \) terms are generated from the multiple terms in eq. (2.12) with different weights, leading to non-aligned phases generically. With an \( O(1) \) phase for the combination, constraints from the electric dipole moment of electron, for example, push the mass limits of supersymmetric particles to be \( O(10) \) TeV \[12\].
There is another serious problem in cosmology. Due to the terms in eq. (2.13), the scalar component of $S$ cannot carry any (even approximately) conserving charge. In this case, there is a moduli problem \cite{9, 13}. The value of $S$ after the inflation is displaced from the minimum due to the deformation of the $S$ potential during inflation, and at a later time $S$ finds its true minimum and starts coherent oscillation about the true minimum. The energy density of the oscillation then dominates over the universe unless the displacement is much smaller than the Planck scale. The decay of $S$, in turn, either destroys the success of the BBN \cite{9} or overproduce gravitinos \cite{16} depending on the mass range of $m_S$ (see \cite{14, 15} for earlier works). There is no range of $m_S$ which is consistent with the cosmology \cite{16}.

It has been widely accepted that the lightest neutralino accounts for dark matter of the universe in gravity mediation scenarios. Abundance of thermally produced neutralinos can be calculated without information on the detail history of the universe, and we obtain the correct order of magnitude. However, once we take into account the existence of the superpartner of the Goldstino, $S$, which always exists, the successful cosmology is spoiled. We argue that an assumption made in the standard calculation that the universe was normal up to temperatures of $O(100)$ GeV is inconsistent with the structure of underlying models.

2.3.2 Gauge mediation

Some of shortcomings in gravity mediation can be cured in gauge mediation models. We assume in gauge mediation that the size of the supersymmetry breaking, $F_S$ (and therefore $m_{3/2}$), is much smaller than that in gravity mediation. The contributions to the soft supersymmetry breaking terms come from

$$K_{\text{gauge}}^{(\text{matter})} = -\frac{4g^4 N_{\text{mess}}}{(4\pi)^4} C_2(R) (\log |S|)^2 \Phi^\dagger \Phi,$$

$$f_{\text{gauge}} = \frac{1}{2} \left( \frac{1}{g^2} \frac{2N_{\text{mess}}}{(4\pi)^2} \log S \right) W^\alpha W^\alpha,$$  \hspace{1cm} (2.14)  \hspace{1cm} (2.15)

where $C_2(R)$ is the quadratic Casimir factors for fields $\Phi$. These terms are generated by integrating out $N_{\text{mess}}$ numbers of messenger fields, $f$ and $\bar{f}$ in the fundamental representation, which have couplings to $S$ in the superpotential, $W \ni kSff$. Singularities at $S = 0$ indicate that the messenger fields become massless at the point.\footnote{In this discussion, we have defined the origin of $S$ to be the point where the messenger particles become massless. It is not necessarily the same definition in subsection 2.1. We will discuss a whole set-up together with the $S$-sector shortly.} Therefore, the theory makes sense only if the potential of $S$ has a (local) minimum at $S \neq 0$. Low energy parameters depend on the coupling constant $k$ only through a logarithmic function. The dependence is encoded as the messenger scale $M_{\text{mess}} = k(S)$ at which gauge mediation effects appear.

The contributions from gauge mediation are much larger than those from gravity mediation in eqs. (2.11), (2.12), (2.13) provided that the value of $S$ is stabilized at $S \ll M_{\text{Pl}}$. Since there is no flavor dependent terms in eq. (2.14), due to the flavor blindness of the
gauge interactions, constraints from flavor violating processes can be easily satisfied when \( m_{3/2} \lesssim O(1) \) GeV.

Another interesting feature is the enhancement of the \( S \) couplings to the MSSM particles \(^{17}\). The scalar component of \( S \) now has couplings to gauginos, \( \lambda \):

\[
\mathcal{L} \ni \frac{m_{1/2}}{\langle S \rangle} S \lambda \lambda + \text{h.c.},
\]

which can be much larger than the coupling to gravitinos, \( \psi_{3/2} \):

\[
\mathcal{L} \ni \frac{F_{S}}{\Lambda^{2}} S \psi_{3/2} \psi_{3/2} + \text{h.c.},
\]

depending on the value of \( \langle S \rangle \). Therefore, the branching fraction of the \( S \) decay into gravitinos is suppressed and the situation of gravitino overproduction from the \( S \) decay can be ameliorated.

However, unfortunately, by lowering the gravitino mass \( m_{3/2} \), we have lost the natural mechanism for generating a \( \mu \)-term. The contributions from eq. (2.12) to the \( \mu \)-term are too small. It is possible to obtain a correct size of \( \mu \)-term by assuming a direct coupling between \( S \) and Higgs fields such as

\[ W \ni \epsilon S H_{u} H_{d}, \]

with a small coefficient \( \epsilon \). This term, however, predicts too large \( B\mu \) term, \( B\mu/\mu \sim (4\pi)^{2} m_{1/2} \), which is unacceptable from the electroweak symmetry breaking. The conclusion is the same if we try to generate a \( \mu \)-term from Kähler terms, e.g.,

\[ K \ni \frac{1}{(4\pi)^{2}} S \psi_{3/2} \psi_{3/2} + \text{h.c.}, \]

This term can be generated by integrating out messenger fields if there is an interaction between the Higgs and the messenger fields \(^{18,19}\). Although this term induces the correct size of \( \mu \)-term, \( \mu \sim m_{1/2}, B\mu/\mu \) is again larger than \( m_{1/2} \) by a one-loop factor.

The \( \mu \)-problem in gauge mediation models cannot be solved by going to the next to minimal supersymmetric standard model (NMSSM). The correct electroweak symmetry breaking is not achieved without further extensions of the model \(^{4,20}\).

We cannot discuss the supersymmetric CP problem without specifying the mechanism for \( \mu \)-term generation because the physical phase \( \arg(m_{1/2} \mu (B\mu)^{\ast}) \) is not determined.

\(^{3}\)There is a logical possibility of generating terms like

\[
K \ni \frac{1}{(4\pi)^{2}} H_{u} H_{d} \log \langle S \rangle + \text{h.c.},
\]

if the Higgs fields have interactions with messenger fields. In this case, the \( B\mu \)-term is not generated, whereas the \( \mu \)-term is generated with the same size as the gaugino masses. This is perfectly consistent with the electroweak symmetry breaking and also the absence of the CP phase in \( m_{1/2} \mu (B\mu)^{\ast} \) as \( B\mu = 0 \). This is the only term which can be written down if \( S \) carries an approximately conserving charge. However, the authors are not aware of an explicit model to realize this situation.
Although it is slightly model dependent, there is another issue in gauge mediation models. In many supersymmetry breaking models, $S$ carries a conserving charge. For example, in the O’Raifeartaigh model, there is an unbroken $R$-symmetry where $S$ carries charge 2. In this case, as discussed in subsection 2.1, the $S$ field is stabilized at $S = 0$ where we cannot integrate out the messenger fields (it is the singular point of the effective Lagrangian). Additional model building efforts to shift the minimum of the $S$ potential have been needed in this type of models.

More explicitly, what we need is to spontaneously or explicitly break the $R$-symmetry in supersymmetry breaking models. If we break it explicitly in the Lagrangian, the theorem of [21] says that a supersymmetric minimum appears somewhere in the field space. Recently, there have been extensive studies on this subject, and many simple models with explicit breaking of $R$-symmetry have been proposed [22 – 25] by allowing a meta-stable supersymmetry breaking vacuum [26]. (See also [27 – 29] for recent models with spontaneous breaking of $R$-symmetry.) An obvious possibility is to add an $R$-breaking cubic term in eq. (2.3) with a small coefficient $\epsilon$,

$$\delta K = \frac{\epsilon}{\Lambda} S^2 \, S^{\dagger} S^{\dagger} + \text{h.c.}$$

(2.20)

This term shifts the minimum of $S$ to $S = \epsilon \Lambda / 2$. This is equivalent to give a small mass term to the messenger fields $W \ni \epsilon \Lambda f \bar{f}$ [24] by a field redefinition $(S - \epsilon \Lambda / 2) \rightarrow S$. A small mass term for $S$, $W \ni \epsilon S^2$ also shifts the minimum of $S$.

In fact, it has been known that these ad hoc deformations of the model were not necessary once we take into account supergravity effects [22]. The gravity mediation effects generate a linear term of $S$ in the potential,

$$V \ni 2 m_{3/2} m^2 S + \text{h.c.}$$

(2.21)

This is a soft supersymmetry breaking term associated with the linear term in the superpotential in eq. (2.2). By balancing with the mass term, $V \ni m_S^2 |S|^2$ with $m_S$ in eq. (2.4), we obtain

$$\langle S \rangle = \sqrt{\frac{3}{6}} \frac{\Lambda^2}{M_{Pl}}.$$  

(2.22)

This shift is due to the fact that $R$-symmetry must be broken explicitly in the supergravity Lagrangian (by the constant term in the superpotential) in order to cancel the cosmological constant [13]. By taking a large $\Lambda$, the shift can be arbitrarily large. Note here that the shift is not suppressed by the gravitino mass which characterizes the effects of gravity mediation. This phenomenon has been known as the tadpole problem for singlet fields [2, 31, 31]. Small soft supersymmetry breaking terms destabilize the hierarchy if there is a gauge singlet field. However, this is not a problem at all for the field $S$ and it is even better in gauge mediation to have a large enough vacuum expectation value of $S$. Since this effect always exists, it is the most economical way of having $S \neq 0$.

[4] The origin of $S$ is now uniquely determined once we assign a charge to $S$.  

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2.3.3 Anomaly mediation

If there is no direct coupling between $S$ and the MSSM particles even including Planck scale suppressed operators, the leading contribution to the sfermion/gaugino masses comes from anomaly mediation effects:

$$m_{1/2} = \frac{g^2 b}{(4\pi)^2} m_{3/2},$$

$$m_{\text{scalar}}^2 = \frac{1}{2} \frac{d\gamma}{d\log \mu_R} m_{3/2}^2,$$

where $b$ and $\gamma$ are the beta function coefficient and the anomalous dimension, respectively, and $\mu_R$ is the renormalization scale [8]. For having $m_{1/2} = O(100) \text{ GeV}$, a large gravitino mass $m_{3/2} \sim 10 - 100 \text{ TeV}$ is needed. There are several good features of this scenario. Because of flavor blindness of the mediation mechanism, there is no supersymmetric flavor problem. The large value of $m_{3/2}$ enhances the decay rate of the gravitino. This makes the gravitino cosmologically harmless as it decays before the BBN starts. The cosmological moduli problem is also absent. The $S$ field can have any conserving charges, and thus it is reasonable to assume that $S$ has stayed at the symmetry enhanced point, $S = 0$, during and after inflation so that there is no large initial amplitude.

Unfortunately, the minimal model is inconsistent with the observation. The scalar leptons have tachyonic masses which would cause a spontaneous breaking of $U(1)_{\text{em}}$ and makes the photon massive. Therefore, we need a modification of the model.

Also there is a $\mu$-problem which is very similar to the situation in gauge mediation. One can assume small couplings between $S$ and the Higgs fields to give a $\mu$-term, but it causes a too large $B\mu$-term, $B\mu/\mu \sim m_{3/2}$ with $m_{3/2} \sim 10 - 100 \text{ TeV}$ which is unacceptable.

2.4 Sweet spot supersymmetry

We have encountered many problems in gravity, gauge and anomaly mediation models. Those are summarized as follows:

- **Gauge mediation ($m_{3/2} \ll 100 \text{ GeV}$)**
  Problems: $\mu$, (CP),

- **Gravity mediation ($m_{3/2} \sim 100 \text{ GeV}$)**
  Problems: Flavor, CP, moduli,

- **Anomaly mediation ($m_{3/2} \sim 10 - 100 \text{ TeV}$)**
  Problems: $\mu$, tachyonic sleptons, (CP).

There have been many attempts to circumvent these problems. For example, in ref. [4, 20] it has been proposed to extend a model of gauge mediation to the NMSSM by introducing a new singlet field. However, for the successful electroweak symmetry breaking, further extension of the model were necessary such as introduction of vector-like matters. Similar attempts have been done in ref. [32, 33] in anomaly mediation models. The gaugino mediation [34] is a variance of the gravity mediation and known to be a successful framework for solving the flavor problem. However, since the model relies on the
Figure 1: Schematic picture of mediation mechanisms. Different mechanism works for different values of gravitino masses. A sweet spot exists at $m_{3/2} \sim 1$ GeV where there is no phenomenological or cosmological problem.

$SW^{\alpha}W_{\alpha}$ term for the gaugino masses, the moduli problem and the CP problem remain unsolved. In ref. [35], a mixture of anomaly and gauge mediation is proposed as a solution to the tachyonic slepton problem (see also [26]). The idea is to modify the structure of the anomaly mediation by introducing an additional light degree of freedom, $X$. It is claimed that the $\mu$-problem and the tachyonic slepton problem can be solved by assuming appropriate couplings of $X$ to the messenger and Higgs fields. However, it is unclear whether such a light degree of freedom is consistent with cosmological history.

It is interesting to notice here that gauge and gravity mediation scenarios do not share problems. This fact motivates us to think of theories in between gauge and gravity mediation. The idea is to solve flavor and moduli problem by reducing $m_{3/2}$, and solve the $\mu$-problem in a similar fashion to the gravity mediation models. The CP problem can also be solved because we can have an approximate PQ symmetry to forbid the $B\mu$-term so that $\text{arg}(m_{1/2}\mu(B\mu)^*) = 0$.

The sweet spot exists at $m_{3/2} \sim 1$ GeV (see figure 1). The interaction terms between matter/gauge field and $S$ are the same as those in gauge mediation (eqs. (2.14), (2.15)). For $m_{3/2} \sim 1$ GeV, possible flavor violating contributions from gravity mediation in eq. (2.11) are sufficiently small. The couplings of $S$ to Higgs fields are

$$K^{(\text{Higgs})} = \left( \frac{S^\dagger H_u H_d}{\Lambda} + \text{h.c.} \right) - \frac{S^\dagger S(H_u^\dagger H_u + H_d^\dagger H_d)}{\Lambda^2}. \tag{2.24}$$

Here we replaced the Planck scale in eq. (2.12) with the “cut-off” scale $\Lambda$ introduced in eq. (2.3). The correct size of $\mu$-term is obtained if $\Lambda \sim 10^{16}$ GeV. The form of the Kähler potential is implicitly suggesting that the Higgs fields have some interactions with the supersymmetry breaking sector mediated by particles with masses of $O(\Lambda)$. We also assumed that there is an approximate PQ symmetry discussed in subsection 2.2 with $\text{PQ}(S) = 2$. With the PQ symmetry, we cannot write down any other terms. Since $S$ carries a charge, the Kähler potential for $S$ is restricted to be the form in eq. (2.3). The term in eq. (2.3) represents the explicit but small breaking of the PQ symmetry.

We here summarize the set-up. We consider the effective Lagrangian written in terms
of the Goldstino multiplet $S$ and the MSSM matter/gauge fields:

$$K = S^\dagger S - \frac{c_S S^\dagger S S}{\Lambda^2} + \left( \frac{c_H S^\dagger H_u H_d}{\Lambda} + {\text{h.c.}} \right) - \frac{c_H S^\dagger S (H_u^\dagger H_u + H_d^\dagger H_d)}{\Lambda^2} + \left( 1 - \frac{4g^4 N_{\text{mess}}}{(4\pi)^4} C_2(R)(\log |S|)^2 \right) \Phi^\dagger \Phi,$$

$$W = W_{\text{Yukawa}}(\Phi) + m^2 S + w_0,$$

$$f = \frac{1}{2} \left( \frac{1}{g^2} - \frac{2N_{\text{mess}}}{(4\pi)^2} \log S \right) W^\alpha W_\alpha.$$

The chiral superfield $\Phi$ represents the matter and the Higgs superfields in the MSSM, and $W_{\text{Yukawa}}$ is the Yukawa interaction terms among them. We defined $O(1)$ valued coefficients $c_S$, $c_H$, and $\Lambda$. We normalize the $\Lambda$ parameter so that $c_S = 1$ in the following discussion. The parameters $c_H$ and $\Lambda$ take real values whereas $c_H$ is a complex parameter. We consider the supergravity Lagrangian defined by the above Kähler potential $K$, superpotential $W$, and gauge kinetic function $f$. This is a closed well-defined system. The linear term of $S$ in the superpotential represents the source term for the $F$-component of $S$. The last term in the superpotential, $w_0$, is a constant, $|w_0| \approx m^2 M_{\text{Pl}}/\sqrt{3}$, which is needed to cancel the cosmological constant. The scalar potential has a minimum at $\langle S \rangle \sim \Lambda^2 / M_{\text{Pl}}$ which avoids the singularity at $S = 0$. The Lagrangian possesses softly broken PQ and $R$-symmetries discussed before. Both symmetries are explicitly broken by the $m^2 S$ term and the $w_0$ term in the superpotential, respectively. Since the PQ and $R$-symmetries are anomalous with respect to the standard model gauge interaction, the PQ transformation shifts the $\theta$ angle as we can see in the function $f$. The set-up includes the dynamics of supersymmetry breaking and mediation. By expanding fields from their vacuum expectation values, we can obtain all the mass spectrum and interaction terms.

When we write down the Lagrangian of the standard model we usually include the Higgs potential, $V = \lambda H^\dagger H - v^2$, and the gauge interaction terms of the Higgs boson instead of just giving bare mass terms to the $W$ and $Z$ bosons. Analogous to that, the system above contains dynamics of the supersymmetry breaking and a mechanism of its mediation instead of simply writing down soft supersymmetry breaking terms. In this sense, this way of construction is essential for the model to be called the MSSM in a true meaning.

The effective Lagrangian is defined at the scale where the messenger fields are integrated out. The messenger scale, $k(\langle S \rangle)$, is not necessary to be $O(\langle S \rangle)$. The $k$ parameter originally comes from superpotential terms like, $W \ni k S f \bar{f}$. If the $S$ field is a composite operator above the scale $\Lambda$ as is often the case in dynamical supersymmetry breaking scenarios, the $k$ parameter is suppressed by a factor of $(\Lambda/M_{\text{Pl}})^{d(S)-1}$, where $d(S)$ is the dimension of the operator $S$ above the scale $\Lambda$. Therefore, the size of $k$ depends on the actual mechanism of the supersymmetry breaking.

\footnote{Our construction should not be confused with the spurion method of writing down the soft terms. The field $S$ is a propagating field and obeys the equation of motion.}
We can see very nontrivial consistencies in this simple set-up. First, the $\mu$-term is generated by the Kähler term, $S^1 H_u H_d/\Lambda$:

$$\mu = \frac{c_\mu F_S}{\Lambda} \sim m_{3/2} \left( \frac{M_{\text{Pl}}}{\Lambda} \right). \quad (2.25)$$

With the shift of $\langle S \rangle$ in eq. (2.22), the gaugino masses are

$$m_{1/2} = \frac{g^2}{(4\pi)^2} \frac{F_S}{\langle S \rangle} = \frac{g^2}{(4\pi)^2} \cdot 6m_{3/2} \left( \frac{M_{\text{Pl}}}{\Lambda} \right)^2. \quad (2.26)$$

Here and hereafter, we take a minimal model with $N_{\text{mess}} = 1$. The qualitative discussion does not change for different values of $N_{\text{mess}}$. Similar sizes of scalar masses are obtained from the Kähler terms. Finally, the moduli problem now turns into a mechanism for the production of dark matter. The energy density of the coherent oscillation of $S$ dominates over the universe, and the reheating process by decays of the $S$-condensation later produces gravitinos through a rare decay process $S \to \psi_{3/2} \bar{\psi}_{3/2}$. The amount can be expressed in terms of $m_{3/2}$ and $\Lambda$ [17]:

$$\Omega_{3/2} h^2 = 0.1 \times \left( \frac{m_{3/2}}{500 \text{ MeV}} \right)^{3/2} \left( \frac{\Lambda}{1 \times 10^{16} \text{ GeV}} \right)^{3/2}. \quad (2.27)$$

Here we have assumed that the decay of $S$ into two Higgs bosons, $S \to hh$, is the dominant decay channel. The phenomenological requirements that $\mu \sim m_{1/2} \sim O(100) \text{ GeV}$, and $\Omega_{3/2} h^2 \approx 0.1$ can all be satisfied when $m_{3/2} \sim 1 \text{ GeV}$ and $\Lambda \sim 10^{16} \text{ GeV}$.

We can see the non-trivial success of this framework in figure 2, where we see how $O(1) \text{ GeV}$ gravitino mass is selected. The bands of $100 \text{ GeV} < \tilde{\mu} < 500 \text{ GeV}$, $100 \text{ GeV} < m_{\tilde{B}} < 500 \text{ GeV}$, and $0.08 < \Omega_{3/2} h^2 < 0.12$ are shown, where we defined $\tilde{\mu} \equiv m_{3/2} M_{\text{Pl}}/\Lambda$ and $\Omega_{3/2} h^2$ by eq. (2.27). The Bino mass $m_{\tilde{B}}$ is the mass of the U(1)$_Y$ gaugino. Surprisingly, these three bands meet at $m_{3/2} \sim 1 \text{ GeV}$ and $\Lambda \sim M_{\text{GUT}} \sim 10^{16} \text{ GeV}$.

The fact that $\Lambda$ coincides with the unification scale, $M_{\text{GUT}}$, is also quite interesting. In grand unified theories (GUTs), such as in SU(5) or SO(10) models, we need to introduce colored Higgs fields in order for models to be consistent with gauge invariance. The colored Higgs fields, however, need to get masses through the spontaneous breaking of SU(5) or SO(10). This suggests that the Higgs multiplets have some interactions with the GUT-breaking sector whose typical mass scale is, of course, $M_{\text{GUT}}$. Therefore, it is quite natural to have $M_{\text{GUT}}$ suppressed interactions in the low energy effective theory. The same “cut-off” scale $\Lambda$ for $S$ suggests that the dynamics of GUT breaking is responsible for the supersymmetry breaking as well. The picture of unification of the Higgs sector, the supersymmetry breaking sector and the GUT breaking sector naturally comes out. Although it sounds like a very ambitious attempt to build a realistic model to realize this situation, it is quite possible and even very simple to build such a dream model by using a recent

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6The band of $\Omega_{3/2} h^2$ does not represent the dark matter density once we deviate far from the region of $m_{3/2} \sim 1 \text{ GeV}$. For $m_{\tilde{B}} \ll \tilde{\mu}$ or $m_{\tilde{B}} \gg \tilde{\mu}$, the successful electroweak symmetry breaking cannot be achieved, and we cannot perform a sensible calculation of the $S \to hh$ decay width.
Figure 2: Phenomenologically required values of the Higgsino mass $\mu$ (with an $O(1)$ ambiguity, see text), the Bino mass $m_\tilde{B}$ and the gravitino energy density $\Omega_{3/2}h^2$. These three quantities have different dependencies on parameters $m_{3/2}$ and $\Lambda$. The three bands meet around $m_{3/2} \sim 1$ GeV and $\Lambda \sim \Lambda_{GUT}$. The quantity $\Omega_{3/2}h^2$ is defined in eq. (2.27). It represents the energy density of the non-thermally produced gravitinos through the decays of $S$ if $S \rightarrow hh$ is the dominant decay channel.

This quite simple framework summarized in eq. (2.25), gauge mediation with direct couplings between supersymmetry breaking sector $S$ and the Higgs fields at the GUT scale, solves all the problems we mentioned before. We discuss these one by one here.

**Supersymmetric flavor problem.** The gravity mediated contributions to the sfermion masses squared are of $O(m_{3/2}^2)$. Therefore, the flavor mixing in sfermions are at most of $O(10^{-4})$ level for $m_{3/2} \sim 1$ GeV. For example, the constraints from the $\mu \rightarrow e\gamma$ decay and $\mu \rightarrow e$ conversion process in nuclei put bounds on the mixing to be

$$\langle \delta_{12} \rangle_{LR,RL} \sim \left( \frac{\mu_{LR,RL}}{m_{SUSY}} \right) \left( \frac{m_{\mu} \tan \beta}{m_{SUSY}} \right) \lesssim 10^{-6}, \quad (2.28)$$

where $m_{SUSY}$ is a typical sfermion/gaugino mass scale and $m_{\mu}$ is the muon mass. By using the fact that the value of $\tan \beta (\equiv \langle H_u \rangle/\langle H_d \rangle)$ is predicted to be $O(30 - 40)$ (see discussion in the next section), this bound is marginally satisfied with sfermion masses of $O(100)$ GeV. If gravitational dynamics maximally violates flavor conservation, future or on-going experiments have good chances to see the effects.
The flavor mixings from the high-scale dynamics such as physics at the GUT scale \[40\] and the effect of right-handed neutrinos \[41\] are small as is always the case in gauge mediation.

**Supersymmetric CP problem.** There are two physical phases in the MSSM:

\[
\arg\left(\frac{m_{1/2}}{m_{\text{SUSY}}} \tan \beta \right), \quad \arg\left(\frac{m_{1/2}}{m_{\text{SUSY}}} A^*\right).
\]

(2.29)

From the Kähler term in eq. (2.24), \(A\)- and \(B\)-terms of \(O(m_{3/2})\) are generated, but these will be overwhelmed by one-loop renormalization group (RG) contributions below the messeenger scale. Since the RG contributions are proportional to the gaugino masses, the physical phases above are approximately vanishing.

In fact, the phases of the original \(O(m_{3/2})\) contributions are also aligned with those of gaugino masses. The phases of three complex parameters in the Lagrangian, i.e., \(m^2\), \(c_\mu\), and \(w_0\), can all be taken to be the same by a field redefinition via \(U(1)_R\) and \(U(1)_{\text{PQ}}\) transformations.

Even if there are \(O(1)\) phases in the \(O(m_{3/2})\) contributions, which is possible if the PQ symmetry is maximally violated by operators suppressed by the Planck scale, the above physical phases are of \(O(1\%)\) which again marginally satisfies the experimental constraints. The upper bound on the electric dipole moment of the electron, for example, gives a constraint \[12\]:

\[
\left(\frac{m_3}{m_{\text{SUSY}}}\right) \left(\frac{m_e \tan \beta}{m_{\text{SUSY}}}\right) \lesssim 10^{-7},
\]

(2.30)

where \(m_e\) is the electron mass. The bound corresponds to \(m_{\text{SUSY}} \gtrsim 300\) GeV for \(m_{3/2} \sim 1\) GeV.

**\(\mu\)-problem.** There are three kinds of \(\mu\)-problem in the MSSM, i.e., “Why \(\mu \ll M_{\text{PQ}}\)?”, “Why \(\mu^2 \sim m_{H_u}^2\)?”, and “Why \(\mu \sim m_{1/2}\)?” The second and third ones are related because there is a one-loop correction to the \(m_{H_u}^2\) parameter proportional to \(m_{H_u}^2\).

The first one was answered by the approximate PQ symmetry. The \(\mu\)-term is forbidden by symmetry, but induced by a small explicit breaking term, \(W \ni m_{\text{S}}^2\).

We can naturally obtain the relation, \(\mu^2 \sim m_{H_u}^2\), once we assume the form of the Kähler potential to be the one in eq. (2.24). This is a generalization of the Giudice-Masiero mechanism in gravity mediation \[11\]. The relation is independent of the “cut-off” scale \(\Lambda\). We discuss a possible origin of the Kähler terms later.

The final relation, \(\mu \sim m_{1/2}\), is realized when \(\Lambda \sim M_{\text{GUT}}\) as we can see in figure 2. From eq. (2.24) and (2.26), the relation between \(\bar{\mu}\) and the Bino mass, \(m_{\tilde{B}}\), is

\[
\frac{\bar{\mu}}{m_{\tilde{B}}} = 0.6 \times \left(\frac{\Lambda}{1 \times 10^{16} \text{ GeV}}\right).
\]

(2.31)

Although it is an ‘accident’ to have similar values of \(\mu\) and the gaugino masses, the value we need, \(\Lambda \sim M_{\text{GUT}}\), is motivated by two other independent physics, i.e., grand unification and dark matter of the universe.
Cosmological moduli/gravitino problem. As we have already discussed, the energy density carried by the coherent oscillation of \( S \) would not cause a problem. The decay of \( S \) reheats the temperature of the universe to of \( O(100) \) MeV for \( m_{3/2} \sim 1 \) GeV and \( \Lambda \sim 10^{16} \) GeV. This is high enough for the standard BBN. The non-thermal gravitino production from this decay gives the largest contribution to the matter energy density of the universe. The amount in eq. (2.27) is, amazingly, consistent with the observation.

The baryon asymmetry existed before \( S \) decays is diluted by the entropy production. If we assume the initial amplitude of \( S \) to be of \( O(\Lambda) \), the dilution factor is estimated to be of order \( 10^{-4}(T_R/10^8 \text{ GeV})^{-1} \) with \( T_R \) the reheating temperature after inflation. Therefore, a larger amount of baryon asymmetry is needed to be generated if baryogenesis happened above the temperature of \( O(100) \) MeV.

If the stau is the NLSP as in the case we will study later, staus are also non-thermally produced through the \( S \) decays if it is kinematically allowed. The pair annihilation process reduces the amount but the abundance ends up with of \( O(50) \) times larger than the result of the standard calculation of the thermal relic abundance. There are constraints on the decay of the staus into gravitinos from the BBN. Recent calculations including the catalyzing effects give an upper bound on the life-time of stau to be \( O(1000) \) seconds [42–45]. Although the lifetime is extremely sensitive to the stau mass \((\propto m_\tilde{\tau}^5)\), the typical lifetime with \( m_{3/2} \sim 1 \) GeV is on the border of this constraint. This coincidence may be interesting for the Lithium abundance of the universe [43, 46].

Unwanted axion? It is common in gauge mediation scenarios that there is an approximate U(1) symmetry which is spontaneously broken. Therefore there is a (possibly unwanted) Goldstone boson associated with it [13]. In the scenario we are discussing the vacuum expectation value of \( \langle S \rangle \) breaks the approximate PQ symmetry spontaneously. The axion associated with the symmetry breaking is actually the scalar component of \( S \) itself. The \( S \) scalar has a mass of the order of 100 GeV (see eq. (2.4)) because of the linear term in the superpotential. Interactions between PQ currents and the axion \( S \) are suppressed by the scale of the symmetry breaking \( \langle S \rangle \sim 10^{14} \) GeV. There is no experimental or astrophysical constraint on such a particle. As we discussed above, the \( S \) scalar even plays an essential role in cosmology.

Dimension-four and five proton decay problem. The dimension-four operators which violate the baryon number conservation are forbidden by an unbroken \( \mathbb{Z}_2 \) subgroup of the PQ symmetry. This is identical to the \( R \)-parity.

Dimension five operators, such as \( QQQL \), are allowed to appear at low energy because the PQ symmetry is spontaneously broken. In particular, if there are following terms in the superpotential:

\[
SQQQL, \quad SUUDE, \quad (2.32)
\]

the dangerous terms like \( QQQL \) and \( UUDE \) appear by substituting the vacuum expectation value of \( S \sim \Lambda^2/M_{\text{Pl}} \sim 10^{14} \) GeV. In GUT models, these effective operators can be generated by diagrams with colored-Higgs exchange. In this case, the coefficients of
the above operators will typically be of $O(f_u f_d/M_{GUT}^2)$ where $f_u$ and $f_d$ are the Yukawa coupling constants of up- and down-type quarks. By substituting $\langle S \rangle$, this becomes effectively $QQQL$ or $UUDE$ operators suppressed by $f_u f_d / M_{Pl}$. The prediction to the proton life-time is on the border of the experimental constraints with such coefficients [47].

**UV completion.** The discussion so far is based on the low energy effective theory defined in eq. (2.25). This effective theory is valid up to the messenger scale $k\langle S \rangle$. Although it is not necessary for the discussion of low energy physics to specify UV models, an existence proof of an explicit UV completion supports our ansatz in eq. (2.25).

It is straightforward to UV complete the theory above the messenger scale by simply assuming a presence of messenger particles $f$ and $\bar{f}$ which carry the standard model quantum numbers, and an interaction term $kS\bar{f}f$. The full model is $K \ni f^\dagger f + \bar{f}^\dagger \bar{f}$ and $W \ni kS\bar{f}f$ instead of terms involving $\log S$ in eq. (2.25).

The model with messenger fields now has a supersymmetric and hence stable vacuum at $S = 0$ and $\bar{f} = f = \sqrt{-m^2/k}$. However, as it has been shown in ref. [22], there is a meta-stable minimum at $\langle S \rangle \sim \Lambda^2 / M_{Pl}$ where supersymmetry is broken and messenger fields are massive. The effective theory in eq. (2.25) correctly describes physics around the meta-stable vacuum.

Above the mass scale $\Lambda$, we need a further UV completion. The simplest model is the O’Raifeartaigh model [10]:

$$K = S^\dagger S + X^\dagger X + Y^\dagger Y, \quad (2.33)$$

and

$$W_S = m^2 S + \frac{\kappa}{2} S X^2 + M_{XY} X Y, \quad (2.34)$$

where $\kappa$ and $M_{XY} (\gg m)$ are a coupling constant and a mass for $X$ and $Y$, respectively. There is an approximate PQ symmetry with charges $PQ(X) = -1$ and $PQ(Y) = 1$. By integrating out massive fields $X$ and $Y$, we obtain the Kähler term $-(S^\dagger S)^2 / \Lambda^2$ with

$$\frac{1}{\Lambda^2} = \frac{|\kappa|^4}{12(4\pi)^2} \frac{1}{M_{XY}^2}, \quad (2.35)$$

**Figure 3:** Feynman diagrams to generate higher dimensional operators in a UV model.
at one-loop level (see figure 3). The Higgs fields can directly couple to this system so that we obtain effective operators in eq. (2.23). The terms are generated by introducing following interaction terms in the superpotential:

$$W_{\text{Higgs}} = h H_u \bar{q} X + \bar{h} H_d q X + M_q \bar{q} q,$$

where $h$ and $\bar{h}$ are coupling constants. Again, the PQ symmetry is preserved for $PQ(q) = PQ(\bar{q}) = 0$. The supersymmetry breaking still happens in this extended model. After integrating out $q$ and $\bar{q}$, we obtain the $c_\mu S^\dagger H_u H_d / \Lambda$ term with

$$c_\mu / \Lambda = \kappa^* h \bar{h} (M_{XY}^2 / M_q^2),$$

where

$$f(x) = \frac{1 - x + \log x}{(1 - x)^2}.$$

The term $-c_H S^\dagger S H_u H_d / \Lambda^2$ is also generated with

$$c_H / \Lambda^2 = \frac{|\kappa|^2 |\hbar|^2}{(4\pi)^2 M_q^2} \cdot g \left( \frac{M_{XY}^2}{M_q^2} \right),$$

where

$$g(x) = \frac{-3 + 4x - x^2 - 2\log x}{2(1-x)^3}.$$

These are obtained by calculating Feynman diagrams in figure 3. No other unwanted terms are generated because of the approximate PQ symmetry in the model.

We can obtain the relation $c_\mu \sim c_H \sim 1$ for $M_{XY} \sim M_q$ if the values of $\kappa$, $h$ and $\bar{h}$ are relatively large. In particular, we find

$$\frac{|\mu|^2}{m_{H_u}^2} = \frac{|c_\mu|^2}{c_H^2} = \frac{|\bar{h}|^2}{(4\pi)^2} \cdot \frac{f(x)^2}{g(x)},$$

where $x = M_{XY}^2 / M_q^2$. The $\mu$-term squared is suppressed by a one-loop factor compared to the soft mass term $m_{H_u}^2$ for $M_{XY} \sim M_q$. The function $f(x)^2 / g(x)$ never exceeds $O(1)$ values even for general relations between $M_{XY}$ and $M_q$. To avoid a too large hierarchy, the loop expansion parameter $|\bar{h}|^2 / (4\pi)^2$ should not be too small, i.e., the model should be (semi) strongly coupled. This fact suggests that this O’Raifeartaigh model itself is an effective theory of some dynamical supersymmetry breaking models.

Indeed, there is an incredibly simple dynamical model which provides the above O’Raifeartaigh model as an effective description. The model is also embeddable into an

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Note that this is not the same situation as the discussion around eq. (20). The one-loop factor enhancement there, $B_{\mu/\mu} = \mu_1^2 / (g^2 / (4\pi)^2)$, is always large due to the perturbativity of the standard model gauge coupling $g$. 
SU(5) unified model in a straightforward way. The same dynamics spontaneously breaks SU(5) gauge symmetry and supersymmetry [37].

The model is based on a strongly coupled gauge theory where $S$ and the Higgs fields appear at low energy as massless hadrons. The constituent ‘quarks’ of these hadrons are $Q$, $\bar{Q}$ and $T$, all of which transform as a vector representation of a strong SO(9) gauge group. Also $Q$ and $\bar{Q}$ carry standard model quantum numbers ($5$ and $\bar{5}$ under SU(5)) and $T$ is singlet under SU(5) but carries $PQ$ ($T \equiv T$)

\[ \left( \begin{array}{c} T_T \\ T_{\bar{T}} \end{array} \right), \]

where the Higgs fields in the $5$ and $\bar{5}$ representation, $H$ and $\bar{H}$, contain $H_u$ and $H_d$ as SU(2) doublet components, respectively.

This is an SO(9) gauge theory with eleven flavors, and the SU(5) gauge group is identified with a subgroup of the SU(11) flavor symmetry. We can write down superpotential terms:

\[ W_{\text{GUT}} = \mu_T T^2 + M_Q Q \bar{Q} - \frac{1}{M_X} (QQ)^2 + \cdots, \]

where $\mu_T$ ($\sim 1 - 10 \text{ GeV}$) corresponds to the small explicit breaking of the PQ symmetry. This term is going to be the $m_S^2$ term in eq. (2.25) at low energy. Once we ignore the superpotential (in the limit of $\mu_T, M_Q \rightarrow 0$ and $M_X \rightarrow \infty$), the SO(9) 11 flavor theory is on the edge of the conformal window [48]. Therefore, at some scale $\Lambda_*$ the gauge coupling constant flows into the infrared fixed point. Although it becomes a strongly coupled conformal field theory (CFT) near the fixed point, there is a dual weakly coupled CFT description with which we can perform perturbative calculations. The dual gauge group is SO(6) and the superpotential above is replaced with

\[ W_{\text{GUT}} = \kappa_* S + \bar{q} \mu_T T^2 + \kappa_* \bar{H} \bar{q} + \cdots (2.44) \]

The field $M$ is a composite meson $M \sim (QQ)$ which transforms as $1 + 24$ representation under SU(5). The fields $t$, $q$, and $\bar{q}$ are dual quarks which are charged under SO(6), and $\kappa_*$ and $\kappa_*$ are the coupling constants at the fixed point ($\kappa_* = \kappa_* \sim (4\pi)/N$ with $N = 6$). At a vacuum where the gauge group is broken down to the standard model gauge group, $\langle M \rangle = \text{diag}(0,0,0,v,v)$ and $\langle q_C \rangle \neq 0$ ($q_C$ : colored components of $q$), this model becomes exactly the same as the O'Raifeartaigh model in eqs. (2.34) and (2.36) with the identification of $t \rightarrow X$, $H_C \rightarrow Y$ ($H_C$ : the colored Higgs field), $\mu_T \Lambda_* \rightarrow m^2$, $\kappa_* (q_C) \rightarrow M_{XY}$ and $\kappa_* (M) \rightarrow M_q$. (See figure 3 for particles to describe the effective theory in each energy interval.)

Once we take into account non-perturbative effects, there appears a supersymmetric minimum far away from the origin of $S$. However, it has been shown in ref. [26] that the vacuum near $S = 0$ is meta-stable. We can see in eq. (2.35) that the $S$ mass squared, $m^2_S = +4F^2_S/\Lambda^2$, is indeed positive.
The loop expansion parameter, $|h^*_\mu|^2 N/(4\pi)^2$ where $N = 6$, is $1/N_F$ at the fixed point in this model ($N_F = 11$). Therefore, we obtain

$$\frac{\mu^2}{m^2_{H_u}} \sim \frac{1}{N_F} \quad (N_F = 11).$$

(2.45)

Although it looked problematic to have a hierarchy in eq. (2.41) in perturbative models, similar sizes of $\mu$ and $m_{H_u}$ can be obtained in this semi strongly coupled theory: $\mu/m_{H_u} \sim 1/3$.\(^8\)

\(^8\)The relation is not a precise prediction of the model. Depending on the ratio of the mass parameters $M_q(\equiv h_\mu(M))$ and $M_{XY}(\equiv h_\mu(q_C))$, which are independent parameters in the superpotential, there can be $O(1)$ deviation from the relation (see eq. (2.41)). If $M_q \lesssim M_{XY}$, we can reliably use eq. (2.41) (by multiplying a factor of $N$) with $h^2 N/(4\pi)^2 \approx 1/N_F$ as the leading order result of the $1/N$ expansion. However, once $M_q/M_{XY}$ becomes too large, such as a factor of three or so, we lose a perturbative control of the calculation. In this case, we can first integrate out $q_D$ and $\bar{q}_D$ (the doublet part of $q$ and $\bar{q}$), and then by taking the Seiberg duality \(^9\) of this dual picture again the theory becomes (semi) weakly coupled. (It is an SO(5) 7 flavor model. See ref. \(^38\).) Although there are $O(1)$ ambiguities in model parameters through the matching between two theories, the naive dimensional analysis \(^5\) gives the same result as the relation in eq. (2.45) even in that case.

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Doublet-triplet splitting problem. The model above completely solves the doublet-triplet splitting problem in GUT models. By the vacuum expectation value of the colored component of $q$ and $\bar{q}$, the gauge group SO(6) $\times$ SU(5) is broken down to the standard model gauge group. The $Hqt$ and $\bar{H}qt$ couplings in eq. (2.44) then give mass terms only for the colored Higgs fields [37]. This dual picture is similar to an SO(10) model proposed in ref. [51].

As discussed before, the dimension five operators for proton decays are sufficiently suppressed thanks to the approximate PQ symmetry.

Supersymmetric fine-tuning problem. The experimental lower limit on the Higgs boson mass from LEP-II experiment, $m_h > 114$ GeV [52], has put a threat on supersymmetric models. In order to satisfy the experimental bound, we need either a heavy scalar top quark (stop) or a large $A_t$-term (the stop-stop-Higgs coupling) since a significant one-loop contribution to $m_h$ is necessary [53]. On the other hand, once we have large $m_t$ or $A_t$, it induces a large one-loop contribution to the soft mass term $m^2_{H_u}$. This immediately means that there is a fine-tuning in the electroweak symmetry breaking as we can see in the condition:

$$\frac{M^2_Z}{2} \simeq -\mu^2 - m^2_{H_u}(\Lambda) - \delta m^2_{H_u},$$  \hspace{1cm} (2.46)

where $\delta m^2_{H_u}$ is the contribution from the radiative correction, and $M_Z$ is the $Z$ boson mass ($M_Z = 91.2$ GeV). If $|\delta m^2_{H_u}| \gg M^2_Z$, we need cancellation between $\delta m^2_{H_u}$ and either $\mu^2$ or $m^2_{H_u}(\Lambda)$ to reproduce a correct value of the $Z$ boson mass. A cancellation of at least $O(1-5\%)$ is necessary to satisfy the bound on the Higgs boson mass in generic gravity or gauge mediation models (see for review [54]).

Although the framework in eq. (2.24) does not avoid the problem, there is an interesting consistency. As we have observed in an example of the UV completion before, the ratio of $\mu^2/m^2_{H_u}(\Lambda)$ is predicted to be small (at least a factor of a few) if the theory has (semi) perturbative description above the scale $\Lambda$. In general, without specifying UV models there is a good reason to believe that the description should be (semi) perturbative. First, we are implicitly assuming that quarks and leptons remain to be elementary particles (weakly coupled) all the way up to the Planck scale, otherwise we reintroduce the flavor problem. If quarks and leptons are strongly coupled above the scale $\Lambda$, it is expected to have interaction terms such as $S^\dagger S \Phi^\dagger \Phi/\Lambda^2$ which induce mixing terms of $O(1)$ in sfermion mass matrices. On the other hand, we must write down the Yukawa coupling constant for the top quark which is of $O(1)$. This indicates that the Higgs fields should not be replaced by a composite operator with a large dimension. If the dimension of the Higgs operator above the scale $\Lambda$ was $d(H) > 1$ and the top quark is an elementary particle as discussed above, the Yukawa coupling would be suppressed by a factor of $(\Lambda/M_P)^{d(H)-1}$. The $O(1)$ Yukawa coupling constant suggests that dimension of the Higgs operator is $d(H) \simeq 1$, i.e., the Higgs fields are not (very) strongly coupled. The loop expansion should then make sense in the calculation of $\mu$ and $m^2_{H_u}(\Lambda)$. The smallness of $\mu^2/m^2_{H_u}(\Lambda)$ is, therefore, a generic feature of the model.

Another amusing point to notice is that the function $g(x)$ in eq. (2.44) is positive valued for $x > 0$. This means $m^2_{H_u}(\Lambda) > 0$ if there is a (semi) perturbative description.
Now with positive $m_{H_u}^2(\Lambda)$ and $\mu^2 \ll m_{H_u}^2(\Lambda)$, the condition of the electroweak symmetry breaking in eq. (2.46) implies that $\delta m_{H_u}^2$ must be negative and large. Indeed, the contributions from the stop-loop diagrams are negative and are proportional to $m_t^2$. Therefore, the “little hierarchy”, i.e., a heavy stop is predicted in this model. The tight experimental bound on the Higgs boson mass is not a big surprise.

**Strong CP problem.** Although the approximate PQ symmetry introduced in the framework does not provide us with a solution to the strong CP problem in a usual way by the axion mechanism [55] (because it is explicitly broken), there is an interesting connection.

The PQ symmetry is anomalous with respect to the SU(3) strong interaction of the standard model. If we demand the PQ symmetry to be non-anomalous, there are two options to take. The first one is to simply assume that the $u$-quark is massless. By combining $U(1)_{PQ}$ with the chiral symmetry, we can make the PQ symmetry non-anomalous.

Another option is to introduce an axion chiral superfield $A$ which has a coupling to the gauge fields:

$$f \ni A \frac{W^\alpha W_\alpha}{f_A}.$$ (2.47)

The kinetic term for $A$ is

$$K \ni (A + A^\dagger)^2.$$ (2.48)

With a PQ transformation of $A$, $A \rightarrow A + i \theta$, we can cancel the gauge anomaly.

Each of two options, massless $u$-quark and the axion, solves the strong CP problem.9

3. Low energy predictions

The set-up in eq. (2.25) provides a characteristic spectrum of the supersymmetric particles. It is different from conventional gauge or gravity mediation models. Since the Higgs sector directly couples to the supersymmetry breaking sector at the GUT scale, the soft mass terms for the Higgs fields are generated at the GUT scale. The gaugino masses and sfermion masses are, on the other hand, generated at the messenger scale. This hybrid feature provides interesting predictions on the low energy spectrum.

We discuss a parametrization of the model defined in eq. (2.25), with which we can calculate the low energy spectrum and interaction terms. As we will see below, we can parametrize the model by three quantities. These three define a theoretically well-motivated hypersurface in the large dimensional MSSM parameter space.

3.1 Parametrization

We first count the number of the parameters in the model. The soft supersymmetry breaking terms for the Higgs sector:

$$m_H^2, \mu,$$ (3.1)

9The scalar component of the axion chiral superfield $A$, the saxion, obtains a mass of the order of 1 GeV by gravity mediation effects [56]. With a sufficiently small decay constant $f_A$, the saxion does not cause a moduli problem as it decays into a pair of gluon much earlier than the decay of $S$.  

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are generated at the scale $\Lambda$. We take the scale $\Lambda$ to be the unification scale $M_{\text{GUT}}$. We assumed the same soft mass terms for $H_u$ and $H_d$ ($m_{H_u}^2(M_{\text{GUT}}) = m_{H_d}^2(M_{\text{GUT}}) = m_H^2$) as motivated by the UV completion discussed before. Gaugino masses, $A$-terms, $B$-term, and sfermion masses are vanishing at the GUT scale.

Below the GUT scale, RG evolutions of the soft terms induce sfermion masses through the Yukawa interactions. The gaugino masses, $A$- and $B$-terms remain vanishing. At the messenger scale, $M_{\text{mess}}$, the messenger fields decouple. The threshold corrections (i.e., gauge mediation effects) contribute to the gaugino masses, sfermion masses and also the Higgs masses squared. Those are calculable with a single parameter, $\bar{M} \equiv \frac{1}{(4\pi)^2} F_S \langle S \rangle$, as we can read off from eq. (2.23). This parameter controls the overall scale of the supersymmetry breaking parameters. For example, the gluino mass is $M_3 = g_3^2 \bar{M}$ [48]. The $A$- and $B$-terms are still vanishing (up to higher order loop corrections [57]) at the messenger scale, but the RG evolution below the messenger scale generates those through one-loop diagrams.

All the soft supersymmetry breaking parameters at the electroweak scale can be expressed in terms of these four parameters, $m_{H_u}^2$, $\mu$, $M_{\text{mess}}$, and $\bar{M}$, by the procedure described above. One combination of the parameters should be fixed by the condition for the electroweak symmetry breaking, i.e., $M_Z = 91.2$ GeV. We take the $m_{H_u}^2$ parameter as an output of the calculation. The model parameters are now defined by $(\mu, M_{\text{mess}}, \bar{M})$. Here we take the running $\mu$ parameter at the scale $M_{\text{SUSY}} \equiv (m_{\tilde{t}_L}^2 m_{\tilde{t}_R}^2)^{1/4}$ as an input parameter.

We show in figure 6 an example of the RG evolution of soft supersymmetry breaking parameters for $(\mu, M_{\text{mess}}, \bar{M}) = (300 \text{ GeV}, 10^{10} \text{ GeV}, 900 \text{ GeV})$. The horizontal axis is the RG scale. We have used the top quark mass, $m_t = 170.9$ GeV [59]. The constraint from the electroweak symmetry breaking fixes the $m_{H_u}^2$ parameter to be $(817 \text{ GeV})^2$. The choice of parameters is motivated by the discussion in the last section. The positive value of $m_{H_u}^2$ and a relatively small value of $\mu(M_{\text{GUT}})$ compared to $\sqrt{m_{H_u}^2}$ are realized with this set of parameters. The lightest Higgs boson mass is calculated to be 115 GeV. We will use this set of parameters in a collider study in section 4.

In the left panel of figure 8, scalar masses and the $\mu$-parameter are plotted. We have defined mass parameters $\bar{m}_X \equiv \text{sgn}(m_X^2)|m_X^2|^{1/2}$ for each scalar mass parameter $m_X^2$. Several interesting things are happening here. With non-zero positive values of $m_{H_u}^2$ the Yukawa interactions induces negative masses squared for sfermions in the third generation. The positive values are motivated by the positivity of the function $g(x)$ in eq. (2.40). The negative contributions to the sfermion masses are compensated by the positive contributions from the gauge mediation effects at the messenger scale. This behavior of the RG evolution gives smaller values of the stau mass, $m_{\tilde{\tau}_R}$, at the electroweak scale compared to
Figure 6: The RG evolution of the supersymmetry breaking parameters. RG equations at one-loop level are used. A parameter set \((\mu, M_{\text{mess}}, M) = (300, 10^{10}, 900) \, \text{[GeV]}\) is chosen. The left panel shows the evolution of the soft masses for \(\tilde{t}_R\) (dotted), \(\tilde{\tau}_R\) (dot-dashed), and \(H_u\) (solid). The \(\bar{m}_X\) parameter is defined by \(\bar{m}_X \equiv \text{sgn}(m^2_X)|m^2_X|^{1/2}\) for each chiral superfield \(X\). The evolution of the \(\mu\)-parameter (dashed) is also shown. Negative contributions for \(\tilde{t}_R\) and \(\tilde{\tau}_R\) above the messenger scale comes from the one-loop contribution through the Yukawa interactions. Threshold effects (gauge mediation) at the messenger scale contribute to sfermion and the Higgs mass parameters. The \(m^2_{H_u}\) parameter is driven to a negative value by the stop-loop diagrams. In the right panel, gaugino masses, \(A\)-, and \(B\)-parameter are shown. The gaugino masses are generated at the messenger scale, and induces \(A\)- and \(B\)-terms by the one-loop running. For the phase convention of \(A\)- and \(B\)-terms, we have used the one defined in ref. [58].

As is clear from eq. (2.46) the value of the \(\mu\)-parameter is approximately obtained by \(\mu \simeq -\bar{m}_{H_u}|_{\mu R \sim \text{TeV}}\) Therefore, we can easily understand from this figure that the value of \(\mu\) is smaller for larger initial values of \(m^2_{H_u}\). On the other hand, the stau mass is also smaller for large \(m^2_{H_u}\) because it receives more negative contributions. This correlation between the \(\mu\)-parameter (the Higgsino masses) and the stau masses is an interesting prediction of the model. We will discuss this relation more in subsection 3.3.

The evolution of gaugino masses, \(A\)- and \(B\)-terms are shown in the right panel of figure 6. We have used the sign convention of those parameters defined in ref. [58]. Those \(R\)-charged parameters are generated at the messenger scale by gauge mediation effects. We see a peculiar behavior of the \(B\)-term. It starts from zero at the messenger scale, goes to negative once and flips its sign to positive later. The negative contribution comes from a loop diagram with the SU(2) gaugino, and a stop-loop diagram with the \(A_t\) term gives a positive contribution. The behavior can be understood by the fact that the \(A_t\) term starts at zero but its absolute value becomes larger at low energy. The positivity of the \(B\)-term remains to be true unless the messenger scale is extremely low such as \(M_{\text{mess}} \lesssim 10^5 \, \text{GeV}\).
In this case, the two physical signs are predicted to be
\[ \text{sgn}(M_i \mu (B \mu)^*) = +1, \quad \text{sgn}(M_i A_i^t) = -1 \quad (i = 1, 2, 3). \quad (3.4) \]

Unlike other models used in literatures for collider studies, there is no choice of these signs. The former predicts a positive sign for the supersymmetric contributions to the anomalous magnetic moment of muon. Also, the latter determines the sign of the chargino-loop contribution to the amplitude of the $b \to s \gamma$ decay to be opposite to the one from loop diagrams with the charged Higgs boson. Interestingly, both of these signs are preferred by the experimental constraints on these quantities.

### 3.2 Electroweak symmetry breaking

It is highly non-trivial whether we can have successful electroweak symmetry breaking with the limited number of parameters. We demonstrate here that the correct Z-boson mass can be obtained without spoiling the perturbativity of the Yukawa interactions for the top and bottom quarks up to the GUT scale.

The left panel in figure 7 shows the value of $\bar{m}_H$ (defined at the GUT scale) required by the correct electroweak symmetry breaking with respect to the running $\mu$-parameter at $M_{\text{SUSY}}$. Curves for different messenger scales $M_{\text{mess}}$ are shown. The $\bar{M}$ parameter is fixed to be 900 GeV. For other values of $\bar{M}$, say $x\bar{M}$ with an arbitrary positive number $x$, we can obtain curves, as a good approximation, by rescaling the both axes by the factor of $x$. As is clear from the behavior of the RG evolution shown in figure 6, larger values of $\bar{m}_H$ are...
necessary for having smaller values of $\mu$. Each lines are terminated at some small value of $\mu$, where the stau becomes too light ($m_{\tilde{\tau}} < 98$ GeV) due to the negative contribution from the running between the GUT scale and the messenger scale. This negative contribution is significant only when $\tan \beta$ is large. It is indeed the case as we will see later.

As we discussed in the previous section, the UV completion of the theory above the GUT scale suggests that the Higgs fields do not get strongly coupled. The discussion is based on the requirement of the $O(1)$ Yukawa coupling constant for the top quark. This indicates a (small) hierarchy between the $m_H$-parameter and the $\mu$-parameter at the GUT scale since the ratio $\mu^2/m_H^2$ turns out to be the loop-expansion parameter. We indicate in the plot the region with $0 \leq \mu^2/m_H^2 < 1/100$ to be ‘weakly coupled’, $1/100 \leq \mu^2/m_H^2 < 1/4$ to be ‘semi perturbative’, and $\mu^2/m_H^2 \geq 1/4$ or $m_H^2 < 0$ to be ‘strongly coupled’ for illustration. (There is no concrete meaning in precise locations of border lines.) We only find solutions with small $\mu$ compared to the overall scale $\tilde{M}$ if we impose (semi) perturbativity. The light Higgsino is, therefore, one of the predictions of the model. Also, a high messenger scale, such as $M_{\text{mess}} \gtrsim 10^8$ GeV is required to be in that region.

There is a possibility that the Higgs fields are fully strongly coupled and the top Yukawa coupling is generated by some strong dynamics by making the top quark also involved in the strong sector. Since there is no stringent constraint for flavor violation in the top-quark physics, this is not a dangerous assumption although the embedding to GUT models may be more difficult. In this case, the stop mass squared at the GUT scale becomes an additional parameter of the model. We do not consider this modification of the framework in this paper.

We show in the right panel the predicted values of $\tan \beta$. Again, we take $\tilde{M} = 900$ GeV. The rescaling of the horizontal axis gives a curve for different values of $\tilde{M}$ with a good accuracy. For small values of $\mu$ preferred by UV completions, $\tan \beta \sim 30 - 40$ is predicted. The values in this range do not cause a Landau pole of the Yukawa coupling constants below the GUT scale. Therefore, the perfectly consistent electroweak symmetry breaking is achieved without any extension of the model. Large values of $\tan \beta$ are due to the fact that the $B\mu$ term is generated only through radiative corrections below the messenger scale. The Yukawa coupling constant for the tau lepton is large for large $\tan \beta$, which affects the running of $m_\tau^2$.

3.3 Light $\tilde{\tau}$ and light Higgsino

The characteristic RG evolution of the supersymmetry breaking parameters in this framework provides interesting predictions in low energy physics. In particular, the correlation of the $\mu$-parameter and the stau mass gives a large impact on collider physics.

The plot in figure shows the correlation for different values of $M_{\text{mess}}$. For small values of $\mu$, the stau is lighter than the Bino and Higgsinos. Therefore, for values of $\mu$ in the (semi) perturbative region (see left panel of figure) there is a big chance that the stau becomes the NLSP (remember that the gravitino is the LSP). The collider physics of $\tilde{\tau}$

\footnote{Precisely speaking, the hierarchy is predicted for the ratio of $\tilde{m}_H$ and the $\mu$ parameter at the GUT scale whereas what is plotted is the $\mu$-parameter at $M_{\text{SUSY}}$. However, the RG running of the $\mu$ parameter changes its value only by $O(10\%)$ (see figure).}
Figure 8: Correlation of the $\mu$ parameter and the lighter stau mass. The overall scale $\tilde{M} = 900$ GeV is set. Below the lines of the Bino mass and the Higgsino mass, the stau is the NLSP. We can clearly see the positive correlation.

The stau mass bound at $m_\tau = 170.9$ GeV.

this region will be very different from the scenario with the neutralino NLSP as there is no missing $E_T$ associated with escaping neutralinos. (The lifetime of the stau is typically of $O(1000)$ seconds with which the stau is regarded as a completely stable particle for the time scale of collider experiments.) Also, the light Higgsino changes the pattern of cascade decays of supersymmetric particles. We will discuss in the next section the overall feature of this scenario at the LHC and demonstrate a way of confirming/excluding the framework.

4. LHC signatures

The theoretical success of the sweet spot supersymmetry motivates us to consider what will be the experimental signatures at the LHC experiments. We show in this section that there are several unique features. We present a way of confirming/excluding the model in the case where the lighter stau is the NLSP.

4.1 Overview of supersymmetric events with $\tilde{\tau}$ NLSP

As we have seen in the last section, it is plausible that the lighter stau is the NLSP. A small value of the $\mu$-parameter is a natural consequence of UV physics, and that makes $\tilde{\tau}$ light through the RG evolution. If it is the NLSP, the lifetime of stau is of $O(1000)$ seconds with our assumption of the $O(1)$ GeV gravitinos. The LHC signals with such a long-lived stau will be quite different from ones with the usual assumption of the neutralino LSP. There have been many studies on collider signatures for the quasi-stable $\tilde{\tau}$-NLSP scenario,
We select the following benchmark point for the collider study:

\[ \mu = 300 \text{ GeV}, \quad M_{\text{mess}} = 10^{10} \text{ GeV}, \quad \bar{M} = 900 \text{ GeV} \, . \quad (4.1) \]

This set represents the most theoretically motivated region of the parameter space as we have discussed before. As we can see in figure 8 the NLSP is the stau with this set of parameters. We have calculated the spectrum by solving RG equations at one-loop level. The running parameters at the scale \( M_{\text{SUSY}} \equiv (\frac{m_{\tilde{t}_L}^2}{m_{\tilde{t}_R}^2})^{1/4} = 1053 \text{ GeV} \) have been used for the calculation of the spectrum. We have ignored the QCD finite corrections at the low energy threshold, which would amount to about 10%. In table 1, we listed masses of superparticles and Higgs bosons. We used \( m_t = 170.9 \text{ GeV} \). The stau mass is 116 GeV, and its lifetime is calculated to be about 7000 seconds with the gravitino mass determined by the dark matter density, \( m_{3/2} = 500 \text{ MeV} \). The running gaugino mass parameters at \( M_{\text{SUSY}}, M_i = g_i^2 M_i \), are \( M_1 = 195 \text{ GeV}, M_2 = 364 \text{ GeV}, \) and \( M_3 = 1013 \text{ GeV} \). The lightest neutralino \( \chi_1^0 \) is, therefore, mostly the Bino, \( \chi_2^0 \) and \( \chi_3^0 \) mainly consist of the Higgsino components, and the Wino is the heaviest, \( \chi_4^0 \). The lighter and heavier charginos, \( \chi_1^{\pm} \) and \( \chi_2^{\pm} \), are mainly the Higgsino and the Wino, respectively. The gluino and squark masses are about 1 TeV. The Higgs boson mass, 115 GeV, is calculated by using a one-loop effective potential with taking into account leading two-loop corrections by appropriately choosing

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**Table 1:** Masses of superparticles and Higgs bosons in GeV for our benchmark point, \( \mu = 300 \text{ GeV}, \) \( M_{\text{mess}} = 10^{10} \text{ GeV} \) and \( \bar{M} = 900 \text{ GeV} \). The gravitino mass is fixed to account for the observed dark matter density (see eq. (2.27)). Here, the masses of the squarks and sleptons of the second generation are omitted, since they are equal to the ones of the first generation. We use the notation for the superparticles and Higgs bosons in the MSSM in ref. [78].

| \( \tilde{g} \) | 1013 | \( \tilde{\nu}_L \) | 543 |
| \( \tilde{\chi}_1^0 \) | 270 | \( \tilde{\ell}_1 \) | 955 |
| \( \tilde{\chi}_2^0 \) | 404 | \( \tilde{\ell}_2 \) | 1177 |
| \( \tilde{\chi}_3^0 \) | 187 | \( \tilde{b}_1 \) | 1128 |
| \( \tilde{\chi}_4^0 \) | 276 | \( \tilde{b}_2 \) | 1170 |
| \( \tilde{t}_1 \) | 307 | \( \tilde{\tau}_1 \) | 116 |
| \( \tilde{t}_2 \) | 404 | \( \tilde{\tau}_2 \) | 510 |
| \( \tilde{u}_L \) | 1352 | \( \tilde{\nu}_r \) | 502 |
| \( \tilde{u}_R \) | 1263 | \( h^0 \) | 115 |
| \( \tilde{d}_L \) | 1354 | \( H^0 \) | 770 |
| \( \tilde{d}_R \) | 1251 | \( A^0 \) | 765 |
| \( \tilde{e}_L \) | 549 | \( H^\pm \) | 775 |
| \( \tilde{e}_R \) | 317 | \( G \) | 0.5 |
The total cross section of the superparticle production at the benchmark point is 1.4 pb for the center-of-mass energy of the LHC. The cross section is dominated by pair productions of \( \tilde{g} \tilde{g} \), \( \tilde{q} \tilde{g} \), and \( \tilde{q} \tilde{q} \). The subsequent decays of these colored particles generate hard jets and other supersymmetric particles such as neutralinos and charginos. The decays of these non-colored superparticles, in the end, produce two quasi-stable \( \tilde{\tau} \)'s for each supersymmetric event. Most of the stau pairs escape a detector and leave two charged tracks.

The decay cascades start with the the decays of \( \tilde{g} \) and \( \tilde{q} \) as shown in figure 9. We have used ISAJET 7.69 to calculate the branching ratios. Since the gluino is lighter than squarks, it decays into a neutralino or a chargino through three-body decay modes. The dominant channel is the decay into a pair of third generation quarks and a Higgsino, \( \chi_{1}^{\pm} \) or \( \chi_{2,3}^{0} \), through the Yukawa interaction of the top quark. The main decay mode of the squarks are \( \tilde{q} \to \tilde{g} + q \), followed by the gluino decay. Therefore, for each supersymmetric event, many hard jets are produced. Especially, a significant number of \( b \)-jets are produced by the gluino decays (and also by the subsequent decays of the top quarks). This is an interesting feature of the model, but at the same time, the large number of jets makes it difficult to trace back the decay chains in the actual analysis because it is hard to specify which jet is the one originated from a particular decay process.

At the end of the decay chain, three lighter neutralinos (Bino and Higgsinos) decay into \( \tilde{\tau} \) and \( \tau \) (figure 10). The heaviest neutralino and the heavier chargino (Winos) decay...
into $\chi_1^\pm$ which in turn decays into a stau and a neutrino. We can measure the mass and the momentum of $\tilde{\tau}_1$’s in the final state by using information on the charged tracks in the muon system. With the full reconstruction of the four momentum of the staus, three of neutralino masses can in principle be measured as the invariant mass of $\tilde{\tau}_1$ and $\tau$.

Sleptons except for $\tilde{\tau}_1$ do not appear in the decay cascades. That means we do not have clear lepton signals such as two opposite-sign same-flavor leptons from the $\chi_0^2$ decay often used as a tool for precision measurements \cite{83}. What we typically have are a lot of third generation quarks ($b$-jets) and leptons ($\tau$-jets) in the final states. It is again interesting but not exciting situation for measurements of the mass spectrum.

4.2 Reconstruction of neutralino masses

We demonstrate here a way of reconstructing the neutralino masses by using the decay modes $\chi_{1,2,3}^0 \rightarrow \tilde{\tau}_1 \tau$, and show that it is possible to determine the three model parameters.

There have been studies on the mass reconstruction of the stau and the neutralinos in stau NLSP scenarios. Especially, in ref. \cite{68}, a detailed study of the stau mass measurement is performed including detector effects. The method is measuring the velocity ($\beta$) of the stau by the time-of-flight and the momentum ($p_{\tilde{\tau}_1}$) from the track. We can then calculate the mass by the formula,

$$m_{\tilde{\tau}_1} = \frac{p_{\tilde{\tau}_1}}{\beta \gamma}.$$  \hspace{1cm} (4.2)

It is concluded that the stau mass can be measured with the accuracy of at least about 100 MeV for $m_{\tilde{\tau}_1} \simeq 100$ GeV. The standard model background is estimated in the paper. Since the staus with high velocities are indistinguishable from muons, a tight selection cut on the velocity, $\beta \gamma < 2.2$, is imposed in the analysis. We reproduced the analysis of the stau mass measurement and found that it is indeed possible to measure it with a very good precision at the benchmark point. We follow the selection cuts proposed in this paper in the analysis of the reconstruction of neutralino masses as well.

The reconstruction of the neutralino masses has been discussed in ref. \cite{66} and also in a recent paper \cite{71}. Once we determine the stau mass, we know the four-momentum of the stau on event-by-event basis. By combining with the four-momentum of $\tau$, we can extract the neutralino masses. There are two things needs to be done in the analysis. Since every supersymmetric event contains two staus in the final state, there are two candidate staus to be combined with for each $\tau$. The invariant mass coincides with a neutralino mass only if we choose a correct combination. We need a strategy to select the correct one. The other thing is that we do not know the four-momentum of $\tau$. The $\tau$ particle decays inside the detector and an invisible neutrino in the decay products always carries away some amount of energy.

In the analysis in ref. \cite{66}, it has been assumed that the selectron and the smuon are also quasi-stable. Therefore, by selecting events with only one stau there is no combinatorial background. A method of fully reconstructing the four-momentum of $\tau$ has been discussed in the paper. If the direction of missing momentum is aligned with that of the $\tau$-jet, it can be identified with the neutrino. By adding missing momentum to the $\tau$-jet momentum, four
momentum can be reconstructed. Even without trying to reconstruct the four-momentum of $\tau$, the authors found that there appear sharp edges at the neutralino masses in the distribution of the $\tilde{\tau} - \tau$ invariant mass by using hadronic decay modes of $\tau$.

We follow this endpoint analysis with hadronically decayed $\tau$’s for the measurement of the neutralino masses. Since there are always two staus in the final state in contrast to the data set analyzed in ref. [66], we need to consider a way of reducing the combinatorial background. Also, with updated experimental bound on the Higgs boson mass, the overall scale of the supersymmetric particles should be higher than the points studied in ref. [66]. This significantly reduces the statistics, which are essential for the endpoint analysis. Also, with the cut on the stau velocity discussed above, the number of signal events are again significantly reduced with the relatively light stau, 116 GeV, at the benchmark point. It is, therefore, non-trivial whether the method works in our model.

It has been recently proposed to reconstruct the energy of neutrinos by decomposing the missing momentum into two directions of $\tau$’s which we know from directions of leptons or $\tau$-jets [71]. Also, by using information of the electric charge of staus and leptons in the final state, we can select the correct combination for the events with $\tilde{\tau}^+$ and $\tilde{\tau}^-$. However, in our model, there are often other neutrinos in the events, e.g., from chargino decays and top-quark decays. The decomposition fails in the presence of such neutrinos. In the leptonic decays of $\tau$, neutrinos tend to carry away majority of $\tau$ energy due to the kinematics and the $V - A$ current structure of the weak interaction. Therefore, all of the uncertainties in the reconstruction of the neutrino momentum reflect to the mass measurement. For the background rejection, the authors have used a loose cut on the stau velocity, $\beta\gamma < 6$, and they checked that it is enough to reject the standard model background from misidentifications of muons as staus. However, with a light stau at our benchmark point, it is non-trivial whether such a loose cut can effectively suppress the background. The overlap between the signal and background regions in the ($\beta\gamma, m_{\tilde{\tau}}$) plane gets larger for a light stau. Once we impose a tighter cut, $\beta\gamma < 2.2$, we could not obtain enough number of events for the analysis. Therefore, we do not pursue this direction.

In the following, we discuss a method of the simulation and selection cuts for reducing the standard model background. The actual analysis will be presented in 4.2.2.

4.2.1 Event generation and selection cuts

We have generated 42,900 events of the superparticle productions in proton-proton collision at the LHC energy by using a event generator HERWIG 6.50 [84] with the MRST [85] parton distribution function. The number of the events is equivalent to the integrated luminosity of 30 fb$^{-1}$. We have used the package TAUOLA 2.7 for $\tau$ decays [86].

For a detector simulation, we have used a package AcerDET-1.0 [87], which is a fast simulator for high $p_T$ physics at the LHC. The AcerDET program identifies isolated leptons, isolated photons and isolated jets out of the final state of each generated event. The cluster selections and the smearing of the four-momentum of leptons, photons and jets are implemented. Leptons and photons are considered to be isolated if they are far from other clusters by $\Delta R > 0.4$, and the transverse energies deposited in cells in a cone $\Delta R = 0.2$ around the cluster are less than 10 GeV. A cluster is recognized as a jet by a cone-based
algorithm if it has $p_T > 15 \text{ GeV}$ in a cone $\Delta R = 0.4$. The package also implements a calibration of jet four-momenta using a flavor independent parametrization, optimized to give a proper scale for the di-jet decay of a light Higgs boson. Each jet is labeled either as a light jet, $b$-jet, $c$-jet or $\tau$-jet, using information of the event generators. We have used default values of the parameters for clustering, selection, isolation, calibration, and labeling processes. For the $\tau$-jet identification, we further implement $\tau$-tagging efficiency of 50% per a $\tau$-labeled jet.

A parametrization of the resolution of the stau velocity is shown in ref. [68]:

$$\frac{\sigma(\beta)}{\beta} = 2.8\% \times \beta.$$  \hfill (4.3)

We smeared the velocity by using this resolution. The same resolution is assumed for the measurement of the muon velocity.

For the smearing of the stau momentum, we have used the momentum resolutions shown in ref. [71]:

one from the sagitta measurement error,

$$\frac{\sigma(p_{\tilde{\tau}_1})}{p_{\tilde{\tau}_1}} = 0.0118\% \times (p_{\tilde{\tau}_1}/\text{GeV}),$$  \hfill (4.4)

one from a multiple scattering term,

$$\frac{\sigma(p_{\tilde{\tau}_1})}{p_{\tilde{\tau}_1}} = 2\% \times \sqrt{1 + \frac{m_{\tilde{\tau}_1}^2}{p_{\tilde{\tau}_1}^2}},$$  \hfill (4.5)

and one from the fluctuation of energy loss in the calorimeter,

$$\frac{\sigma(p_{\tilde{\tau}_1})}{p_{\tilde{\tau}_1}} = 89\% \times (p_{\tilde{\tau}_1}/\text{GeV})^{-2}.$$  \hfill (4.6)

We have smeared the stau momentum according to these resolution width $\sigma(p_{\tilde{\tau}_1})$.

If the measured velocity of the stau is high enough, such as $\beta \gamma > 0.9$ [71], the stau will be identified with a muon and can be used as a trigger. However, for slow staus, we need to rely on other triggers. For the simulation of the triggering, we have chosen only events passing one of the following conditions [88]: one isolated electron with $p_T > 20 \text{ GeV}$, one isolated photon with $p_T > 40 \text{ GeV}$, two isolated electrons/photons with $p_T > 15 \text{ GeV}$, one muon with $p_T > 20 \text{ GeV}$, two muons with $p_T > 6 \text{ GeV}$, one isolated electron with $p_T > 15 \text{ GeV}$ and one isolated muon with $p_T > 6 \text{ GeV}$, one jet with $p_T > 180 \text{ GeV}$, three jets with $p_T > 75 \text{ GeV}$, and four jets with $p_T > 55 \text{ GeV}$. Here, isolated electrons/photons, isolated muons and jets must be in the central regions of pseudorapidity $|\eta| < 2.5, 2.4,$ and 3.2, respectively. Following [71], we treated staus with $\beta \gamma > 0.9$ as muons in the simulation of triggering.

For the event selection, we require two stau candidates for each event. Since the stau mass can be precisely determined, a stau identification can be performed by testing if its

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\[\text{According to the paper, the original study has been done by G. Polesello and A. Rimoldi, in ATLAS Internal Note ATL-MUON-99-06, but it is not publicly available.}\]
measured mass by eq. (4.2) is consistent with the actual mass. For the consistency test, we took a window of the measured velocity, $\beta_{\text{meas}}$:

$$\beta' - 0.05 < \beta_{\text{meas}} < \beta' + 0.05,$$

(4.7)

where $\beta'$ is a velocity calculated from the measured momentum, $p_{\text{meas}}$, by assuming the stau mass, i.e., $\beta' = \sqrt{p^2_{\text{meas}}/(p^2_{\text{meas}} + m^2_{\tilde{\tau}})}$ (see [68]). To reduce the standard model background from mis-identifications of muons as staus, we required one of the stau candidate selected above to have $\beta \gamma < 2.2$. The transverse momentum cut, $p_T > 20$ GeV, is also imposed.

The lower limit on the velocity $\beta \gamma > 0.4$ is imposed to ensure the stau to reach the muon chamber. As for the isolation of stau, we have used the same criterion with that of the muon.

The standard model background can be further suppressed by requiring the presence of the two charged stable particles as well as a large effective mass [69]. The effective mass $M_{\text{eff}}$ is defined by the scalar sum of the transverse momentum of the four leading jets $p_{T,i}$ ($i = 1 - 4$), and the missing transverse momentum $E_{T}^{\text{miss}}$:

$$M_{\text{eff}} = p_{T,1} + p_{T,2} + p_{T,3} + p_{T,4} + E_{T}^{\text{miss}}.$$

(4.8)

The effective mass distribution of supersymmetric events has a peak around 1 TeV. We impose a cut $M_{\text{eff}} > 800$ GeV. With all of the selection cuts discussed above, the standard model background is reduced to a negligible level [68].

Events with only one $\tau$-tagged jet is selected for the reconstruction of the neutralino mass. We require $p_T > 40$ GeV for the $\tau$-jet. If we allow two $\tau$-tagged jets, the number of signal events increases by about 10%, but it also increases the probability of selecting a fake $\tau$-jet. The effects of the fake $\tau$-jets are taken into account by assuming the probability of the mis-tagging of non-$\tau$-labelled jets to be 1% per a jet [89]. We also discuss the significance of the fake events by comparing with a case with a 5% mis-tagging probability.

For simplicity, we took the tau-tagging efficiency and the mis-tagging probability to be independent of jet $p_T$, $\eta$, and the decay modes of $\tau$.

### 4.2.2 Invariant mass analysis

With the selection cuts described above, the total number of events are reduced to 2,000 in which we have 1,563 events with a true $\tau$-jet and a true stau pair. With these limited statistics, we need to develop an effective method to reduce the combinatorial background.

The combinatorial background can be reduced by choosing a stau which gives the smaller value of the invariant mass $M_{\tilde{\tau}\tilde{\tau}}$ for every $\tau$ candidate. Since the neutralinos produced by the decay of gluinos or squarks are likely to be highly boosted, the stau and $\tau$ tend to be emitted in a similar direction, and therefore the invariant mass is likely to be much larger than the neutralino masses if we choose a wrong combination. To see how it works, we show the invariant mass distribution $M_{\tilde{\tau}\tilde{\tau}}$ of the combination selected by the above strategy. We use the four-momentum of $\tau$ extracted from the event generator (figure 11). Here, no information on the charge of $\tau$ is used. As we see from the figure, the lowest mass combination shows peaks at the masses of $\chi_{1,2,3}^0$. The correct combinations are chosen with the probability of about 70%.
Figure 11: The $\tilde{\tau} - \tau$ invariant mass distribution. The combination with the lowest invariant mass is chosen. The four-momentum of $\tau$-lepton extracted from the event generator is used.

Figure 12: The distribution of the $\tau$-jet energy fraction $E_{\tau\text{-jet}}/E_\tau$ in the hadronic decay modes of $\tau$ in supersymmetric cascade decays. In the left panel, we show the energy fractions for $\tau$'s which originate from three species of neutralinos, $\chi^0_1$, $\chi^0_2$ and $\chi^0_3$, respectively. They are rescaled so that the number of events are the same for three neutralinos. In the right panel, we did not distinguish the origin of $\tau$. Shaded histograms are the distribution of the energy fraction for the two-body decays, $\tau \rightarrow \pi \nu$, $\tau \rightarrow \rho \nu$ and $\tau \rightarrow a_1 \nu$ assuming stable mesons. Energy calibration of the $\tau$-jets is performed by AcerDET.

In the actual experiment, however, the four-momentum of $\tau$-lepton is not available.
Instead, the four-momentum of the $\tau$-jet from the hadronic $\tau$ decays is available. If we use the four-momentum of the $\tau$-jet in the $M_{\tilde{\tau}\tilde{\tau}}$ analysis, the peaks shown in figure 11 are smeared by the effect of the missing energy in the $\tau$ decays. The left panel of figure 12 shows the distribution of the energy fraction of $\tau$-jet, $E_{\tau\text{-jet}}/E_{\tau}$, in the neutralino decays. We plotted histograms for each neutralino, $\chi_1^0$, $\chi_2^0$, and $\chi_3^0$. We rescaled the histograms so that the number of events are the same for each neutralino. Energies are measured in the laboratory frame. With the HERWIG event generator and the TAUOLA package, effects of the polarization of $\tau$ are taken into account. As we can see, there are sharp edges in the distribution at $E_{\tau\text{-jet}}/E_{\tau} = 1$. Especially, the edge is sharper for $\chi_1^0$ compared to $\chi_2^0$ and $\chi_3^0$. This can be understood as an effect of the polarization of $\tau$. Since the stau is mostly right-handed, the chirality of $\tau$ from the neutralino decay is right-handed (left-handed) if the neutralino is gaugino-like (Higgsino-like). By the $V - A$ current structure of the weak interaction, neutrinos tend to be emitted in the opposite (same) direction to the $\tau$ direction if $\tau$ is right-handed (left-handed), and that makes the edge sharper (broader) \cite{90}. With this structure, we can expect that the $M_{\tilde{\tau}\tilde{\tau}}$ distribution reconstructed with $\tau$-jet four-momentum shows sharp edges at three neutralino masses although the Higgsino edges become slightly weaker.

In the right panel of figure 12 we plotted the same quantity, $E_{\tau\text{-jet}}/E_{\tau}$, from all the neutralino decays. The overall shape, monotonically increasing function and has a sharp edge at $E_{\tau\text{-jet}}/E_{\tau} = 1$, can be understood from the distribution of $E_{\tau\text{-jet}}/E_{\tau}$ in the two-body decays of $\tau$,

$$
\tau \rightarrow \pi\nu \ (11\%), \quad \tau \rightarrow \rho\nu \ (26\%), \quad \tau \rightarrow a_1\nu \ (18\%)
$$

The $\rho$ and $a_1$ mesons subsequently decay into two pions and three pions, respectively. Here the percentages of each mode denote the branching ratios. The branching ratio of the leptonic modes are 35%, and the other 10% comes from more than five-body decay modes or the modes with $K$ mesons. When we ignore the width of the mesons, the energy fraction $E_{\text{meson}}/E_{\tau}$ from each decay distributes uniformly between $(m_{\text{meson}}^2/m_{\tau}^2, 1)$ in the relativistic limit of $\tau$ ($E_{\tau} \gg m_{\tau}$). In the figure, we show the distributions of $E_{\text{meson}}/E_{\tau}$ as shaded histograms. The energy fraction in other hadronic modes tends to pile up near the edge because of the kinematics of the many body final state. The distribution of $E_{\text{meson}}/E_{\tau}$ well resembles the properties of the distribution of $E_{\tau\text{-jet}}/E_{\tau}$. The thresholds at each meson mass are smeared by the effects of their finite decay widths (see for e.g., \cite{90}).

It is important to notice that the distribution of $E_{\tau\text{-jet}}/E_{\tau}$ has a tail in the unphysical region, $E_{\tau\text{-jet}}/E_{\tau} > 1$. These entries come mainly from the calibration of the $\tau$-jet energy used in the detector simulation. Especially, we should note that the distribution is slightly biased toward the $E_{\tau\text{-jet}}/E_{\tau} > 1$ region. The detailed shape of the distribution, of course, depends on the actual algorithm for the calibration. We performed a fitting of the

\footnote{We thank L. Dixon for pointing out the possibility of having polarization effects.}
Figure 13: Left) The distribution of the lowest invariant mass combination of $\tilde{\tau}_1$ and $\tau$-jet. The shaded histogram shows the events with a mis-identified $\tau$-jet which is simulated by assuming a mis-tagging probability of a non-$\tau$-labelled jet to be 1%. The small allows and dashed lines denote the input values of three neutralino masses. Three curves are fitting functions of three endpoints which correspond to the endpoints of $\chi_{1,2,3}^0$ from left to right, respectively. The third endpoint is statistically not very significant. Right) The same as the left figure but we assumed the mis-tagging probability to be 5% per a non-$\tau$-labelled jet. The endpoints of $\chi_1^0$ and $\chi_2^0$ are visible whereas the significance of $\chi_3^0$ events are reduced due to the shape of the background events. The bin size is 10 GeV in the right figure.

distribution around the peak with a smeared jagged function $f(x)$,

$$f(x; x_0, \sigma, C_1, C_2) = \int_{-\infty}^{\infty} dx' g(x-x'; x_0) \sqrt{\frac{2\pi \sigma^2}{2}} \exp \left( -\frac{x'^2}{2\sigma^2} \right),$$

$$g(x; x_0) = \begin{cases} 
C_1 x, & (0 < x < x_0), \\
C_2 x, & (x_0 < x) 
\end{cases}$$

with four fitting parameters, the position of the edge $x_0$, the smearing factor $\sigma$, and two slopes $C_1$ and $C_2$. The fitting gives the position of the edge $x_0$ to be $x_0 = 1.049 \pm 0.003$, about five percent larger than unity. Since we will identify the position of the edge in the $M_{\tau\tau}$ distribution as the neutralino mass, the bias ends up with systematic errors of the mass measurement toward larger values. Therefore, in the actual analysis of the LHC data, we need to understand the shift of the edge location caused by the calibration of the $\tau$-jets energy.

Having understood the edge structure of the $E_{\tau\tau}$ distribution, we try to reconstruct the neutralino masses. Figure 13 shows the distribution of the smaller invariant mass out of two possible combinations of $\tilde{\tau}_1$ and the $\tau$-jet. We took the mis-tagging probability to be 1% (left) and 5% (right). As we expected, we can clearly see the edge structures in the left panel. We can determine the masses of $\chi_{1,2,3}^0$ from the location of the edges. The
tail of the distribution for \(M_{\not\tau} \gtrsim 350\) GeV stems from the mis-tagging of \(\tau\)-jets (shaded histograms). In addition to the background from mis-tagging of \(\tau\)-jets, the histogram includes events with a fake stau from muons in supersymmetric events. The standard model background is assumed to be negligible with the selection cut discussed before [68].

In the left figure, we can see the structure that the shape in figure 11 is smeared according to the \(E_\tau\)-jet/\(E_T\) distribution in figure 12. The edge structure of the third neutralino \(\chi_3^0\) is not very clear with the bin size of 5 GeV. In order to extract the masses of \(\chi_1^0, \chi_2^0\), we have fitted the each endpoints with the smeared jagged function \(f(x - m_{\tilde{\tau}_1}; m_{\text{edge}} - m_{\tilde{\tau}_1}, \sigma_m, C_1, C_2)\) in eq. (4.10) plus a constant \(a_0\) which represents the background events around the edges. The results of the fitting are given in table 2.

By taking into account the above physics as well as systematic uncertainties such as dependence on the calibration algorithm for the \(\tau\)-jet momentum, we conclude that the masses of first two (possibly three) neutralinos can be measured with an accuracy of, at least, about 5% level.

If we use a loose strategy for the identification of \(\tau\), the background will significantly affects the edge structure. In the right panel of figure 13, we have used the mis-tagging probability to be 5% per a non-\(\tau\)-labelled jet. The edge structure is not significant for \(\chi_1^0\) whereas we can see the edges of \(\chi_2^0\) and \(\chi_3^0\). (We changed the bin size to 10 GeV in this figure.) This situation will improve when we use a looser cut on \(p_T\) of \(\tau\)-jets such as \(p_T > 20\) GeV. A similar accuracy for the neutralino mass measurement is possible even in that case.

### 4.3 Parameter determination

It is straightforward to determine the model parameters \((\mu, M_{\text{mess}}, \bar{M})\) from the measurement of \(m_{\tilde{\tau}_1}, m_{\chi_1^0}, \) and \(m_{\chi_2^0}\). Performing \(\chi^2\) analysis would either give best fit values of these parameters or exclude the model.

In most cases, a simpler analysis than the global fit is possible. First, by assuming that the model is correct, we can find that one of the two neutralinos we measured in the previous section should be Higgsino-like since their masses deviate from a GUT relation between \(M_1\) and \(M_2\). Secondly, we can neglect the \(\tan\beta\) dependence in the neutralino masses. With a large value of \(\tan\beta\) (see figure 7), corrections are of \(O(1/\tan\beta)\). Thus, the neutralino masses depend merely on the messenger scale \(M_{\text{mess}}\). The parameters \(\mu\) and \(\bar{M}\) can be determined at the level of 5% from the measurement of two leading neutralino masses. If we can also measure the mass of \(\chi_3^0\), we can check the consistency of the GUT

<table>
<thead>
<tr>
<th>(\chi_1^0)</th>
<th>(m_{\text{edge}}) [GeV]</th>
<th>(\sigma_m) [GeV]</th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(a_0)</th>
<th>(m_{\chi_0}^{\text{input}}) [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\chi_1^0)</td>
<td>194 (\pm) 2</td>
<td>3 (\pm) 2</td>
<td>0.93 (\pm) 0.35</td>
<td>0.37 (\pm) 0.35</td>
<td>19 (\pm) 20</td>
<td>187</td>
</tr>
<tr>
<td>(\chi_2^0)</td>
<td>279 (\pm) 3</td>
<td>13 (\pm) 4</td>
<td>0.33 (\pm) 0.14</td>
<td>-0.014 (\pm) 0.086</td>
<td>20 (\pm) 17</td>
<td>276</td>
</tr>
<tr>
<td>(\chi_3^0)</td>
<td>314 (\pm) 1</td>
<td>1 (\pm) 1</td>
<td>0.066 (\pm) 0.027</td>
<td>-0.018 (\pm) 0.018</td>
<td>8.3 (\pm) 4.5</td>
<td>307</td>
</tr>
</tbody>
</table>

Table 2: The fitting parameters of the function \(f(x - m_{\tilde{\tau}_1}; m_{\text{edge}} - m_{\tilde{\tau}_1}, \sigma_m, C_1, C_2)\) in eq. (4.10) plus a constant \(a_0\) around each edge. The final column shows the actual masses of \(\chi_1^0, \chi_2^0, \chi_3^0\).
Figure 14: Left) The stau mass \( m_{\tilde{\tau}_1} \) as a function of \( M_{\text{mess}} \) for four values of the \( \mu \)-parameter. The overall scale is set for \( \tilde{M} = 900 \text{ GeV} \). The dashed horizontal line corresponds to \( \tilde{\tau}_1 \) mass, \( m_{\tilde{\tau}_1} = 116 \text{ GeV} \), at the benchmark point. The thick vertical line denotes the value of \( M_{\text{mess}} \) determined by assuming 5% precisions of \( \mu \) and \( \tilde{M} \). Right) The pseudo-scalar Higgs boson mass \( m_A \) as a function of \( M_{\text{mess}} \) for four values of the \( \mu \)-parameter. The overall scale is set for \( \tilde{M} = 900 \text{ GeV} \). The dashed horizontal line corresponds to the prediction of \( m_A \) for \( M_{\text{mess}} = 10^{10} \text{ GeV} \). The thick vertical line denotes the value of \( M_{\text{mess}} \) determined from the stau mass measurement (see the left panel). The arrow on the \( m_A \) axes denotes the error of the prediction including the error \( \Delta \tilde{M} \).

relation between \( M_1 \) and \( M_2 \) from the mass splitting between \( \chi^0_2 \) and \( \chi^0_3 \), which provides a non-trivial check of GUT theories.

We can then determine \( M_{\text{mess}} \) from the measured stau mass. We demonstrate in the left panel of figure 14 the determination of the messenger scale \( M_{\text{mess}} \). Once we know the value of \( \mu \) and \( \tilde{M} \), \( m_{\tilde{\tau}_1} \) can be calculated as a function of \( M_{\text{mess}} \). Since we measure \( m_{\tilde{\tau}_1} \) at a few permille level, and \( \mu \) and \( \tilde{M} \) at 5% level, we can read off the corresponding value of \( M_{\text{mess}} \) from the figure. We find that the exponent of \( M_{\text{mess}} \) is determined with an accuracy of \( \pm 0.2 \). Therefore, at the benchmark point, the parameters are determined with the precision of,

\[
\Delta \mu \sim 20 \text{ GeV}, \quad \Delta \tilde{M} \sim 50 \text{ GeV}, \quad \Delta \log_{10} M_{\text{mess}} \sim 0.2. \tag{4.12}
\]

Finally, once all the parameters are determined, we can make a prediction of any other physical quantities. The simplest test is the peak location of the \( M_{\text{eff}} \) distribution which depends on the squark masses \([91, 83, 92]\). As a more non-trivial example, we show a prediction of the mass of the pseudo-scalar Higgs boson, \( m_A \), in the right panel of figure 14. We can predict a value of \( m_A \) with an uncertainty of 5% level:

\[
\Delta m_A \sim 40 \text{ GeV}, \tag{4.13}
\]

around \( m_A = 765 \text{ GeV} \). At the LHC, the heavy Higgs boson \( H^0/A^0 \) will be discovered up to \( m_{H/A} \sim 800 \text{ GeV} \) for \( \tan \beta \sim 40 \) assuming and integrated luminosity 30 fb\(^{-1}\) \([93]\). Therefore, we can perform a non-trivial check of the model from the \( m_A \) measurement.
5. Conclusions

In studies of LHC signals of supersymmetric theories, setting the model parameters is the first non-trivial task which needs to be done. At the LHC experiments, the main production process of the supersymmetric particles is a pair production of colored objects, such as a pair production of gluinos and squarks. It is thus essential to know how these particles decay. Also, since there are many kinds of particles, it is often the case that a measurement of supersymmetric parameters suffers from background processes which also come from supersymmetric events. In order to estimate the amount of the background, we need to set all of the parameters in the Lagrangian.

There are, on the other hand, over 100 parameters in the Lagrangian of the MSSM. It is practically impossible to study every point in the 100 dimensional parameter space. Therefore, parametrizations such as the mSUGRA model and the “gauge mediation” model (the gauginos and scalar masses from the formula of the gauge mediation \[4 – 6\] and \[θ\] and \[B\] parameters as free parameters) have been proposed and used as standard benchmark models. The number of the parameters is reduced to be a few in these models.

As a purpose of the study of generic signatures of the supersymmetric models and for development of methods to extract physical quantities, these parametrizations have played a significant role in studies of collider physics \[91, 83, 62, 66\]. However, it is dangerous to rely too much on these parametrizations. Interesting parameter regions in the MSSM can be precluded by assumptions made without theoretical motivations. Remember that they are simply convenient parametrizations of the more than 100 unknown parameters.

The sweet spot scenario we have presented provides an example of a simple parametrization of the MSSM (by only three parameters), and it is theoretically supported. It is the first example of such a simple parametrization which has in background a well-defined closed framework of the supersymmetry breaking and mediation with no phenomenological/cosmological problems.

There are new features in the collider signatures. For example, a relatively light Higgsino is preferred and that makes the decays of gluino into third generation quarks to be the dominant channel. Many numbers of \(b\)-jets will show up in each supersymmetric event. Also, the light stau is predicted if the Higgsinos are light. If it is the NLSP, two charged tracks left by escaping staus and \(τ\)-jets from the \(χ^0 \rightarrow \tilde{τ}τ\) decays can be used to reconstruct the three parameters in the Lagrangian as we have demonstrated. The model confirmation/exclusion is possible by measuring any other quantities such as the mass of the pseudo-scalar Higgs boson.

Acknowledgments

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