Flipped cryptons and ultrahigh energy cosmic rays

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Cryptons are metastable bound states of fractionally-charged particles that arise generically in the hidden sectors of models derived from heterotic string. We study their properties and decay modes in a specific flipped SU(5) model with long-lived four-particle spin-zero bound states called tetrons. We show that the neutral tetrons are metastable, and exhibit the tenth order nonrenormalizable superpotential operators responsible for their dominant decays. By analogy with QCD, we expect charged tetrons to be somewhat heavier, and to decay relatively rapidly via lower-order interactions that we also exhibit. The expected masses and lifetimes of the neutral tetrons make them good candidates for cold dark matter, and a potential source of the ultrahigh energy cosmic rays which have been observed, whereas the charged tetrons would have decayed in the early Universe.

I. INTRODUCTION

Metastable particles of mass $O(10^{12−15})$ GeV whose lifetime is greater than the age of the Universe would be appealing candidates for cold dark matter (CDM), and their decays might provide the observed ultra-high-energy cosmic rays (UHECRs) [1,2]. A perfect candidate for such particles is provided by “cryptons” [3–5], bound states that appear in the hidden sectors of unified superstring models. It has been pointed out that the hidden sectors of compactifications of the heterotic string generically contain fractionally-charged particles [6,7]. Since there are very stringent limits on the abundances of fractionally-charged particles [8], it is desirable to confine them, just as occurs for quarks in QCD. This is exactly what happens to the fractionally-charged states in the flipped SU(5) free-fermionic string model [9], where this solution to the problem of fractionally-charged states was first pointed out [3,4], and which remains the only example in which this solution has been worked out in detail.

In flipped SU(5), the cryptons are bound states composed of constituents with electric charges $\pm \frac{1}{5}$ that form 4 and 4 representations of a hidden non-Abelian gauge group, SO(6) $\sim$ SU(4) [9]. This confines the fractionally-charged states into integer-charged cryptons that may be either mesonlike 44 combinations or baryon-like states containing four 4 or 4 states, that we term tetrons, at a characteristic mass scale $\Lambda_4 \sim 10^{11−13}$ GeV [4]. It is known that superheavy particles $X$ with masses in the range $10^{11}$ GeV $\leq m_X \leq 10^{14}$ GeV might well have been produced naturally through the interaction of the vacuum with the gravitational field during the reheating period of the Universe following inflation in numbers sufficient to provide superheavy dark matter [10]. As was pointed out in [5], cryptons have just the right properties to be produced in this way, in particular, because their expected masses $\sim \Lambda_4$ fall within the preferred range.

In general, tetrons may decay through Nth-order nonrenormalizable operators in the superpotential, which would yield lifetimes that are expected to be of the order of

$$\tau \approx \frac{\alpha^{2−N}}{m_X^2} \left( \frac{M_S}{m_X} \right)^{2(N−3)},$$

where $m_X$ is the tetron mass and $M_S \sim 10^{18}$ GeV is the string scale. The $\alpha$-dependent factor reflects the expected dependence of high-order superpotential terms on the effective gauge coupling $g$. If some tetrons can decay only via higher-order interactions with $N \geq 8$, the tetron might be much longer-lived than the age of the Universe, in which case it might be an important form of cold dark matter [11]. However, no significant fraction of the astrophysical cold dark matter could consist of charged tetrons, as these would have been detected directly [12–14]. If the neutral tetrons are close to the experimental limit in $(m_X, \tau_X)$ space, with lifetimes in the range $10^{15}$ years $\leq \tau_X \leq 10^{22}$ years [11], an additional possibility is that their decays might explain the UHECRs observed by the Akeno Giant Air Shower Array collaboration [1], if these turn out to exceed significantly the Greisen-Zasputin-Kuzmin cutoff [2,5,15].

We make in this paper a detailed study of cryptons in the minimal flipped SU(5) string model [9]. A survey of nonrenormalizable superpotential terms up to tenth-order enables us to investigate whether neutral tetrons might live long enough to constitute cold dark matter, and whether charged tetrons are likely to have had lifetimes short enough to avoid being present in the Universe today. We also study whether the decays of neutral tetrons could...
generate the UHECR. We indeed find that charged tetrons would have decayed into neutral tetrons in the early universe through lower-order interactions, while neutral tetrons decay through higher-order interactions with a lifetime that makes them a potential source for the UHECRs, as well as being attractive candidates for cold dark matter.

II. FIELD AND PARTICLE CONTENT IN THE FLIPPED SU(5) MODEL

Already before the advent of string models, flipped SU(5) attracted interest as a grand unified theory in its own right, principally because it did not require large and exotic Higgs representations and avoided the straitjacket of minimal SU(5) without invoking all the extra gauge interactions required in larger groups such as SO(10) [16,17]. Interest in flipped SU(5) increased in the context of string theory, since simple string constructions could not provide the adjoint and larger Higgs representations required by other grand unified theories. Moreover, it was observed that flipped SU(5) provided a natural “missing-partner” mechanism for splitting the electroweak-doublet and color-triplet fields in its five-dimensional Higgs representations [9]. We now review the properties of the favored version of the flipped SU(5) model derived from string theory, before discussing how, as an added bonus, it can accommodate UHECRs.

In a field-theoretic “flipped SU(5) ⊗ U(1) model the Standard Model states occupy \( \tilde{5}, 10 \), and 1 representations of the 16 of SO(10), with the quark and lepton assignments being “flipped” \( u_L^i \leftrightarrow d_L^i \) and \( \nu_L^i \leftrightarrow e_L^i \) relative to a conventional SU(5) grand unified theory (GUT):

\[
\begin{pmatrix}
  u^i_L \\
  d^i_L \\
  e
\end{pmatrix}
= 
\begin{pmatrix}
  F_{10} \\
  F_{\tilde{10}}
\end{pmatrix}
\begin{pmatrix}
  u^i_L \\
  d^i_L \\
  e
\end{pmatrix}.
\]

In particular, this results in the 10 containing a neutral component with the quantum numbers of \( \nu_{L}^i \). Spontaneous GUT symmetry breaking can be achieved by using a 10 and \( \tilde{10} \) of superheavy Higgs where the neutral components develop a large vacuum expectation value (VEV), \( \langle \nu_{H}^i \rangle = \langle \nu_{\tilde{H}}^i \rangle \).

\[
H_{10} = \{Q_H, d_{H}^{i}, \nu_{H}^{i}\}; \quad H_{\tilde{10}} = \{Q_{\tilde{H}}, d_{\tilde{H}}^{i}, \nu_{\tilde{H}}^{i}\},
\]

while the electroweak spontaneous breaking occurs through the Higgs doublets \( H_2 \) and \( H_{\tilde{2}} \),

\[
h_{5} = \{H_{2}, H_{3}\}; \quad h_{\tilde{5}} = \{H_{\tilde{2}}, H_{\tilde{3}}\}.
\]

The presence of a neutral component in the 10 and \( \tilde{10} \) of Higgs fields provides a very economical doublet-triplet splitting mechanism which gives a large mass to the Higgs triplets \( (H_2, H_{\tilde{3}}) \) while keeping Higgs doublets \( (H_1, H_{\tilde{1}}) \) light through trilinear superpotential couplings of the form,

\[
FFh \rightarrow d_H^i \langle \nu_{H}^i \rangle H_3
\]

\[
\tilde{F} \tilde{F} \tilde{h} \rightarrow \tilde{d}_{\tilde{H}}^i \langle \tilde{\nu}_{\tilde{H}}^i \rangle H_3.
\]

Thus, in contrast to GUTs based upon other groups such as SU(5), SO(10), etc., flipped SU(5) does not require any adjoint Higgs representations. As a direct consequence of this, it is the only unified model that can be derived from string theory with a \( k = 1 \) Kac-Moody algebra [3]. As an added bonus, this dynamic doublet-triplet splitting does not require or involve any mixing between the Higgs triplets leading to a natural suppression of dimension five operators that may mediate rapid proton decay.

String-derived flipped SU(5) was created within the context of the free-fermionic formulation, which easily yields string theories in four dimensions. This model belongs to a class of models that correspond to compactification on the \( Z_2 \times Z_2 \) orbifold at the maximally symmetric point in the Narain moduli space [18]. At the string scale, the full gauge symmetry of the model is SU(5) ⊗ U(1) ⊗ U(1)^4 ⊗ SO(10)_h ⊗ SU(4)_h, and the spectrum contains the following massless fields [9].

A. Observable-sector

This comprises three 16 representations of SO(10), that contain the SU(5) ⊗ U(1) chiral multiplets \( F_i(10, \frac{1}{2}), \tilde{F}_i(\tilde{5}, \frac{3}{2}), l_i(1, \frac{1}{2}) \) \((i = 1, 2, 3)\); extra matter fields \( F_{4i}(10, \frac{1}{2}), F_{ai}(\tilde{5}, \frac{3}{2}), \tilde{l}_i(1, -\frac{1}{2}), \tilde{F}_{ai}(\tilde{5}, -\frac{1}{2}) \); and four Higgs-like fields in the 10 representation of SO(10), that ⊆ SU(5) ⊗ U(1) representations \( h_i(5, -1), \tilde{h}_i(\tilde{5}, 1), i = 1, 2, 3, 45.\)

A viable string-derived flipped SU(5) model must contain the Standard Model in its light, low-energy spectrum, while all other observable fields should have masses sufficiently high to have avoided production at particle accelerators or observation in cosmic rays. Additionally, there must be two light Higgs doublets. As we discuss below, these two objectives have been achieved in some specific variants [19,20] of the flipped SU(5) model, although the exact flavor assignments of these states corresponding to those of the standard model particle content is rather model-dependent. However, a convenient choice for the flavor assignments of the fields up to mixing effects is as follows:

\[
\tilde{f}_{i}; u, \tau; \quad \tilde{f}_{2}; \tilde{e}, e/\mu; \quad \tilde{f}_{3}; \tilde{\mu}, \mu/e
\]

\[
F_{3}; Q_{2}, \tilde{s}; \quad F_{3}; \tilde{Q}_{1}, \tilde{d}; \quad F_{4}; Q_{3}, \tilde{b}
\]

\[
l_{i}; \tilde{\tau}, l_{3}; \tilde{\tilde{e}}; \quad l_{5}; \tilde{\tilde{u}}.
\]
B. Singlets

There are ten gauge-singlet fields \( \phi_{45}, \phi^+, \phi^-, \phi_i (i = 1, 2, 3, 4) \), \( \Phi_{12}, \Phi_{23}, \Phi_{34} \), their ten 'barred' counterparts, and five extra fields \( \Phi_i (i = 1 \cdots 5) \).

C. Hidden-Sector

This contains 22 matter fields in the following representations of \( SO(10)_h \otimes SU(4)_h \): \( T_i (10, 1), \Delta_i (1, 6) \times (i = 1 \cdots 5); \tilde{F}_1 (1, 4), \tilde{F}_i (1, 4) (i = 1 \cdots 6) \). Flat potential directions along which the anomalous combination of hypercharges \( U(1)_h \) is cancelled induce masses that are generally near the string scale for some, but not all, of these states. Depending upon the number of \( T_i \) and \( \Delta_i \) states remaining massless, the \( SO(10) \) condensate scale is \( 10^{14-15} \) GeV and the \( SU(4) \) condensate scale is \( 10^{11-13} \) GeV [21]. The \( \tilde{F}_{3,5} \) and \( \tilde{F}_{3,5} \) states always remain massless down to the condensate scale. The \( U(1) \) charges and hypercharge assignments are shown in Table I below.

In order to preserve D and F flatness, many of the singlet fields can develop vacuum expectation values, as can some of the hidden-sector fields. Many of these flat directions have been studied in detail [22]. Typically, we have \( \langle \Phi_{23}, \Phi_{31}, \Phi_{23}, \Phi_{31}, \Phi_{45}, \Phi^+, \phi^- \rangle \neq 0 \), while it can be shown that there is no solution unless \( \langle \Phi_3, \Phi_{12}, \Phi_{12} \rangle = 0 \). The phenomenological details of a particular model depend upon the flat direction which is chosen.

### Table I

<table>
<thead>
<tr>
<th>State</th>
<th>( SU(4) \otimes SO(10) )</th>
<th>( U_1 (1) )</th>
<th>( U_2 (1) )</th>
<th>( U_3 (1) )</th>
<th>( U_4 (1) )</th>
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<tbody>
<tr>
<td>( \Delta_1 )</td>
<td>( (6, 1)^0 )</td>
<td>0</td>
<td>-1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta_2 )</td>
<td>( (6, 1)^0 )</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta_3 )</td>
<td>( (6, 1)^0 )</td>
<td>-1/2</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>( \Delta_4 )</td>
<td>( (6, 1)^0 )</td>
<td>0</td>
<td>-1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta_5 )</td>
<td>( (6, 1)^0 )</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>( (1, 10)^0 )</td>
<td>0</td>
<td>-1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>( (1, 10)^0 )</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>( (1, 10)^0 )</td>
<td>-1/2</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>( T_4 )</td>
<td>( (1, 10)^0 )</td>
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<td>-1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( T_5 )</td>
<td>( (1, 10)^0 )</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( \tilde{F}_1 )</td>
<td>( (4, 1)^{+5/4} )</td>
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<td>-1/2</td>
<td>0</td>
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<td>1/2</td>
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<tr>
<td>( \tilde{F}_3 )</td>
<td>( (4, 1)^{-5/4} )</td>
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<td>1/2</td>
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<tr>
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<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>( \tilde{F}_5 )</td>
<td>( (4, 1)^{+5/4} )</td>
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<td>-1/2</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>( \tilde{F}_6 )</td>
<td>( (4, 1)^{+5/4} )</td>
<td>0</td>
<td>-1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
</tbody>
</table>

The superheavy Higgs \( H_{10} \) can, in general, be a linear combination of \( F_1, F_2, F_3, \) and \( F_4 \), while \( H_{10} = F_5 \). The Higgs doublet matrix takes the following form, including terms up to 5th order in the superpotential:

\[
m_h = \begin{pmatrix}
0 & \Phi_{12} & \tilde{F}_{12} & T_{2}^{2} \tilde{F}_{45} \\
\Phi_{12} & 0 & \Phi_{23} & \Delta_{2}^{2} \Phi_{45} \\
\Phi_{31} & \Phi_{23} & 0 & \Phi_{45} \\
\Delta_{2}^{2} & T_{2}^{2} \tilde{F}_{45} & \Phi_{45} & 0
\end{pmatrix}
\]  

(10)

If only all-order contributions generated by singlet VEVs are considered, \( H_1, H_{245} = \cos \theta H_2 - \sin \theta H_{45}, R_{12} = \cos \theta H_1 - \sin \theta H_2, \) and \( H_{45} \) light, where tan\( \theta \) = \( \langle \Phi_{23} \rangle / \langle \Phi_{45} \rangle \) and \( \tan \theta = \langle \Phi_{31} \rangle / \langle \Phi_{23} \rangle \). The \( \langle TT \rangle \) in the Higgs doublet matrix give additional structure. With the choice \( \langle \Phi_{12}, \Phi_{12} \rangle = 0 \) and with the additional constraints \( \langle \Delta_{2}^{2} \rangle = 0 \) and \( \langle T_{2}^{2} \rangle = 0 \), the massless Higgs doublet eigenstates are identified as \( H_2 = H_1 \) and \( H_2 = H_{45}. \) Similarly, the Higgs triplet mass matrix can be formed, and it is found that all of the Higgs triplets become massive [19].

If the state \( F_\beta \propto -(F_3)F_1 + (F_1)F_3 \) is the linear combination of the quarks and leptons with the specific string representations can be made

\[ t b\nu_{\tau} : Q_4 \bar{d}_4 u_3 \bar{s}_1 l_1 \bar{l}_1, \]  

(11)

\[ c s \mu \nu_{\mu} : Q_2 \bar{d}_2 u_3 \bar{s}_1 l_2 \bar{l}_2, \]  

(12)

\[ u d e \nu_{\tau} : Q_3 \bar{d}_3 u_3 \bar{s}_1 l_3 \bar{l}_3. \]  

(13)

In addition to the above states which have been identified with those of the Standard Model, there are extra states \( \tilde{f}_3 \) and \( \tilde{l}_3 \), as well as 'exotic states' \( f_4 \) and \( \tilde{l}_4 \), which should not appear in the light spectrum. In particular, there are 5th order superpotential terms that contain \( f_3 \) and \( \tilde{l}_3 \) which can generate dimension-five operators leading to rapid proton decay. Fortunately, there are superpotential terms [19] of the form

\[ f_4 \sum_i \alpha_i \tilde{f}_i \tilde{l}_i \sum_i \alpha_j f_j, \]  

(14)

which allow these states to become heavy.

The singlet fields also may potentially obtain masses. The relevant trilinear couplings involving the singlet fields are

\[ \frac{1}{2}(\phi_{45} \bar{\phi}_{45} \Phi_3 + \phi^+ \bar{\phi}^+ \Phi_3 + \phi^- \bar{\phi}^- \Phi_3 + \phi_i \bar{\phi}_i \Phi_3) \]

\[ + (\phi_1 \bar{\phi}_2 + \bar{\phi}_1 \phi_2) \Phi_4 + (\Phi_{12} \Phi_{23} \Phi_{31}) \]

\[ + \Phi_{12} \phi^+ \phi^- + \Phi_{12} \phi_i \phi_j + h.c., \]  

(15)

from which it is clear that having \( \langle \Phi_3 \rangle \neq 0 \) would give trilinear mass terms for \( \phi_{45}, \phi^+, \phi^- \), \( \phi_i \) and their barred counterparts. However, \( \langle \Phi_3 \rangle = 0 \) is required. Moreover, we have the result [19]
III. CRYPTON BOUND STATES

Since the strong-interaction scale for the SU(4) factor in the hidden-sector is expected to lie below that for the SO(10) factor, we concentrate on the states bound by the hidden-sector SU(4) interactions. These include “holomorphic” mesons with the contents $T_i T_j, \Delta_i \Delta_j$ and $\tilde{F}_i \tilde{F}_j$, “nonholomorphic” mesons with the contents $T_i T'_j, \Delta_i \Delta'_j$ and $\tilde{F}_i \tilde{F}'_j$, baryons with the contents $\tilde{F}_i \tilde{F}_j \Delta_k$ and $\tilde{F}_i \tilde{F}_j \Delta_k$, and quadrilinear tetrons, with the contents of four $\tilde{F}_i$ and/or $\tilde{F}_i$ fields and/or their complex conjugates. We assume that the baryons are heavier than the lightest tetrons, which are expected to be Bogomolnyi-Prasad-Sommerfield-like holomorphic states with the quantum numbers of $\tilde{F}_i \tilde{F}_j \tilde{F}_k \tilde{F}_{l'}$ and $\tilde{F}_i \tilde{F}_j \tilde{F}_k \tilde{F}_{l'}$, where $i, j, k, l = 3, 5$. Nonholomorphic tetrons with the quantum numbers of $\tilde{F}_i \tilde{F}_j \tilde{F}_k (\tilde{F}_i^\dagger)^* (\tilde{F}_j^\dagger)^* (\tilde{F}_k^\dagger)^*$, etc., are generally expected to be heavier, although this remains to be proven. We assume that, by analogy with QCD, these excited states have short lifetimes.

Crypton bound states occur in cryptospin multiplets with different permutations of confined constituents, analogous to the flavor SU(3) and SU(4) multiplets of bound states in QCD. We recall that the observable-sector non-Abelian gauge interactions do not act on the hidden-sector supermultiplets, and assume masses $\gg \Lambda_4$ for all the U(1) gauge supermultiplets except that in the Standard Model, in which case they also do not contribute significantly to the cryptospin mass splittings. Two classes of diagrams are likely to contribute to the mass differences between cryptospin partners: electromagnetic “self-energy” diagrams and the photon-exchange “Coulomb potential” diagrams. We do not enter here into a discussion of which of these classes of diagrams is likely to dominate for which cryptospin multiplets, as this is not essential for our purposes.

We expect these diagrams to have the following orders of magnitude:

$$O\left(\frac{\alpha}{\pi}\right) \Lambda_4 \times \{a) \Sigma_i Q_i^2, b) Q_T^2\},$$

where the $Q_i$ are the charges of the tetron constituents, and $Q_T$ is the total tetron charge. It is easy to check the well-known fact that both of these terms make positive contributions to both the $\pi^+ - \pi^0$ and $p - n$ mass differences. The former agrees with experiment in sign and order of magnitude, and the difference of the latter from experiment is explained by the difference between the $u$ and $d$ quark masses, so one may have some confidence in the qualitative estimates in (17).

Each of the dependences in (17) would give $m_{\tau^+} > m_{\tau^0}$. We therefore expect the doubly-charged tetrons

$$\Psi^{--} = \tilde{F}_3 \tilde{F}_3 \tilde{F}_3 \tilde{F}_3, \quad \Psi^{++} = \tilde{F}_3 \tilde{F}_3 \tilde{F}_3 \tilde{F}_3, \quad \Psi^{-+} = \tilde{F}_3 \tilde{F}_3 \tilde{F}_3 \tilde{F}_3,$$

and for the singly-charged states

$$\Psi^+ = \tilde{F}_3 \tilde{F}_3 \tilde{F}_3 \tilde{F}_3, \quad \Psi^- = \tilde{F}_3 \tilde{F}_3 \tilde{F}_3 \tilde{F}_3, \quad \Psi^0 = \tilde{F}_3 \tilde{F}_3 \tilde{F}_3 \tilde{F}_3.$$

IV. THE DECAYS OF THE LIGHTEST SU(4) MESONS

We first discuss the decays of the lightest hidden-sector SU(4) bound-state mesons. In analogy with QCD chiral symmetry breaking, it is expected that there will be an isorotiplet of crypto-pions that could play the role of pseudo-Nambu-Goldstone bosons, with masses that are small compared to $\Lambda_4$. Specifically, the charged SU(4) pion states

$$\pi^\pm = (\tilde{F}_3 \tilde{F}_3 \tilde{F}_3),$$

are expected to have masses

$$m_{\pi^\pm}^2 = \Lambda_4 \times (m_3 + m_5),$$

where $m_{3,5}$ are the bare masses of the fractionally-charged constituents, which are expected to be $< \Lambda_4$, as we discussed above. The neutral SU(4) pion state

$$\pi^0 = \frac{1}{\sqrt{2}} (\tilde{F}_3 \tilde{F}_3 \tilde{F}_3 - \tilde{F}_3 \tilde{F}_3 \tilde{F}_3),$$

is expected to be lighter by an amount

$$m_{\pi^0}^2 = \left(\frac{\alpha}{\pi}\right) \Lambda_4^2 \ln (\Lambda_4^2 / m_{\pi^\pm}^2).$$

The cryptospin-zero state

$$\eta^0 = \frac{1}{\sqrt{2}} (\tilde{F}_3 \tilde{F}_3 + \tilde{F}_3 \tilde{F}_3)$$

is expected to be significantly heavier because of a $U_A(1)$ anomaly.
We find that there are $N = 3$ superpotential terms of the form
\[ \tilde{F}_1 \tilde{F}_3 \Phi_3 - \tilde{F}_2 \tilde{F}_3 \Phi_{12} \] (28)
that would allow the crypto-$\pi^0$ and -$\eta^0$ mesons to decay very rapidly. Additionally, we expect the crypto-$\pi^0$ and -$\eta^0$ states to have couplings to pairs of photon supermultiplets, analogous to those of the QCD $\pi^0$ and $\eta^0 \rightarrow \gamma \gamma$. These couplings would be described in an effective supergravity Lagrangian by terms in the chiral gauge kinetic function $f$ of the form $\alpha \pi / \Lambda_4$ and $\alpha \eta / \Lambda_4$, where $\Pi$, $\eta$ denote composite superfields and $\Lambda_4$ is the scale at which the hidden-sector SU(4) interactions become strong. As in the case of the QCD $\pi^0$ decaying to $\gamma \gamma$, these couplings would give very short lifetimes for the crypto-$\pi^0$ and -$\eta^0$ states. It is also possible that in some variant models the crypto-$\pi^0$ and -$\eta^0$ might have additional decays, analogous to those of the QCD $\eta^0$, which would further shorten their lifetimes.

In the case of the charged crypto-pions, we find terms of the form
\[ \pi^- (F_2 F_3 F_4 \tilde{h}_{4s} + F_3 F_4 \Phi_4 f_{4s} + F_4 \Phi_4 h_{1s} \tilde{f}_5^s + F_3 h_{45} \tilde{f}_{2s}^* + F_5 h_{45} \tilde{f}_{5s}^*), \] (29)
and
\[ \pi^+ (F_4 \Phi_3 \tilde{f}_5 + \tilde{h}_{45} f_{4s} \tilde{f}_4^s + \tilde{h}_{45} f_{4s} \tilde{f}_5^s). \] (30)
that would allow the $\pi^+$ states to decay fairly rapidly.

There would also be a complex spectrum of heavier nonholomorphic SU(4) bound-state mesons, analogous to the $\rho$ and heavier mesons of QCD, but we expect them all to be very unstable, and do not discuss them further. Likewise, we do not discuss mesons made of the higher SU(4) representations $\Delta_1$, or $FF\Delta$ cryptobaryons, or SO(10) bound states, as these have been studied previously in [4].

V. THE FATE OF THE NEUTRAL TETRONS

As discussed above, we expect the lightest tetrons to be the electrically neutral states. These can decay only through higher-order nonrenormalizable superpotential terms, for which the first candidates appear at eighth order
\[ \Psi^0 F_4 \phi^- \tilde{h}_{2s} \tilde{f}_5. \] (31)
\[ \Psi^0 \phi^+ \tilde{h}_{45} f_{4s} \tilde{f}_4^s. \] (32)
At ninth order, we find terms involving neutral tetrons of the following forms:
\[ \Psi^0 (\Phi_{31} f_{4s} \tilde{f}_{3s} \tilde{f}_5 + \Phi_{31} f_{4s} \tilde{f}_{3s} \tilde{f}_5 + \Phi_{31} \tilde{f}_{4s} \tilde{f}_{3s} \tilde{f}_5), \] (33)
\[ \Psi^0 (F_1 \phi_1 \phi^- \tilde{h}_{2s} \tilde{f}_5 + F_2 \phi_2 \phi^- \tilde{h}_{2s} \tilde{f}_5 + F_2 \phi_4 \phi^- \tilde{h}_{2s} \tilde{f}_5 + F_2 \phi_4 \phi^- \tilde{h}_{2s} \tilde{f}_5). \] (34)
All of these eighth and ninth order terms contain fields which are expected to have large masses, so we do not expect that these decay modes would be kinematically accessible. The next terms yielding possible neutral tetron decays are of tenth order. There are a large number of such terms, of which the following are those containing only fields that are light in the model:
\[ \Psi^0 (F_2 F_3 \Phi_{31} \tilde{f}_{45} \phi^- h_{1s} + F_2 F_3 \phi_4 \phi_4 \tilde{h}_{45} \tilde{f}_5^s + F_4 \Phi_3 \phi_4 \phi_4 \tilde{h}_{45} \tilde{f}_5^s + (\Phi_{31} \phi_4 \phi^- + \Phi_{32} \phi_4 \phi^+) h_{1s} (f_{3s} \tilde{f}_2^s + \tilde{f}_3 \tilde{f}_5^s), \] (35)
\[ \Psi^0 (F_2 F_4 \phi_4 \phi^- h_{1s} + F_2 F_4 \phi_4 \phi^- h_{1s} + F_2 F_4 \phi_4 \phi^- h_{1s} + F_4 \Phi_3 \phi_4 \phi_4 \tilde{h}_{45} \tilde{f}_5^s + (\Phi_{31} \phi_4 \phi^+ h_{1s} + h_{1s} \tilde{h}_{45} \tilde{f}_5^s + \Phi_{32} \phi_4 \phi^+ h_{1s} + \tilde{h}_{45} \tilde{h}_{45} \tilde{f}_5^s + \phi^- h_{1s} \tilde{h}_{45} \tilde{f}_5^s) \] (36)
These tenth order interactions would have a lifetime $\sim 10^{17} - 10^{32}$ years for the mass range $\sim \Lambda_4 = 10^{12} - 10^{13}$ GeV and $M_\tau = 10^{17} - 10^{18}$ GeV. These interactions involve multiparticle decays involving both particles and supersymmetry (SUSY) partners, within the constraints of R-parity and charge conservation. Although there are many of these decay interactions some general comments can be made. Almost all of them contain Higgs fields which would tend to decay (depending upon what the mass of the Higgs turns out to be) into $W^\pm$, quark-antiquark pairs (neutral Higgs) or $\tau$ leptons (charged Higgs), or remain as Lightest Supersymmetric Particle (LSP) if they are Higgsinos, assuming Higgsinos compose a fraction of LSP. Since the Higgs couple to heavier particles, we would expect $\tilde{H}_{45}$ to decay most strongly to a pair the heaviest up-type quark allowed by kinematics, which is expected to be the c-quark. Similarly, we would expect the $H_{1s}$ to decay most strongly to $\tau^\pm$, and to pairs of b-quarks. Furthermore, most of the decay interactions contain many Higgs fields as well as $10$ and $5$ fields which may also produce quarks and antiquarks. Thus, several such pairs are expected to be created. These decay interactions also all involve several singlet fields which could decay into observable particles if their mass is great enough, or remain as hot-dark matter if is not.
VI. THE FATE OF THE CHARGED TETRONS

The lifetimes and abundances of charged tetrons have recently been discussed by Coriano et al. [23], who have raised questions about their lifetimes and abundances relative to those of the neutral tetrons. In particular, they pointed out that if the only ways for the charged tetrons to decay are through the same higher-order nonrenormalizable operators that govern the decays of the neutral tetrons, then, if the neutral tetrons are long-lived, so also would be the charged tetrons, and they would probably have comparable cosmological abundances. Since there are very strong constraints on stable charged matter [12–14], it was argued in [23] that tetrons could not be good candidates for dark matter.

Indeed, we do find ninth order superpotential terms involving charged tetrons that correspond to the annihilations of their constituents:

\[ \tilde{\Psi}^{\pm+}(\Phi_{31}\tilde{\phi}^{\pm}\tilde{\phi}^{-}\tilde{l}_{4}^{\pm}\tilde{l}_{4}^{\pm} + \Phi_{23}\phi^{\pm}\phi^{-}\tilde{l}_{4}^{\pm}\tilde{l}_{4}^{\pm}), \]  
\[ \tilde{\Psi}^{-+}F_{3}\phi^{+}h_{1}f_{4}^{1}, \]  
\[ \Psi^{-}(\Phi_{31}\tilde{\phi}^{+}\tilde{\phi}^{-}\tilde{l}_{4}^{+}\tilde{l}_{4}^{+} + \Phi_{31}\tilde{\phi}^{+}\tilde{\phi}^{-}\tilde{l}_{4}^{+}\tilde{l}_{4}^{+} + \Phi_{33}\phi^{+}\phi^{-}\tilde{l}_{4}^{+}\tilde{l}_{4}^{+} + \Phi_{23}\phi^{+}\phi^{-}\tilde{l}_{4}^{+}\tilde{l}_{4}^{+} + \Phi_{12}\phi^{+}\phi^{-}\tilde{l}_{4}^{+}\tilde{l}_{4}^{+}), \]

where the \( \tilde{\Psi}^{\pm+} \) and \( \Psi^{-} \) are the only ways for charged tetrons to decay, they would have lifetimes similar to those of the neutral tetrons. Moreover, there are no superpotential terms corresponding to decays of \( \Psi^{++}, \Psi^{--}, \Psi^{+}, \) or \( \tilde{\Psi}^{-} \) that appear before tenth order, which would correspond to even longer lifetimes.

However, there is another mechanism which enables the heavier (charged) members of cryptospin multiplets to decay relatively rapidly into the lightest (neutral) isospin partner, analogous to the \( B \) decay of the neutron into its lighter isospin partner in QCD, the proton. We recall that neutron decay is generated by a four-fermion interaction of the type \( (du\bar{u}e)/m_{N}^{2} \), which leads to an effective neutron decay interaction of the form \( (\tilde{n}\tilde{p}\bar{e}/m_{N}^{2}) \). This then leads to a neutron decay rate \( \Gamma_{n} \approx (\delta m)^{3}/m_{N}^{2} \), where \( \delta m \) is the neutron-proton mass difference. In the case of charged-crypton decay, we expect there to exist a cryptostrong-interaction of the form

\[ \frac{\tilde{C}^{+}\tilde{C}^{-}\cos(\pi^{+}\delta\pi^{0})}{\Lambda_{4}^{2}}. \]  

(42)

where the \( C^{+,0} \) are charged and neutral-crypton fields. If the \( C^{+,0} \) mass difference \( \Delta M \) were larger than \( m_{\pi^{+}} + m_{\pi^{0}} \), the \( C^{+} \) decay rate would be very rapid: \( \Gamma_{C^{+}} \approx (\Delta M)^{3}/\Lambda_{4}^{2} \). However, we expect that \( \Delta M < m_{\pi^{+}} + m_{\pi^{0}} \), in which case the two crypto-pions must be virtual. In this case, the lowest-order decay interaction becomes

\[ \alpha\Delta M\tilde{C}^{+}\tilde{C}^{0}\bar{F}\bar{F}B_{1}B_{2}. \]  

(43)

where \( F \) denotes the Maxwell field strength and \( \bar{F} \) its dual, \( B_{1,2} \) denote generic Minimal Supersymmetric Standard Model bosons, \( M_{s} \) is the string scale, and \( \alpha = \alpha(\Lambda_{4}) \). If the \( \pi^{+} \) can only decay through higher-order interactions, (43) would be replaced by effective interactions with more inverse powers of \( M_{s} \). Setting \( \Delta M \approx \alpha(\Lambda_{4}) \) as suggested by (17), and assuming the minimum values \( m_{\pi^{+}}, m_{\pi^{0}} \approx \alpha(\Lambda_{4})^{2} \) allowed by (26), interactions of the form (43) would yield decay rates of order

\[ \Gamma_{C^{+}} \approx \frac{\alpha^{11}\Lambda_{4}^{3}}{\epsilon_{s}M_{s}^{2}}. \]  

(44)

with additional factors of \( (\Delta M/M_{s})^{2} \approx (\alpha(\Lambda_{4})/M_{s})^{2} \) for higher-order \( \pi^{+} \) decay interactions. In the case of the interactions (18) in our particular flipped SU(5) model, we would pick up an extra factor of \( (\alpha(\Lambda_{4})/M_{s})^{4} \).

In this case, we estimate a charged-crypton lifetime \( \tau_{\pm} \approx 10^{2} - 10^{9} \) years for \( \Lambda_{4} \approx 10^{13} - 10^{12} \) GeV and \( \Lambda_{s} \approx 10^{17} \) GeV. For the same range of \( \Lambda_{4} \) and \( \Lambda_{s} \approx 10^{18} \) GeV, we estimate a charged-crypton lifetime of \( \tau_{\pm} \approx 10^{8} - 10^{14} \) years. These charged-tetron lifetimes are much shorter than what we expect for the neutral tetrons. For comparison, with the same values of \( \Lambda_{4} \) and \( \Lambda_{s} \), taking \( M_{X} = \Lambda_{4} \) and assuming a ninth-order neutral-crypton decay interaction, we estimate a neutral-crypton lifetime \( \tau_{0} \approx 10^{13} - 10^{26} \) years for \( \Lambda_{4} = 10^{13} - 10^{12} \) GeV and \( M_{s} = 10^{17} \) GeV, and \( \tau_{0} \approx 10^{25} - 10^{38} \) years for the range of neutral mass and \( M_{s} = 10^{18} \) GeV. In particular, \( \tau_{0} \approx 10^{5} \) years and \( \tau_{0} \approx 10^{10} \) years if \( 3 \times 10^{12} \) GeV \( \leq \Lambda_{4} \leq 2 \times 10^{13} \) GeV with \( M_{s} = 10^{17} \) GeV and \( 3 \times 10^{13} \) GeV \( \leq \Lambda_{4} \leq 2 \times 10^{14} \) GeV with

1In QCD, the \( W^{-} \) couples to \( \tilde{n}\tilde{p} \) via a strongly-interacting vector-meson \( \rho^{-} \). By an analogous vector-meson dominance argument, one could consider the interaction (42) as being mediated by the exchange of a nonholomorphic crypto-\( \rho \) meson.
\( M_\ell = 10^{18} \text{ GeV} \). In fact, it is possible to choose a value for \( \Lambda_\ell \) in the expected range such that \( \tau^\pm \approx 10^9 \text{ years} \) and \( \tau^0 > 10^{10} \text{ years} \) for all values of \( M_\ell \) between \( 10^{17} - 10^{18} \text{ GeV} \). Thus, it is always possible to choose reasonable values of these parameters such that neutral tetrons will have a lifetime longer than the present age of the Universe while the charged tetrons will have decayed prior to photon-matter decoupling. Therefore, neutral tetrons will have a lifetime longer than the present age of the Universe. Thus, the flipped SU(5) string model does not predict the existence of any charged cold dark matter. Time will tell whether the UHECRs are in fact due to the decays of ultraheavy particles, but the flipped SU(5) string model seems to have the appropriate characteristics for this to be possible, as well as providing possible cold dark matter candidates in the forms of its neutral tetron bound states. We believe that these properties along with the other successes of string-derived flipped SU(5) such as dynamic double-triplet splitting and natural suppression of dimension-5 operators that mediate rapid proton decay make this model particularly attractive and should strongly motivate future study.

**VII. CONCLUSION**

We have made in this paper a detailed study of crypton decays in a specific flipped SU(5) string model. We have shown that there are neutral tetrons that are naturally metastable in this string model, with lifetimes long enough to make perfect candidates for cold dark matter and possibly act as sources of UHECRs. Moreover, their charged cryptospin partners naturally decay much more rapidly, with lifetimes that may be much shorter than the age of the Universe. Thus, the flipped SU(5) string model does not predict the existence of any charged cold dark matter. Time will tell whether the UHECRs are in fact due to the decays of ultraheavy particles, but the flipped SU(5) string model seems to have the appropriate characteristics for this to be possible, as well as providing possible cold dark matter candidates in the forms of its neutral tetron bound states. We believe that these properties along with the other successes of string-derived flipped SU(5) such as dynamic double-triplet splitting and natural suppression of dimension-5 operators that mediate rapid proton decay make this model particularly attractive and should strongly motivate future study.

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