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MAINTAINING THE GEOMETRICAL ALIGNMENT OF A
LARGE ACCELERATOR

A Preliminary Theoretical Study

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In the planning of a very large alternating gradient type accelerator, it is generally expected that some means for checking the geometrical alignment at regular intervals of time, and of making the indicated mechanical corrections will have to be provided. If a means can be devised such that we can look forward to being able to correct the alignment easily and with little loss in machine running time, the effort of finding a site with suitable structure, of testing the stability of the site and of designing a foundation for the machine will be very much lightened.

To try to put into more quantitative terms what we might be willing to accept as satisfactory performance in an alignment scheme, let us adopt the following statements, which, although arbitrary, seem to be reasonable. (1) We will be willing to devote up to half a day every second week to routine checking and adjustment. This amount of time will be available in the

normal course of scheduling in such a way that no loss will be caused specifically by the alignment operation.

(2) The amount of misalignment that we can allow to accumulate between adjustments will be .02". Theory indicates that this is a reasonably safe guess. The figure .02" applies to the constancy of the radius, in going around the machine, and to altitude, or the degree of flatness. .02" per two weeks will amount to half an inch a year. This may seem at first sight to be a very large amount of movement, but actually, it is just the amount which has been recorded in the measurements on the CERN site at Geneva (CERN-PS/AED 2, July 1955 and CERN-PS/AED 3, Sept. 1955). While it is altogether possible that the site selected by MURA will be more stable than the CERN site, it will not be wise to count on more than one order of magnitude less motion. The alignment mechanism will have to have an intrinsic precision considerably better than the .02". It should be capable of observing changes at least five times as small, or .004".

Our object in this paper will be to look, in some detail, into the question of the feasibility of determining changes in lengths and altitudes with the speed and accuracy just prescribed, and to lay out preliminary outlines for the instrumentation which will have to be incorporated into the accelerator in order to do the job.

The magnet of the accelerator will have the form of a ring which is cut up into a number (40 or so) independent segments, with field-free spaces between them. Each segment of the magnet will be built on a rigid bed plate. The adjacent ends of the bed plates will be kept in alignment relative to one another, so we will have in effect a flexible, circular train. Because the short range alignment will be maintained by the accurate matching of one segment end to the next, it will be necessary to apply absolute radius and altitude measurements only at intervals of several segments, say at at least 8 but at not more than 16, places in the circle. Radius measurements will of course be made from a central hub, which will consist of a fixed concrete pier.

Altitude, or flatness

It is not required that the machine be level, but only that it lie in a plane. There are two standard and obvious lines of approach: an optical method, and the use of the free surface of a liquid. The optical method would employ a telescope of some kind, mounted on a rotatable, vertical spindle at the center of the machine. Making the spindle and its pier good enough mechanically would probably not be difficult. The errors introduced by the varying gradient in the index of refraction of the air would, according to the recent experience of the CERN group, not be serious,

providing the measurements were made at the optimum time of day and in favorable weather. Their measurements indicated that under selected conditions the radius of curvature introduced by the refraction effect was 50,000 Km, which makes an error of only 10^{-4} mm at 500 ft. A more serious problem lies in diffraction. If a 3" lense were used in the telescope, the diffraction pattern of a line viewed 500 ft. away would be such as to give the line an apparent width of somewhat more than 1 millimeter. Further, in order to be able to resolve, visually, .02" at 500 ft., a magnification of about 2500 would be necessary. These figures indicate that the optical apparatus would have to be more elaborate than a simple telescope. However, the design of suitable optical equipment would not be at all impossible.

The free surface of a liquid, as a reference for altitude, seems to offer attractive possibilities. For example a glass tube, say one inch in diameter, in the form of a circle the diameter of the machine, and filled half full of clean mercury would provide the necessary continuous free surface. The tube would be supported on the bed plates or just at the side. Simple screw micrometer heads, mounted on the bed plates extending just to the mercury surface would easily give the elevations to within less than .001 inch. If greater sensitivity or rapidity in reading should be desired,

electronic or electrical sensing devices would be available. A more practical construction might be to have a free surface only at each measuring station, and to have all the measuring stations interconnected by means of small, filled tubes. The pitfall to avoid here would be in allowing too much variation in elevation in the plumbing between stations. The linear expansion coefficient of mercury is $6 \times 10^{-5}/\text{deg C}$. A U-tube of mercury 4 cm in height will show a .001 inch difference in elevation between its two surfaces if the two columns differ in temperature by 10°C . Therefore if in the levelling system suggested the free surfaces and the interconnecting tubes were kept within an altitude variation of 4 cm, the effects of temperature variations would be avoided to a sufficient extent. Small ups and downs in the interconnecting tubes would contribute some additional error, but if their temperature variations were random, their effects would come in only as the square root of the sum of their squares.

The time constant of such a mercury system can be made to have any desired value, either through the choice of the size of the tubing connecting the stations with one another, or by means of adjustable constrictions. Enough flow friction can easily be introduced, if needed, to render the system critically damped.

As a preliminary guess, a favorable value for the time constant might lie in the range between 10 minutes and one hour. Such a time constant would filter

out all elastic earth waves, which would be of no interest in any case, and would produce a steady mercury surface for measurement. If the measurements were to be carried out manually, an hour's time might actually be needed for a complete run since the levels of quite a large number of individual stations would have to be determined and reported. Electronic sensing devices, wired in to a central station, would, of course, reduce the time required for a measurement to almost nothing, in which case a time constant of a few minutes, in the mercury system, would be adequate.

Tide effects caused by the moon would probably be observed. We assume that we would not want to try to average over these effects by means of the mercury time constant, but that we would want to plot them, and take them into account in that way. This would place an upper limit of the order of an hour on the time constant. By plotting the tide effects over a long period of time we would be able to take them into account quite exactly.

The accuracy with which the level of the machine could be determined would depend largely upon the length of time used for the measurement, as is the case in an "signal-to-noise" problem. We would be dealing with a combination of random and periodic disturbances. The known periodic disturbances, mainly

from the moon, could certainly be corrected out of the data, to within .001" or less, on the basis of accumulated observations. Passing wave-trains (elastic waves) would be filtered out by the long time constant of the mercury system. There is no way to estimate in advance the "random noise", but it seems reasonable to expect that, in an hour's average, this would not introduce an error of more than .001". However, this point would have to be verified in a preliminary experiment in the area chosen for the location of the machine.

Radius Measurements

For maintaining the constancy of the radius of the machine, in 8 to 16 directions from the center, a type of glorified measuring rod seems to give promise of fulfilling the requirement. The problems associated with a measuring rod are (a) temperature control and (b) force control. To get into perspective, consider the following figures which are worked out for a rod 1/4 inch in diameter and 500 ft. long.

Material	Change in length per Degree C	Change in length per kilogram tension	Wt/ft	Wt. of 500 ft.
Steel	.06"	.0098"	.093kg	45kg
Fuzed silica	.003"	.026"	.025kg	13kg
Tungsten	.026"	.0053"	.22kg	109kg
Invar	.003"	.0088"	.22kg	46kg

Both invar and silica are excellent from the standpoint of temperature coefficient of expansion. However, in almost any method of supporting a long rod, the uncertainty in the tension will be about proportional to the weight of the rod per unit length. For this reason the figure of merit, in regard to the problem of maintaining constant tension, will be the Young's Modulus divided by the density. Because of its low density, silica wins over all the other in this respect. Since it is also just as good as invar in its temperature expansion coefficient, silica seems to be the best choice. There is one other property which may be of some significance, but on which we have very little information, and that is the slow change in dimensions with time after manufacture, that is, the aging effect. We shall, however, proceed for the present using silica as our material.

In designing the measuring rod we shall aim to keep the variation in length to within $\pm .001$ " due to temperature variation and to within the same limits due to variation of tension. The compound precision will then be about $\pm .0014$ ". This will mean that the temperature of the silica will have to be held within $\pm .3^{\circ}\text{C}$ and that the tension will have to be held within ± 40 grams.

To maintain an entire 500 ft. length of rod at constant temperature to within $\pm .3^{\circ}\text{C}$ would require a rather elaborate thermostat system and housing. It will be much easier, instead, to devise a method of

measuring, and maintaining, the correct effective, or average, temperature over the length of rod, and to allow it to vary in temperature from one point to another. To do this we will wind insulated wire on the rod, spaced uniformly at, say 1/8" per turn. We know that the wire, everywhere, will change its resistance with temperature in a nearly straight line relationship, provided the temperature range is not large. Likewise, the rod will change its length with temperature in a nearly straight line relationship. Over any small local part of the rod the resistance of the wire will be related in a precise way to the length of the same part, since both the resistance and the length change with the temperature approximately linearly. Therefore, if we add all the parts together, the total resistance will give us the total length. If the resistance vs temperature and the length vs temperature were both precisely linear relations, we would then be able to determine the total length from the total resistance without regard to variation in temperature along the rod. The relationships are, of course, not exactly linear. However, while linearity is a sufficient condition, it is not the only condition which will permit an exact measurement. If in the two formulas, for expansion and change in resistivity,

$$L = L_0 (1 + \alpha_1 T + \beta_1 T^2 + \dots)$$

$$R = R_0 (1 + \alpha_2 T + \beta_2 T^2 + \dots)$$

$$\alpha_1/\alpha_2 = \beta_1/\beta_2 = \dots$$

there will be a precise relation between L and R, independently of local variations of temperature along the rod. We would hope, therefore, to find a metal for the wire such that the α and β for resistance change would match the α and β for the expansion of the rod, over the range of temperature to be used. We will show that it will be very easy to achieve this, to such a degree that the error introduced into the measurement by the non-linearity will be not more than a few tenths of .001" in a temperature range of, say 5 deg. C.

A way to estimate the effect of non-linearity is the following. We want to measure, by reading the resistance of the wire, the effective temperature of the rod, to within .3°C, while local parts of the rod vary in temperature over a range of at least 5°C. If, in the equation

$$L = L_0 (1 + \alpha T + \beta T^2)$$

$\beta(5)^2$ is small compared to $\alpha(.3)$, we will be justified in neglecting the effect of the quadratic term over the 5° range. We do not have the value of β for silica, but we do have it for tungsten, which can be taken as typical at least in order of magnitude. β is 6×10^{-10} while α is 4.3×10^{-6} . Thus $\beta(5)^2 = 1.5 \times 10^{-8}$ and $\alpha(.3) = 1.3 \times 10^{-6}$. The value of β in the temperature coefficient of resistivity of copper is not accurately known to us, but some measurements made by us indicate that $\beta < 10^{-3}\alpha$. Thus $\beta(5)^2 \ll \alpha(.3)$, meaning that

here again the effect of the non-linearity, in the 5°C temperature range, is far below our required limits of error. Thus, it appears that, if we keep the temperatures at the various points along the rod within a bracket of 5° , or even 10° , the matching of the higher terms in the equations for expansion of the rod and change of resistivity of the wire is an unnecessary refinement. Or, to approach from the other direction, we may expect that if a very approximate match were made, and the problem were looked into more carefully, it would be found that a 20 or 30 degree variation in temperature along the rod could be allowed.

The precision with which the resistance of the wire must be read in order to determine the length of the rod to $\pm .001"$ is readily computed and, if it is copper, the result is $\pm .14\%$. This is a mild requirement for even a simple bridge circuit.

Now let us talk about forces on the rod. The weight of the $1/4"$ silica rod is 13 kilograms and the tension necessary to change its length $.001"$ is 40 grams. If the rod were rested on a surface or a series of rollers, the coefficient of friction would have to be of the order of 10^{-3} . Therefore a better means of support will have to be devised. Suppose we suspend the rod by a series of threads, spaced, say, at intervals of one foot. Let the rod run along the axis of a copper pipe.

Let the suspending threads be 1" long, attached to the "roof" in the pipe. Now we have to consider the horizontal component of force put into the rod by the threads, whose directions with respect to the vertical will change as the copper pipe and the rod expand by different amounts. In an actual case, the copper pipe will expand as the earth does, because it will be securely fastened down. However, let us take the more unfavorable case and say that it expands freely, which will be .1 inch per deg. C in 500 ft. By how much will this cause the rod to stretch, due to the changes in directions of the suspending threads? By evaluating a simple double integral, which we will not bother with presenting here, the result is .0035". This is not good enough. We note, however, that the silica rod changes by only .035 as much as the pipe from which it is suspended does. We therefore repeat the process and suspend a second rod below the first one. The change in length of the second rod is then only .001 as much as the change in length of the pipe, or .0001" for a 1°C change of temperature of the pipe. The first rod will have to have the same temperature as the second, but the requirement here is not severe. For the temperature differential between the first and second rods to produce a .001" error in the length of the second, the average temperatures of the rods will have to differ by 10° C. This requirement will easily be satisfied by using a copper tube for the enclosure.

To put all that we have computed together, the design for a "measuring stick" to measure 500 ft with $\pm .0014$ " precision shapes up as follows. The enclosure consists of a copper pipe, $1/8$ " wall thickness, and 3" in diameter, insulated with steam-pipe asbestos lagging material. Heating wires and thermostats are placed under the asbestos, in independent lengths of 10 ft. each. The latter requirements are brought about by the fact that part of the pipe will be out-of-doors, where the seasonal, and even diurnal, temperature change may be very large. The temperature control can be a relatively crude one--about 5 degrees precision. The pipe is supported on concrete piers, at intervals of 10 ft. Two silica rods, $1/4$ " in diameter are suspended in the pipe, one from the other on 1" threads spaced 1 ft. The second rod is wound with #30 enameled copper wire, 8 turns to the inch, lacquered down. The assembly, 500 ft. long, can be made (as a rough guess) for \$5000.

We may visualize an entire aligning system as follows: Each sector of the machine is built on a rigid bed plate. Each bed plate is supported at four points by levelling screws, and there is a horizontal (radial) adjustment screw at each end. Provision is made, locally, at each straight section, to keep the two neighboring sector ends in alignment with each other. In the mercury system there are two measuring stations (a free surface and a sensing device) at each straight section; one at the

outer radius and one at the inner radius. There are 8 (or possibly 16) "measuring sticks" running like the spokes of a wheel from a central monument, and the sensing equipment for the measuring sticks is located at the inner, common point.

The initial calibration of the measuring sticks would be carried out as follows: The 500 ft. assemblies would be built in sections of about 10 ft. The 10 ft. pieces of silica would attach to one another by means of precision metal ferruls. Disassembly and reassembly would not change the calibration, because the three important elements, the rods, the resistance wire, and the thread suspensions, would never be separated, but would be left intact in each 10 ft. length. Initially, all the "measuring sticks" would be assembled side by side and calibrated against one another over as great a range of temperature as would be expected in practice. Each rod would then have a calibration curve relating resistance to length. In using the rods for actual checking of the machine, no attempt would be made to bring each rod to a standard temperature, but rather the resistance would be measured and the length read from the calibration curve.

The conclusion of this study is that it will be possible to check both altitude and radius in a machine of 500 ft. radius to well within .0014", and to do the checking and readjustment operation within a half day's time.