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Probing Inclusive Production Mechanisms with the (p,2p) Reaction

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ABSTRACT

Several models for the inclusive production of protons at backward angles are reviewed. From the model amplitudes, predictions for the (p,2p) differential cross section in which one of the protons is emitted in the backward direction are calculated. It is found that the magnitudes and angle dependence of the predicted cross sections vary significantly from one model to the next.

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I. INTRODUCTION

Since Frankel *et al.*¹ first measured the backward inclusive differential cross section of protons produced in proton-nucleus collisions at intermediate energies, there have been several models²⁻⁶ put forward to explain the data. These models differ considerably in their microscopic detail, but they can each generally fit the data for emitted proton angles in the 160° to 180° range. While recent (p,p') data at more forward angles has begun to place limits on the range of validity of some of the models, it does not appear likely that further (p,p') data will be able to distinguish between them. The purpose of this paper is to consider some reactions related to (p,p'), in particular the (p,2p) reaction, in an effort to find a means of testing these models.

It will be useful to have some sort of categorization of the models for our purposes. The models usually involve some sort of primary interaction, for example elastic scattering of the incident proton off a cluster of nucleons inside the nucleus. Because the clusters or residual nuclei can carry excitation energy which may lead to their breakup, the few body intermediate state very likely leads to a many body final state. It is assumed in all of these models that the protons emitted in the de-excitation of the residual nucleus are of such low energy as not to contribute to the 100-400 MeV protons observed by Frankel *et al.* Hence we will categorize the reactions by the number of projectiles there are in the intermediate state following the primary interaction.

We see immediately that two-body intermediate states are unlikely to contribute significantly to the high energy backward events looked at by Frankel since the residual nucleus would have to be carrying off

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an enormous amount of excitation energy. We will therefore concentrate our attention on three- and four-body intermediate states.

By far the largest number of models fall into the three-body intermediate state category. One of the first models proposed in this group involves the incident proton elastically scattering in the forward direction off a target proton, putting the latter on the mass shell. It is this target proton which is observed in the backward direction. The residual nucleus is then simply a spectator. This model was used by Amado and Woloshyn³ for describing inclusive proton production at backward angles, and by Boal and Woloshyn⁷ for describing the inclusive production of alpha particles at backward angles.⁸ We shall call this the direct knockout (DK) model.

A simplification of this model was proposed by Frankel:² the quasi-two-body scaling hypothesis (QTBS). In this model, the phase space integral of the transition matrix element is set equal to a product of one function varying rapidly with and depending only on the minimum momentum of the recoiling nucleus, and a second function which varies slowly with the other kinematical variables. This approximation is not too bad for pp scattering and QTBS successfully describes the inclusive proton data. However, the approximation fails for p-alpha scattering, and indeed QTBS has been shown to be inadequate as a description of the inclusive alpha data. Since QTBS by itself will not allow the prediction of less inclusive processes than (p,p'), it will not be considered further here.

A model in some respects similar to the DK model is one proposed by Weber and Miller.⁴ Here, though, the observed proton is the spectator

and the primary interaction is between the incident proton and the remaining A-1 nucleons of the target nucleus (of mass number A). The need for detailed knowledge of the p + (A-1) collision is avoided because of the inclusive nature of the (p,p') experiment.

The other three-body intermediate state model we wish to discuss is the correlated cluster model of Fujita.⁵ In some sense the CC model incorporated both the DK and MPE models in that the incident proton scatters off everything from a single proton in the nucleus to an (A-1) nucleon cluster. However, the observed proton is now assumed to be the incident proton, scattered through a very large momentum transfer.

Lastly, we take note of a four-body intermediate state model, the cluster recoil model (CR) proposed by Woloshyn.⁶ As in the DK model, the primary interaction is pp scattering. However, whereas the residual nucleus carries off momentum $\vec{k}$ (where the target proton had momentum $-\vec{k}$ in the nucleus) in the DK model, in the CR model $\vec{k}$ is carried off by a "jet" of n nucleons, the remaining A-1-n nucleons remaining roughly at rest in the lab. Although one has to model the relative probability of the nuclear proton with momentum $-\vec{k}$ recoiling against the n-nucleon jet with momentum $\vec{k}$ both as a function of k and n, the overall normalization is fixed by the (p,p') data.

We have outlined above four models for the (p,p') reaction (DK, MPE, CC, CR) which we feel are significantly different from one another that they should be able to be distinguished experimentally. There are other likely models of which we are aware, and doubtless others of which we are not aware, which we have excluded for brevity. In the following sections, the DK, MPE and CC models will be reviewed in turn

(concentrating most heavily on the DK model) and their (p,2p) predictions will be discussed. Unfortunately, the normalization conventions of the various authors are not uniform, nor, of course, are their parametrizations of the data which are fed into the models. The approach taken in this paper will be to use the same data parametrization (e.g. for pp scattering) in every model to facilitate comparison, but to leave the normalization conventions intact as much as possible, for easier reference to the original works.

II. DIRECT KNOCKOUT MODEL

The kinematic labels attached to each participant are shown in Fig. 1. The incident proton has an energy and momentum of E_i and p_i in the lab system, and leaves the reaction with E_f and p_f . The struck nucleon has momentum $\vec{k}$ before the collision, and energy and momentum E_q and q after. We will use $E_k = \bar{E}^* + (k^2 + M_A')^{1/2}$ (where M_A' is the mass of the residual nucleus and $\bar{E}^*$ its average excitation energy) to denote the energy of the residual nucleus, and ϵ_k to mean $(k^2 + M_p)^{1/2}$, where M_p is the proton mass.

Using the phase space normalization conventions of Bjorken and Drell,⁹ and using the closure approximation to replace the sum over final nuclear states by a final state with an excitation energy $\bar{E}^*$, Amado and Moloshyn³ write the (p,p') differential cross section for a target of A nucleons in the form

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{M_p^4}{2(2\pi)^5} \frac{q}{p_i} \int \frac{d^3p_f}{E_f \epsilon_k} \left(n_p(k) |T_{pp}|^2 + n_n(k) |T_{pn}|^2 \right) \delta(E_i + M_A - E_f - E_q - E_k) \quad (1)$$

where the probability of finding a proton with momentum k in a nucleus

with Z protons, $n(k)$, is normalized by

$$2 \int \frac{M_p}{\epsilon_k} n(k) \frac{d^3k}{(2\pi)^3} = Z, \quad (2)$$

and similarly for the neutron momentum distribution $n_n(k)$. The collision between the incident proton and the off-mass-shell proton in the nucleus is assumed to be dominated by elastic scattering, the matrix element for which, T_{pp} , is normalized to

$$\frac{d\sigma}{dt} = \frac{M_p^2}{16 \pi p_{lab}^2} |T_{pp}|^2 \quad (3)$$

where t is the square of the four-momentum transfer. No effort is made to include off-mass-shell effects.

The parametrization for the pp scattering cross section chosen here is to assume that the integrated cross section is constant (as a function of energy) at 24 mb,¹⁰ and that $d\sigma/dt$ has the form

$$\frac{d\sigma}{dt} = Ae^{bt} \quad (4)$$

The parameter, b , has the rough energy dependences over the range of interest, of

$$b = 14.39 T_{lab} - 6.13 \text{ GeV}^{-2} \quad (5)$$

where T_{lab} is the lab kinetic energy of the incident proton in GeV. At the limits of applicability of this formula, b was restricted to be greater than 0.7 GeV⁻² and less than 7.0 GeV⁻². Knowing b for a particular T_{lab} , the integrated version of Eq. (4) can be solved for A . To determine the appropriate T_{lab} for the reaction, the momenta of p_f and q were used to define the Mandelstam variable s , for which the corresponding lab energy was then calculated.

The parametrization of $|T_{pn}|^2$ is somewhat more cumbersome. Over the energy range of interest, $|T_{pn}|^2$ at $u=0$ (backward scattering) was approximated by (in units of mb/GeV²)

$$|T_{pn}|_{u=0}^2 = \begin{matrix} -1.369 \times 10^4 & T_{1ab} + 1.686 \times 10^4 & (T_{1ab} < 0.7 \text{ GeV}) \\ 0.709 \times 10^4 & & (T_{1ab} > 0.7 \text{ GeV}). \end{matrix} \quad (6)$$

The parameter b' appearing in

$$\frac{d\sigma}{du} = A' e^{b'u} \quad (7)$$

was fitted with (in GeV⁻²)

$$b' = \begin{matrix} -94.34 & T_{1ab} + 115.7 & (T_{1ab} < 0.876 \text{ GeV}) \\ 33.0 & & (T_{1ab} > 0.876 \text{ GeV}). \end{matrix} \quad (8)$$

In general, it was found that scattering from neutrons contributed no more than a few per cent of the predicted cross section, so the parametrization chosen is not terribly crucial to the results presented here.

The last ingredient required for the calculation is the form of the momentum distribution. Theoretical understanding of the high momentum components of the momentum distribution is still far from complete, but some recent work by Zabolitzky and Ey,¹¹ as well as most fits to data, point to an approximately exponential shape in the 300-900 MeV/c range. Following Amado and Woloshyn, we try

$$n(k) = \frac{C}{(2 \cosh(k/2 k_0))^2} \quad (9)$$

where k_0 is treated as a parameter and C is determined by the normalization condition, Eq. (2). The same form is used for both protons and neutrons. Hence, the model contains only one free parameter, k_0 .

Shown in Figs. 2-4 are the predictions of this model with $k_0 = 88 \text{ MeV}$ for a variety of targets, the bombarding energy being 800 MeV. In the original paper of Amado and Woloshyn,³ k_0 was fixed by an analysis of (e, e') data, and Eq. (1) was not evaluated exactly. Thus, the agreement was not as good as is shown in these figures. It is clear that the agreement is good for Li and Be, but has begun to deteriorate slightly for Ta. Had the simple exponential been used instead of Eq. (9) for $n(k)$, the predicted results would have been roughly 20% lower.

In this model, multiple scattering effects have been neglected, and it is important to find out just how significant they are. A recent experiment at TRIUMF¹² tried to address this question by using ⁴He as a target. While the data has not yet been finalized for publication, the preliminary results are in rough agreement with the DK predictions.

To test this model further, one would have to go to a less inclusive process than (p, p') . Since the DK model assumes that the incident proton is scattered forward without large momentum transfer, one can easily calculate the DK predictions for the $(p, 2p)$ reaction. A comparison of light vs heavy target could then be used to investigate the importance of multiple scattering.

The differential cross section for two proton coincidence reactions is given by

$$\frac{d^3\sigma}{d\Omega_f d\Omega_q dE_q} = \frac{M_p^4}{2(2\pi)^5} \frac{q}{p_i} \frac{E_k}{\epsilon_k} \left[\frac{p_f^2}{p_f(E_k + E_f) - |\vec{p} \cdot \vec{q}| E_f \cos \theta_f} \right] n(k) |T_{pp}|^2 \quad (10)$$

where θ_f refers to the angle between $\vec{p}_f$ and $\vec{p} - \vec{q}$. For this and other models, ⁹Be has been chosen as a target because it has a small A and is

easy to handle. Shown in Fig. 5 is the DK prediction for the backward proton being observed at 160° . As expected, the peak in the distribution of the forward proton is near 0° ($\theta_{f,lab}$ is the angle between p_f and the beam axis, a positive $\theta_{f,lab}$ being on the opposite side of the beam from θ_q). For $\theta_q = 90^\circ$, shown in Fig. 6, the peak shifts out to about 15° . In both of these calculations, p_f has been chosen to lie in the plane formed by $\vec{p}$ and $\vec{q}$. The cross section decreases as $\vec{p}$ moves out of the plane. The incident proton kinetic energy in both these results is 800 MeV.

III. MULTI-PARTICLE EXCHANGE MODEL

The particle labels used for this model will be the same as those used in the previous section. In order to be able to make the (p,2p) predictions for this model, we will have to go beyond the model of Weber and Miller⁴ and actually construct a matrix element for the p-residual nucleus collision. Thus, the (p,2p) predictions generated here are going to have significantly more model dependence than that proposed by Weber and Miller.

The normalization for the cross section chosen here will be that of Bjorken and Drell⁹ for spinless particles. This is the same normalization taken in our previous work on (p, α) reactions. Denoting the p-residual nucleus matrix element by T_{PR} , the differential cross section for inclusive proton production is given by

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{C}{2^5(2\pi)^5 M_A M_A'} \frac{q}{p_i} M_A' F(q) \int \frac{d^3p_f}{E_f E_k} |T_{PR}|^2 \delta(E_i + M_A - E_q - E_f - E_k). \quad (11)$$

The function $F(q)$ is proportional to $n(k)$ of the direct knockout model. The constant C will be determined by fitting the inclusive data.² Now,

if the kinematic limits for the p_f integration were close to those of the free p-R reaction, then the integral would simply be proportional to the p-R cross section. That means that for fixed E and $|\vec{q}|$, the inclusive differential cross section should vary only slowly with θ_q . Experiment shows a fairly rapid variation as one goes to forward values of θ_q , so the particular version of the MPE model under investigation here is probably limited to large θ_q . To get an idea of the effect of shifting the integration limits as a function of $\vec{q}$, the following parametrization is chosen.

i) $F(q)$ is simply set equal to e^{-q/k_0} . As in the DK model, a value of k_0 of 88 MeV adequately describes the large θ_q data.

ii) The total p-R cross section is assumed to have the same form as given in Weber and Miller, i.e.

$$\sigma = 86 (A-1)^{2/3} \text{ mb}, \quad (12)$$

independent of energy in this region.

iii) The differential cross section for p-R collisions is assumed to have the same t dependence as Eq. 4, and b is chosen by a rough fit to the slope parameters of elastic p-R scattering at intermediate energy:¹³

$$b = 5.5 A^{1.06} \text{ GeV}^{-2}. \quad (13)$$

Lastly, with this normalization, the p-R differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{|T_{PR}|^2}{64\pi M_R^2 p_{lab}^2}. \quad (14)$$

While this is admittedly not the best parametrization available, it will do for the purpose of this paper.

The results for ${}^9\text{Be}(p,p')X$ at incident proton energy of 800 MeV are shown in Fig. 7. The constant C is fixed by the 158° data, and one can see that the cross section predicted for 100° is indeed not too much different.

The $(p,2p)$ predictions are given by Eq. (11) with the $|\vec{p}_f|$ integration performed to take care of energy conservation. The results for a Be target are shown in Figs. 8 and 9. In contrast to the DK model, the peak in the distributions of the MPE model does not shift significantly with θ_q .

IV. CORRELATED CLUSTER MODEL

The last model which we wish to treat in detail is the correlated cluster model of Fujita.⁵ In this model, p_f is replaced by a cluster of N nucleons of energy E_{cc} . The differential cross section then consists of A terms:

$$\frac{d^3\sigma}{d^3q} = \sum_N G_N \int \frac{1}{E_q E_{cc}} |T_{p-cc}|^2 w_N(k) \frac{d^3p_f}{(2\pi)^3} \frac{1}{(8\pi)^2 F} \quad (15)$$

$$\delta(E_i + NM_p - E_q - E_{cc} - \vec{q}^2)$$

where

$$F = NM_p p \quad (16)$$

$$G_N = \frac{A}{N} \frac{1}{N^{N-1}} \left(\frac{V_c}{V} \right)^{N-1} \xi_N \quad (17)$$

$$w_N(k) = \left(\frac{4\pi}{N_V} \right)^{3/2} e^{-k^2/N_2} \quad (18)$$

The reader is referred to the original paper of Fujita for a detailed discussion of the values of the parameters involved. The values of these parameters which Fujita found to give an acceptable fit to the

inclusive data were:

$$v = 32500 \text{ MeV}^2 \quad (19)$$

$$V_c/V = 0.07 \quad (20)$$

$$\xi_N \approx 1 \quad (21)$$

The p -cluster cross section was replaced by the pp cross section by Fujita,⁵ and the amplitude normalized as in Eq. (14). The same criteria for selecting the appropriate s and t , and the same pp cross section parametrization as were used in the DK calculations are used in the results discussed below.

We are interested mainly in the $N=1$ term of Eq. (15), as the recoiling clusters in Fujita's model do not break up, by and large. Evaluation of Eq. (15) then shows that, depending on Eq, scattering off a single nucleon contributes only $\sim 10^{-4}$ to 10^{-5} of the predicted inclusive cross section. It comes as no surprise then, that when the $|\vec{p}_f|$ integration only is performed in Eq. (15) to give the $(p,2p)$ prediction, the results are substantially lower than those found in the DK or MPE calculations. The ${}^9\text{Be}(p,2p)X$ cross sections for one proton at 160° are shown in Fig. 10.

V. DISCUSSION

In the above, we have calculated some $(p,2p)$ predictions for a variety of models. Each model has its own characteristic spectrum which should allow experiment to rule at least one of them out. For example, although the peak in the cross section for the DK and MPE models is grossly the same, the peak should shift as θ_q is varied in the DK model, but remain roughly centred at the beam axis in the MPE model. The reason that the peak is approximately stationary in the

ME model is that the amplitude assumed here for p-R scattering falls off very rapidly with momentum transfer of the incident proton to the residual nucleus. In contrast both of these models to the CC model, the predicted cross section of the CC model is several orders of magnitude lower than the other two.

This same conclusion also holds true for the CR model. Because the CR model was not broken down into elementary amplitudes, we did not use it to make (p,2p) predictions. However, as described in the introduction, one can evaluate the contribution to the (p,p') predictions of the term in which the "jet" comprises all of the residual nucleus. This would then give a very rough measure of the scale factor which could be applied to the DK model results to approximate the CR results. The $n = A - 1$ contribution to the ${}^9\text{Be}(p,p')X$ cross section at incident proton energy of 800 MeV is shown in Fig. 11. As with the CR model, it is observed that the predicted cross section should be much smaller than the DK predictions.

In all of the above calculations, the average excitation energy of the residual nucleus has been set equal to zero. A more realistic calculation would have a non-zero value, but would introduce extra model dependence into the results, a situation we have tried to avoid. Green and Korteling,¹⁴ using heavy fragment yields as constraints on an evaporation calculation, have estimated that at the beginning of the evaporation cascade, the distribution of excitation energies has an average value in the 20-40 MeV range for an incident proton in the 200-500 MeV range. While this is not necessarily the same distribution that one would expect here, it provides us with a rough value for $\bar{E}^*$. This

would manifest itself in the DK model by a downward shift of an amount $\bar{E}^*$ in E_f from the $\bar{E}^* = 0$ value of roughly $E_f - E_q$.

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FIGURE CAPTIONS

1. Energy-momentum labels for direct knockout model.
2. Comparison of direct knockout model predictions with experiment for the reaction ${}^6\text{Li}(p,p')X$ at incident proton energy of 800 MeV, and observed proton angle of 100° and 158° . Data from Ref. 2.
3. Comparison of direct knockout model predictions with experiment for the reaction ${}^9\text{Be}(p,p')X$ at incident proton energy of 800 MeV and observed proton angle of 100° and 158° . Data from Ref. 2.
4. Comparison of direct knockout model predictions with experiment for the reaction ${}^{181}\text{Ta}(p,p')X$ at incident proton energy of 800 MeV and observed proton angle of 100° and 158° . Data from Ref. 2.
5. DK predictions for ${}^9\text{Be}(p,2p)X$ with $\theta_q = 160^\circ$ and 800 MeV incident proton kinetic energy.
6. DK predictions for ${}^9\text{Be}(p,2p)X$ with $\theta_q = 90^\circ$ and 800 MeV incident proton kinetic energy.
7. Comparison of MPE model with experiment for the reaction ${}^9\text{Be}(p,p')X$ at incident proton energy of 800 MeV and observed proton angle of 100° and 158° . Data from Ref. 2.
8. MPE predictions for ${}^9\text{Be}(p,2p)X$ with $\theta_q = 160^\circ$ and 800 MeV incident proton energy.
9. MPE predictions for ${}^9\text{Be}(p,2p)X$ with $\theta_q = 90^\circ$ and 800 MeV incident proton energy.
10. CC predictions for ${}^9\text{Be}(p,2p)X$ with $\theta_q = 160^\circ$ and 800 MeV incident proton energy.
11. Comparison of $n=A-1$ contribution in CR model with total predicted cross section for ${}^9\text{Be}(p,p')X$ at $\theta_q = 160^\circ$, $T_p = 800$ MeV.

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