exclusively on the free, for reasons to be outlined below.

from the J/ψ to ϒ with a silver cluster, we will concentrate our attention

on studying p(ρ,σ,σ) experiment, our results are discussed and conclu-

ded: E' = p(ρ,σ) reaction, as well as predictions for
takes our results for the A(p(ρ)) reaction, at least for the

and outline the single scattering model used here. Section III con-

and identity of the cross-2-body scaling? (q/ξ) approach to this problem.

In Sec. II of this paper, we will discuss the question of the va-

collision being to affect the cluster.

a proton from a transferred cluster in the nucleus; the result of the

stood in terms of a mechanism which involves the single scattering or

shortcuts. We propose that this "non-evaporation," region can be under-

ings physically reasonable model details and excitation energy dis-

ergetic from what one would expect from an evaporation model employ-

regime, the slope of the differential cross section is significant.

essentially explained by conventional evaporation models. Above this

clears, below the transition region of 17–20 MeV the data can be suc-

energy of the emitted fragment; for fragments with momentum 4 and 12

below the observed light fragment as a function of T, kinetic en-

data from a clear peak in the inclusive differential cross section

of other intermediate energy have become available. These

over the past several years, data on the proton induced fragmentation

I. INTRODUCTION

ABSTRACT

a direct knockout model is developed for intermediate energy in-

made for the A(p(ρ)) reaction, where A = 6, 7, 9, and 12, these predictions are

in the backward hemisphere. In particular, the model is applied to

correlation nuclear reactions involving the emission of light fragments.
II. THE DIRECT KNOCKOUT MODEL AND QTBS

The single scattering model has been quite successful in explaining the high energy inclusive spectra of protons produced in the backward hemisphere. It has also been successfully applied to proton emission in high energy heavy ion reactions. Although the single scattering model has theoretical shortcomings it does provide a useful basis for phenomenological analysis and it is natural to extend it to inclusive reactions in which composite fragments are emitted.

In the single scattering picture an incident proton is scattered elastically from the observed secondary particle (a nucleon or composite fragment), the remainder of the nucleus being a spectator. The closure approximation is usually made so that effectively the final state consists of three particles: the incident proton which is scattered largely in the forward direction, the emitted particle and the recoiling nucleus approximated by a single state with some average excitation energy. Our kinematic labels for these particles are given in Fig. 1.

The expression for the differential cross section is

\[
\frac{d\sigma}{d\Omega dE_q} = \frac{qC}{2^{5}(2\pi)^{5}M_A P|P^q} \int \frac{k dk d\Theta_k}{E_k} F(k)|T|^2
\]

(1)

where \(\Theta_k\) is the angle between \(\hat{p} \times \hat{q}\) and \(\hat{k} \times (\hat{P} - \hat{q})\). The quantity \(T\) is the proton plus observed particle elastic scattering amplitude and \(F(k)\) is some effective structure function. Within the single scattering model \(F(k)\) gives the probability that the ejected particle has momentum \(k\) in the nucleus. The normalization constant \(C\) can be interpreted by use of the relation

\[
\frac{C m_q}{(2\pi)^2 M_A} \int \frac{F(k) k^2 dk}{E_k} = n_{eff}
\]

(2)

where \(m_q\) is the mass of the observed particle and \(n_{eff}\) is the effective number of target particles seen by the incoming proton. We note that for inclusive proton data our analysis indicates that the effective number of target protons equals \(Z\) to a good approximation.

Frankel proposed that the expression on the right hand side of (1) could be rewritten as

\[
\frac{d\sigma}{d\Omega dE_q} = \frac{qE_q}{|P^q|} G(k_{min}) f(p,q)
\]

(3)

where \(f\) is a slowly varying function of \(p\) and \(q\) and \(G\) is a function only of the nuclear recoil momentum \(k\), when \(k\) and \(P_f\) are collinear. This is the quasi-two-body scaling (QTBS) hypothesis, and should be distinguished from the single scattering model of Eq. (1). QTBS can be partially derived from (1) when \(|T|^2\) is a much more slowly varying function of \(k\) than \(F(k)\). This is roughly true for backward inclusive proton scattering at intermediate energies, where we would expect a steep fall-off in \(F(k)\) because of the large internal momenta required for the nuclear protons. In essence, QTBS can be treated as an approximation to the single scattering model where the \(\vec{k}||\vec{P}_f\) configuration dominates the cross section. QTBS has enjoyed reasonable success in describing the available backward inclusive proton spectra on a wide variety of targets at intermediate energy.

As a first attempt at the fragmentation problem, we try the QTBS approach. We show in Fig. 2 the function \(G(k_{min})\) for 300 MeV protons on Ag, with the \(a\) particles being observed at 90° and 160°. The ranges of \(k_{min}\) covered at the two angles do not overlap in this experiment, and it is
\( p = q \cdot 0.983 \cdot 1.259 \cdot 0.989 \cdot 1.4 \cdot 7.892 \cdot 1.0.382 + 1.728 \cdot 0.1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \).

kinetic energy range of \( E \) is

The deuteron momentum, our fit to the data in the 250-600 MeV proton

where \( E \) is the total center-of-mass energy squared and \( p \) is the

\[ L = \sum_{i=0}^{n} \frac{E_i}{m_i} \]

Ignoring spin and write

with our normalization for the single scattering expression (1), we

where \( L \) is the four-momentum transfer squared. To be consistent

\[ q^2 = 0 \left( \frac{dp}{d\theta} \right) \]

termed cross section

The last integral for the calculation is the p+ elastic peak.

\[ (\frac{d\sigma}{d\Omega}) \left( p^0 \right) \]

predictions for \( (p^0 \cdot 0) \) and \( (p^0 \cdot 0) \).

Now that we have \( F(p) \) explicitly, we can proceed with the predic-

\[ F(p) \]

range.

form \( \chi^2/\nu \) for both data sets.

We find that, indeed, \( F(p) \) is well represented by \( \exp(-p^2/\kappa^2) \). Hence, we can extract \( F(p) \) directly from the figure data. Interestingly,

\[ \frac{1}{V} \left( \frac{\gamma}{\pi} \right)^{1/2} \text{Im}(F(p^0)) \]

\( F(p) \) is not such a rapid function of \( p, \) than the integral can be ap-

I now show in Sec. I, \( \gamma \) is a rapidly varying function.

where we have approximated \( \gamma \) by \( \gamma', \) since the recoil and excitation

\[ \gamma \]

the \( F(p) \) for the differential cross section at 180°. Then, with \( v \), (1),

To obtain a phenomenological expression for \( F(p) \), we look at the

peaks without correlations, for which \( \kappa \approx 0 \).

With \( \kappa \approx 150 \text{ MeV} \), this fall-off is much less steep than what one ex-

When considering the data, a function similar to the form \( \exp(-p^2/\kappa^2) \)

Single particle contribution to the high momentum part of the disribu-

tractions. Now and later. They find that the correlation modify the

looked at the effect of correlations on the single particle momentum dis-

have been extracted from experiment. In a recent paper, ZAbbyst and EF

For \( F(p) \) or some phenomenological form of \( F(p) \), it is clear that it can be

To proceed with the direct knockout model, we need either a model

\( F(p) \) or \( \chi^2/\nu \) for both data sets.

The \( F(p) \) behind the data, although it gives us a hint that the single

varrying function \( (p^0) \). Hence, \( \langle \gamma \rangle \) is not an acceptable description

the 0° to 180° data cannot all be explained due to the slowly

clear from the figure that the factor of 50 in the normalization of
where \( T_p \) is the lab energy of the incident proton in MeV, and \( |T|_{L=0}^2 \) is in mb - GeV². We are then left with two parameters, \( C \) and \( k_0 \). These are fixed by fitting the backward \( (p,\alpha) \) data at 210, 300 and 480 MeV incident proton energy. A comparison of our fits with the data are shown in Figs. 3 and 4. We find that \( k_0 \approx 78 \) MeV and

\[
N \approx \frac{C}{2^{5(2n)^2}} \frac{N_A}{A'} \tag{10}
\]

has the value 0.20 GeV⁻¹. From this value of \( N \), the effective number of \( \alpha \)-particles seen by the incoming proton can be calculated by means of Eq. (2).

\[
n_{\text{eff}} \approx (8\pi k_0)^3 N_A \tag{11}
\]

which gives 5.6 for the effective number of \( \alpha \)'s.

Similar calculations have been done for \((p,\alpha)\) data taken on \(^9\)Be and \(^{27}\)Al with 500 MeV protons, the \( \alpha \)-particle being detected at 120°. The same slope parameter \( k_0 \) is found for these reactions as well, and the effective number of \( \alpha \)-particle's is found to be \( \approx 0.3 \) for \(^9\)Be and \( \approx 1.4 \) for \(^{27}\)Al. Due to the error on the absolute normalization of the data alone, there is at least a 25% error associated with these numbers.

Since the two parameters of the model have been determined by these fits, the amplitude for the three body final state is now completely defined in this model. This means that the differential cross section for either the recoil nucleus or the forward proton in coincidence with the alpha can be predicted.

In our calculation we have set the average excitation energy of the residual nucleus \( A' \) equal to zero. In reality the residual nucleus will carry some excitation energy. Evaporation calculations indicate that the average excitation is in the 10-20 MeV range so that a large number of residual nuclear states contribute to the inclusive cross section. This would undoubtedly complicate the interpretation of alpha plus recoil nucleus coincidence experiments.

On the other hand the binding energy per nucleon is grossly the same for all the states involved so that

\[
T_p \approx T_{pf} + T_q + \bar{E}^\alpha + T_k. \tag{12}
\]

The last two terms will sum to perhaps 20 MeV, so that, for \( 40 \leq T_q \leq 90 \) MeV, \( T_{pf} \) will be fairly large, especially for large \( T_p \). This relationship then says that \( T_{pf} \) will roughly the same, independent of \( \Theta_{\text{lab}} \), the angle between the forward proton and the beam axis. Hence, a measure of \( T_{pf} \) in the \((p,\alpha)\) reaction should give a good measure of \( \bar{E}^\alpha \).

In light of the above, we focus our attention on the \((p,\alpha)\) reaction. We will calculate \( d\sigma/d\Omega_{q} d\Omega_{f} dE_{\alpha} \), integrating over \( \hat{p} \) and \( |\hat{p}_{f}| \) to get rid of the four dimensional delta function. We find

\[
d\sigma = N \frac{q}{p} \frac{M_A'}{M_A} \frac{p_f^2}{|\hat{p}_f(E_p,\hat{M}_A,\hat{E}_f)|} F(k)|T|^2 \tag{13}
\]

where \( \Theta_{f} \) is the angle between \( \hat{p}_f \) and \( \hat{p}_q \). As a sample calculation, we have looked at 300 MeV protons producing \( \alpha \)-particles at 90° and 160°. The differential cross section for the forward protons is shown in Figs. 7 and 8. We have chosen the kinetic energies of the alpha's to be 40, 70 and 100 MeV. The angle \( \Theta_{f} \) is positive for the proton on the opposite side of the beam from the alpha, and the particles are all taken to be coplanar. As before, \( \bar{E}^\alpha \) is set equal to zero.
ACKNOWLEDGMENTS

and lead to this discrepancy.

It is likely that those scattering effects are more important in (a,2J)

and that they account for higher mass targets. A reaction, there, might be found to increase with $A$ in the range 6 $A$ to

which is different from that described by Bollhoft et al. in the (ca,2J)

a rapidly increasing function of $A$. A more than this behavior of

interpreted as indicating that the probability of cluster formation is

the fact that our $N^2$ increases monotonically with $A$ could then be

cease the cross section should become less important as $N$ increases.

or of alpha particles. We expect that these scattering effects might do-

it is of interest to look at the dependence of the effective num-

also may be significant for higher mass fragments.

closers, it is much lower, what this will be important for alpha parti-

energy, that is, not break up after leaving the nucleus.

The probability that the cluster is emitted with sufficiently small

is slight by the proton.

1. The probability of cluster formation, as discussed above.

2. The probability that the cluster will not be rescattered after it

of these effects:

The constant $\alpha$ is more of an ambiguity. It should really be a product

very dramatically from one emitted cluster to the next.

While this argument is quite crude, it indicates that $\alpha$ should not

\[
\alpha \approx \frac{e^{k/k_0}}{N} \cdot \frac{N}{k/k_0}
\]

be proportional to

of finding nucleons in a cluster with momentum $k$ would be roughly

proportionality of having a nucleon of momentum $k$. Then, the probability

the structure function $F(k) = e^{k/k_0}$ approximately represents the

the value found for the $(p,p')$ reaction. This is not unreasonable if

are worth making. The value of $k$ that was obtained is within the

way of determining these parameters, but (some comments on other works

would have been entirely high). Presently, we do not have any evidence

to the $(p,p')$ data (this is not a least $z^2$ fit, as the competing costs

the two parameters of the model, $c$ and $k_0$, are extracted by a fit.

not too important to the problem.

reasonable test of this model, provided multiple scattering effects are

$2O < 120$, measurement of the $(p,p')$ reaction should provide a

agreement, we feel confident about our predictions for other reactions

backscattered particles is predicted. While our simple calculations for the

the model and multiple scattering cross section for the $(p,p')$ reaction for $O^7$ in the

are well described in terms of a single scattering mechanism. From

we have shown that the inclusive $(p,p')$ data at backward angles

IV. DISCUSSION AND CONCLUSION
REFERENCES


The data for $^9$Be and $^{27}$Al targets were collected specifically to check the normalization constant in the model. The experimental conditions were similar to those described in Green and Korteling, ref. 1.


FIGURE CAPTIONS

1. Direct knockout model notation; the energy and momenta labels of the three particles in the final state are: measured fragment ($E_q, \vec{q}$), forward proton ($E_F, \vec{p}_F$) and residual nucleus ($E_N, \vec{k}$). The average excitation energy of the residual nucleus is denoted as $\bar{E}^n$.
2. The function $G(k_{min})$ for the Ag(p,a) reaction with incident proton energy of 300 MeV.
3. Comparison of the single scattering model predictions for Ag(p,a)X with experiment for the a observed at 90°.
4. Comparison of the single scattering model predictions for Ag(p,a)X with experiment for the a observed at 160°.
5. Comparison of the single scattering model for $^9$Be(p,a)X with experiment. The incident proton energy is 500 MeV and the alpha is observed at 120°.
6. Comparison of the single scattering model for $^{27}$Al(p,a)X with experiment. Same conditions as with Fig. 5.
7. Predictions of the single scattering model for Ag(p,pu)X with 300 MeV protons and the alpha at 160°.
8. Predictions of the single scattering model for Ag(p,pau)X with 300 MeV protons and the alpha at 90°.