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PCAC AND THE NON-RELATIVISTIC PION-NUCLEON VERTEX†

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Abstract

The Foldy-Mouthuysen reduction technique is applied to the sigma model and pseudovector coupling pion-nucleon Lagrangians. In contrast to electrodynamics the requirement that the non-relativistic pion-nucleon Lagrangian have an approximate symmetry under the same global transformation as the relativistic theory does not uniquely fix the Foldy-Mouthuysen transformation. It is shown that the partially conserved axial-vector current relation is satisfied by the non-relativistic theory and the implications of this on the form of the effective pion-nucleon vertex are discussed.

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1. Introduction
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The Foldy-Mouthuysen¹⁾ (FW) transformation provides a systematic way of constructing non-relativistic operators which reproduce relativistic results to arbitrary order²⁾ in $1/m$. The technique has been applied to the pion-nucleon interaction^{3,4)} and the ambiguity in the form of the effective pion production operator has received wide attention⁵⁻⁹⁾. This ambiguity arises from the freedom to make unitary transformations of the reduced (non-relativistic) operator. Friar⁸⁾ has emphasized that in principle such unitary transformations can not affect physical amplitudes since changes in the effective operators are compensated by changes in the wave functions. In practice, however, many calculations^{7,10-13)} of pion production and absorption in nuclear reactions use a model Hamiltonian (e.g. a shell model or phenomenological optical potential) to determine the wave functions. In such applications the form of the effective non-relativistic pion-nucleon vertex becomes an important practical question¹⁴⁾.

In this paper we consider the question of whether the approximate symmetry of pion-nucleon dynamics¹⁵⁾ (chiral invariance), or the partially conserved axial-vector current¹⁶⁾ (PCAC) can be used to restrict the form of the FW transformation and thus of the non-relativistic pion-nucleon vertex. To motivate this work it is useful to consider the example of electrodynamics. In this case the requirement that the non-relativistic Hamiltonian be invariant under the usual local gauge transformation $\psi(x) \rightarrow [1 + ie A(x)]\psi(x)$, $\bar{A}(x) \rightarrow \bar{A}(x) + \nabla A(x)$ uniquely determines the FW transformation³⁾. In principle it is possible to use an FW transformation which is not gauge invariant at the

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expense of changing the meaning of gauge transformation for the non-relativistic system. In practice this option is never used.

Two different pion-nucleon Lagrangians with an approximate *global* symmetry and PCAC are examined. Unfortunately in neither case does the invariance restrict the Foldy-Wouthuysen transformation. In the pseudovector coupling model *all* unitarily equivalent FW transformations preserve the approximate symmetry of the Lagrangian. On the other hand for the sigma model^{17,15} it is *impossible* to find an FW transformation such that the invariance of the non-relativistic theory is the same as of the relativistic theory¹⁸. It is shown however, that PCAC is preserved in the Foldy-Wouthuysen reduction and the low-energy theorem which follows from it is used to restrict the form of the $O(k)$ pieces of the effective non-relativistic pion-nucleon vertex.

The Foldy-Wouthuysen transformation technique is reviewed in sect. 2 and in sect. 3 it is applied to the pseudovector and sigma model Lagrangians. The approximate symmetry of the Lagrangian is discussed and it is shown that it does not restrict the form of the FW transformation. The form of terms generated by unitary transformation is also given. In sect. 4 the low-energy theorem following from PCAC is reviewed. It is shown that the reduced Lagrangians also have a partially conserved axial-vector current and the implications of this on the form of the effective pion-nucleon vertex are discussed. Conclusions are given in sect. 5.

2. The Foldy-Wouthuysen Transformation

In this section the reduction of a relativistic Lagrangian by the Foldy-Wouthuysen technique^{1,2} is reviewed. We prefer to work with the Lagrangian rather than with the Hamiltonian since it is the interaction part of the Lagrangian that enters the perturbation expansion of the S-matrix¹⁹. Consider a Lagrangian density

$$\mathcal{L} = \bar{\psi} \left(i \not{\partial} - m - \Gamma[\phi] \right) \psi + \mathcal{L}_B \quad (2.1)$$

where $\Gamma[\phi]$ is the coupling term and $\mathcal{L}_B$ is the boson part of the Lagrangian. The terms in the fermion part of the Lagrangian can be separated into odd and even operators²⁾

$$\mathcal{L}_F = \psi^\dagger \left(i \frac{\partial}{\partial t} - \gamma_0 m - \mathcal{E} - 0 \right) \psi, \quad (2.2a)$$

$$= \psi^\dagger \left(i \frac{\partial}{\partial t} - L \right) \psi. \quad (2.2b)$$

Transforming the fermion field $\psi' = \exp(iS) \psi$,

$$\mathcal{L}_F = \psi'^\dagger \left\{ i \frac{\partial}{\partial t} - e^{iS} \left(L - i \frac{\partial}{\partial t} \right) e^{-iS} \right\} \psi' \quad (2.3a)$$

$$= \psi'^\dagger \left(i \frac{\partial}{\partial t} - L' \right) \psi'. \quad (2.3b)$$

Taking $S = -i \gamma_0 0/2m$ reduces the odd operators in L' by order $1/m$ compared to those in L . Repeated application of this procedure eliminates the odd terms up to an arbitrary order in $1/m$. The result to $O(1/m^2)$ is

$$\mathcal{L}_F = \psi'^\dagger \left\{ i \frac{\partial}{\partial t} - \gamma_0 m - \frac{\gamma_0 0^2}{2m} - \mathcal{E} + \frac{1}{8m^2} [0, \dot{0}] + \frac{1}{8m^2} [0, [\dot{0}, \mathcal{E}]] \right\} \psi', \quad (2.4)$$

where ψ' is the appropriately transformed fermion field. If $\Gamma[\phi]$ contains odd pieces linear in the boson field the term $[0, \dot{0}]$ will not appear in the Hamiltonian. This is in agreement with the results of

Goldman²⁰⁾ and Nieto⁹⁾ but does not imply that the $[0,0]$ term does not appear in the calculation of the S-matrix.

The operator O can be written as $O = O_1 + O_2$ where $O_1 = i \gamma_0 \vec{\gamma} \cdot \vec{\nabla}$ comes from the fermion kinetic energy term and O_2 from the interaction term. The reduction can be carried out in successive stages^{3,4)} with S taken to be an arbitrary combination of O_1 and O_2 . To order $1/m^2$ the difference between different reduced Lagrangians can be written as

$$L^\lambda - L^0 = i \frac{\lambda}{8m^2} [O_1, O_2] + \frac{\lambda}{8m^2} \left\{ [O_1, [O_2, \mathcal{E}]] - [O_2, [O_1, \mathcal{E}]] \right\}, \quad (2.5)$$

where λ is an arbitrary parameter. For $\lambda = 0$, $S = -i \gamma_0 (O_1 + O_2)/2m$. In this paper we will be concerned mainly with the form of the non-relativistic pion-nucleon vertex so the second term in (2.5) will not be considered since it must involve at least two boson fields.

Finally we note that for electrodynamics (and other gauge theories) only the choice $\lambda = 0$ leads to a reduced Lagrangian which is invariant under the usual gauge transformation³⁾. In this case $S = (i \vec{\gamma} \cdot \vec{\nabla} + e \vec{\gamma} \cdot \vec{A})/2m$ commutes with the local gauge transformation.

3. Application to the pion-nucleon interaction

In this section the FW technique is applied to the pion-nucleon interaction. To some extent this has been discussed previously by Barnhill¹³⁾, Cheon⁴⁾ and others⁵⁻⁹⁾. The new feature examined here is whether the approximate global symmetry of Lagrangians with PCAC is preserved by the non-relativistic reduction.

First consider the pseudovector coupling model. Although it does not have a chiral symmetry it does have a partially conserved axial vector current¹⁵⁾. The Lagrangian is given by

$$\mathcal{L} = \bar{\psi} [i \not{\partial} - m - f \gamma_5 \gamma_\mu \partial_\mu \vec{\tau} \cdot \vec{\phi}] \psi + \frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi} - \frac{\mu_\pi^2}{2} \vec{\phi} \cdot \vec{\phi}. \quad (3.1)$$

Except for the last term $\mathcal{L}$ is invariant under the transformation¹⁵⁾

$$\begin{aligned} T_P: \quad \psi &\rightarrow \psi \\ \phi &\rightarrow \phi + f^{-1} \vec{a}(x) \end{aligned} \quad (3.2)$$

for $\vec{a}(x) = \text{constant}$. Separating the odd and even terms $\mathcal{L}$ can be written

$$\begin{aligned} \mathcal{L} = \psi^\dagger \left\{ i \frac{\partial}{\partial t} - \gamma_0 m + f \gamma_0 \vec{\gamma} \cdot \vec{\nabla}_\pi \vec{\tau} \cdot \vec{\phi} \gamma_5 + i \gamma_0 \vec{\gamma} \cdot \vec{\nabla} + f \gamma_5 \vec{\tau} \cdot \vec{\phi} \right\} \psi \\ + \frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi} - \frac{\mu_\pi^2}{2} \vec{\phi} \cdot \vec{\phi} \end{aligned} \quad (3.3)$$

so that

$$\mathcal{L} = - f \gamma_0 \vec{\gamma} \cdot \vec{\nabla}_\pi \vec{\tau} \cdot \vec{\phi} \gamma_5, \quad (3.4a)$$

$$O_1 = - i \gamma_0 \vec{\gamma} \cdot \vec{\nabla}, \quad (3.4b)$$

and

$$O_2 = f \vec{\gamma} \cdot \vec{\phi}. \quad (3.4c)$$

The odd terms O_1 and O_2 are individually invariant under T_P for constant $\vec{a}(x)$. Unlike electrodynamics therefore, the requirement that the reduced Lagrangian have the same invariance as the relativistic Lagrangian places no restriction on the FW transformation.

Using (2.5) the lowest order λ -dependent pion-nucleon coupling in the reduced theory is

$$\frac{\lambda}{8m^2} f \psi^\dagger \left\{ \gamma_0 \vec{\gamma} \cdot \vec{\nabla} \gamma_5 \vec{\tau} \cdot \vec{\phi} + \vec{\tau} \cdot \vec{\phi} \gamma_5 \gamma_0 \vec{\gamma} \cdot \vec{\nabla} \right\} \psi. \quad (3.5)$$

In momentum space the contribution to the effective vertex takes the form

$$- i \lambda f \frac{\omega_\pi^2}{8m^2} \vec{\sigma} \cdot (\vec{p}_1 - \vec{p}_f) \quad (3.6)$$

where ω_π is the pion energy and $\underline{p}_i(\underline{p}_f)$ is the initial (final) nucleon momentum. This coupling is $O(k^3)$ in pion four-momentum. Within the pseudovector coupling model the $O(k)$ terms of the effective vertex, which includes the so-called Galilean correction term¹²⁾, are unambiguous and not subject to phenomenological adjustment.

We next consider the sigma model¹⁵⁾ given by the Lagrangian

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[i\not{\partial} - m - g(s + i \underline{\tau} \cdot \underline{\phi} \gamma_5)]\psi \\ & + \frac{1}{2} [\partial_\mu \underline{\phi} \cdot \partial^\mu \underline{\phi} - \mu_\pi^2 \underline{\phi} \cdot \underline{\phi}] + \frac{1}{2} [\partial_\mu s \partial^\mu s - \mu_\sigma^2 s^2] \\ & - \lambda^2 v s (s^2 + \underline{\phi}^2) - \frac{\lambda^2}{4} (s^2 + \underline{\phi}^2)^2 - v \mu_\pi^2 s + \epsilon c s \end{aligned} \quad (3.7)$$

where v is the expectation of the sigma field and $s = \sigma - v$. Except for the last term $\mathcal{L}$ is invariant under the transformation¹⁵⁾

$$T_\sigma: \psi \rightarrow (1 - i \underline{a}(x) \cdot \underline{\tau} \gamma_5) \psi, \quad (3.8a)$$

$$\underline{\phi} \rightarrow \underline{\phi} + 2 \underline{a}(x) (s + v), \quad (3.8b)$$

$$s \rightarrow s - 2 \underline{a}(x) \cdot \underline{\phi}, \quad (3.8c)$$

for $\underline{a}(x) = \text{constant}$. The even and odd parts of the fermion Lagrangian are

$$\mathcal{L}^e = g \gamma_0 s, \quad (3.9a)$$

$$O_1 = -i \gamma_0 \underline{\gamma} \cdot \underline{\nabla}, \quad (3.9b)$$

$$O_2 = i g \gamma_0 \gamma_5 \underline{\tau} \cdot \underline{\phi}. \quad (3.9c)$$

and
Since T_σ involves γ_5 and mixes pion and sigma fields it is clear that the reduced theory can not be invariant under the same transformation as the relativistic theory¹⁸⁾. Unlike the pseudovector model there is no invariant combination of O_1 and O_2 .

To $O(1/m)$ the reduced Lagrangian is

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left\{ i \frac{\partial}{\partial t} - \gamma_0 m - g \gamma_0 s - \frac{\gamma_0}{2m} (-i \gamma_0 \underline{\gamma} \cdot \underline{\nabla} + i g \gamma_0 \gamma_5 \underline{\tau} \cdot \underline{\phi})^2 \right\} \psi \\ & + \mathcal{L}_B + \epsilon c (s - 2 \underline{a}(x) \cdot \underline{\phi}). \end{aligned} \quad (3.10)$$

The lowest order λ -dependent pion-nucleon coupling is $O(1/m^2)$ and is given by

$$i \frac{\lambda g}{8m^2} \bar{\psi} \gamma_5 \underline{\tau} \cdot \underline{\nabla} [\underline{\gamma} \cdot \underline{\nabla} \gamma_5 \underline{\tau} \cdot \underline{\phi} + \underline{\tau} \cdot \underline{\phi} \gamma_5 \underline{\gamma} \cdot \underline{\nabla}] \psi, \quad (3.11)$$

which in momentum space contributes

$$-i \frac{\lambda g}{8m^2} \omega_\pi \underline{\sigma} \cdot (\underline{p}_i + \underline{p}_f) \quad (3.12)$$

to the non-relativistic vertex. This has the form of the Galilean correction term^{12,13)}. As we have seen there is nothing that chiral invariance of the sigma model can tell us about its presence or absence. There is however a way of determining whether it is allowable as a phenomenological effective operator.

4. Constraints from the low-energy theorem

No use has yet been made of the fact that the pion-nucleon Lagrangians lead to PCAC. Using PCAC the leading term (in pion four-momentum) of the pion emission amplitude and the axial current of the nucleon can be related²¹⁾. In this section we derive the axial current for the non-relativistic pseudovector and sigma models and discuss the implications on the form of the effective pion-nucleon vertex.

A detailed derivation of the low-energy theorem using PCAC may be found in ref.²¹⁾. Let $\mathcal{M}_j^\mu = \langle b | J_j^\mu | a \rangle$ be the matrix element of the axial current (isospin index j) between states a and b . Now $\mathcal{M}_j^\mu$ can be written as $\mathcal{M}_j^{\mu \text{ext}} + \Delta \mathcal{M}_j^\mu$ where the matrix element for the coupling of the axial-vector current (carrying momentum k) to an external line is given by

$$\mathcal{M}_j^{\mu \text{ ext}} = M_j^{\mu}(-1) + M_j^{\mu}(0) + O(k) + \frac{2m}{g} \frac{9A}{k^2 - \mu_\pi^2} i \frac{k_\mu}{k^2 - \mu_\pi^2} \mathcal{M}_j^{\mu} \quad (4.1)$$

where $\mathcal{M}_j^{\mu}$ is the matrix element for pion emission $\langle b | J_j^{\mu} | a \rangle$ and $M_j^{\mu}(-1)$ ($M_j^{\mu}(0)$) is the $O(k^{-1})$ ($O(k^0)$) amplitude with the axial-vector current coupling to an external nucleon line. PCAC requires

$$k_\mu (\mathcal{M}_j^{\mu \text{ ext}} + \Delta \mathcal{M}_j^{\mu}) = i \frac{2m}{g} \frac{9A}{k^2 - \mu_\pi^2} \frac{\mu_\pi^2}{k^2 - \mu_\pi^2} \mathcal{M}_j^{\mu} \quad (4.2)$$

from which one obtains²¹⁾

$$\mathcal{M}_j^{\mu}(0) = -i \frac{g}{2m} \frac{9A}{gA} k_\mu M_j^{\mu}(-1). \quad (4.3)$$

The $O(k^0)$ contribution to $\mathcal{M}_j^{\mu}$ is thus determined by the $O(k^0)$ terms of the nucleon axial-vector current.

To use the low-energy theorem it is necessary to generate a partially conserved axial-vector current in the FW transformed theory.

This is quite easy for the pseudovector model since T_p commutes with the FW transformation. Using

$$J_j^{\mu} = - \frac{\delta \mathcal{L}}{\delta \partial_\mu \alpha_j(x)} \quad (4.4)$$

one finds that for the non-relativistic version of the pseudovector model that

$$J_j^{\mu} = \psi'^{\dagger} \left(- \frac{i}{2m} [\tilde{\gamma} \cdot \tilde{\nabla} \gamma_5 \tau_j + \tau_j \gamma_5 \tilde{\gamma} \cdot \tilde{\nabla}], -\gamma_0 \tilde{\gamma} \gamma_5 \tau_j \right) \psi' - f^{-1} \partial^\mu \phi_j + \text{higher order terms}. \quad (4.5)$$

Using the equation of motion for the pion field it is easy to show that

$$\partial_\mu J_j^{\mu} = - \frac{\mu_\pi^2}{f} \phi_j, \quad (4.6)$$

that is, PCAC is satisfied. The λ -dependent terms also contribute to J_j^{μ} in higher order of $1/m$. The contribution of (3.5) to J_j^0 is

$$- \frac{\lambda}{8m_\pi^2} \psi'^{\dagger} \left[\frac{\partial}{\partial t} \gamma_0 \tilde{\gamma} \cdot \tilde{\nabla} \gamma_5 \tau_j - \tau_j \gamma_5 \gamma_0 \tilde{\gamma} \cdot \tilde{\nabla} \frac{\partial}{\partial t} \right] \psi'. \quad (4.7)$$

In momentum space the lowest order λ -dependent coupling of the axial current to the nucleon is

$$- \frac{\lambda}{8m_\pi^2} \{ \sigma \cdot \tilde{p}_f (E_f - E_i) - \tilde{\sigma} \cdot (\tilde{p}_i - \tilde{p}_f) E_i \}. \quad (4.8)$$

This is $O(k)$ so it does not contribute to $M_j^{\mu}(-1)$. Hence $\mathcal{M}_\pi(0)$ remains independent of λ . This is expected from the form of (3.5) and supports the notion that in a pseudovector coupling model the relativistic and non-relativistic external pion emission amplitudes by themselves satisfy the soft pion constraint²²⁾.

The sigma model is less straightforward since the chiral transformation does not commute with the FW reduction. Formally one can show that there still is an invariance of the non-relativistic theory and that the appropriate non-relativistic fermion transformation operator is obtained by applying the FW reduction to the operator in (3.8a). Fortunately it is not necessary to know explicitly the transformation properties of the non-relativistic Lagrangian. Equivalent results are obtained by making an FW reduction not of the Lagrangian $\mathcal{L}$ of eq. (3.7) but of the Lagrangian which has been transformed by T_σ , that is, of

$$\mathcal{L}'_\sigma = T_\sigma^{-1} \mathcal{L} T_\sigma, \quad (4.9a)$$

$$= \bar{\psi} \{ i \not{\partial} - m - g(s + i \tilde{\tau} \cdot \tilde{\phi} \gamma_5) \} \psi + \mathcal{L}_B$$

$$+ \bar{\psi} \gamma_\mu \gamma_5 \tilde{\tau} \cdot \psi \cdot \partial^\mu \tilde{\alpha}(x) - 2 \tilde{\phi} \partial^\mu s \cdot \partial_\mu \tilde{\alpha}(x) + 2(s + v) \partial_\mu \phi \cdot \partial_\mu \tilde{\alpha}(x)$$

$$+ e c(s - 2 \tilde{\alpha}(x) \cdot \tilde{\phi}), \quad (4.9b)$$

$$= \psi'^{\dagger} \left\{ i \frac{\partial}{\partial t} - \gamma_0 m - g \gamma_0 s + \gamma_0 \tilde{\gamma} \cdot \tilde{\nabla} \alpha \tilde{\tau} \cdot \tilde{\alpha} \gamma_5 \right. \\ \left. + i \gamma_0 \tilde{\gamma} \cdot \tilde{\nabla} - i g \gamma_0 \gamma_5 \tilde{\tau} \cdot \tilde{\phi} + \gamma_5 \tilde{\tau} \cdot \tilde{\alpha} \right\} \psi$$

$$+ \mathcal{L}'_B + e c(s - 2 \tilde{\alpha}(x) \cdot \tilde{\phi}), \quad (4.9c)$$

where

$$\mathcal{L}'_B = \mathcal{L}_B - 2\phi \partial^\mu s \cdot \partial_\mu \alpha(x) + 2(s+v) \partial^\mu \phi \cdot \partial_\mu \alpha. \quad (4.10)$$

Carrying out the FW transformation with

$$\mathcal{E} = g \gamma_0 s - \gamma_0 \vec{\gamma} \cdot \vec{\nabla}_\alpha \vec{\gamma} \cdot \vec{\alpha} \gamma_5 \quad (4.11a)$$

and

$$0 = -i \gamma_0 \vec{\gamma} \cdot \vec{\nabla} + i g \gamma_0 \gamma_5 \vec{\gamma} \cdot \vec{\phi} - \gamma_5 \vec{\gamma} \cdot \vec{\alpha} \quad (4.11b)$$

gives the reduced Lagrangian

$$\begin{aligned} \mathcal{L}'_\sigma = \psi'^\dagger \left\{ i \frac{\partial}{\partial t} - \gamma_0 m - g \gamma_0 s + \gamma_0 \vec{\gamma} \cdot \vec{\nabla}_\alpha \vec{\gamma} \cdot \vec{\alpha} \gamma_5 \right. \\ \left. - \frac{\gamma_0}{2m} (-i \gamma_0 \vec{\gamma} \cdot \vec{\nabla} + i g \gamma_0 \gamma_5 \vec{\gamma} \cdot \vec{\phi} - \gamma_5 \vec{\gamma} \cdot \vec{\alpha}) \right\} \psi' \\ + \text{terms higher order in } 1/m \\ + \mathcal{L}'_B + \epsilon c (s - 2\alpha \cdot \phi). \end{aligned} \quad (4.12)$$

The axial-vector current is then

$$\begin{aligned} J^\mu_j = \psi'^\dagger \left(-\frac{i}{2m} [\vec{\gamma} \cdot \vec{\nabla} \gamma_5 \tau_j + \tau_j \gamma_5 \vec{\gamma} \cdot \vec{\nabla}] + \frac{g}{m} (\vec{\gamma} \times \vec{\phi})_j, -\gamma_0 \vec{\gamma} \gamma_5 \tau_j \right) \psi' \\ + 2\phi_j \partial^\mu s - 2(s+v) \partial^\mu \phi_j + \text{higher order terms.} \end{aligned} \quad (4.13)$$

The divergence of J^μ_j is easily obtained from $\partial_\mu J^\mu_j = -\delta \mathcal{L}'_\sigma / \delta \alpha_j(x)$.

This is zero except for the contribution of the last term of (4.12)

which is proportional to ϕ_j so that PCAC is satisfied by all unitarily equivalent reduced Lagrangians.

The fermion part of the axial current (4.13) involves only even operators. The terms describing the coupling of the axial current to the nucleon (i.e., terms without a boson field) are identical to those in the pseudovector model. Using the low-energy theorem this means that the leading term in the pion emission amplitude $\mathcal{M}_\pi(0)$ is identical in non-relativistic pseudovector and sigma models just as it is for the relativistic theories.

In higher orders of $1/m$ the FW transformed Lagrangian and hence the axial current will depend on the order in which the odd operators are removed from (4.9c). The form of the lowest order term in which an ambiguity arises is

$$i \frac{\lambda}{8m^2} \psi'^\dagger [-i \gamma_0 \vec{\gamma} \cdot \vec{\nabla}, -\gamma_5 \vec{\gamma} \cdot \vec{\alpha}] \psi', \quad (4.14)$$

This makes a contribution to J^0_j which is identical to the λ -dependent term (4.7) found for the pseudovector model. Since (4.14) is $O(k)$ in momentum space it does not contribute to $\mathcal{M}_\pi(0)$. The same argument applies to λ -dependent terms of higher order in $1/m$.

Taking PCAC into account we see that $\mathcal{M}_\pi(0)$ must be independent of λ . This means that the contribution of the λ -dependent pion-nucleon vertex eq. (3.12) to processes in which a pion is emitted or absorbed on an external nucleon line *must* be cancelled by other λ -dependent terms in the amplitude. In such processes the contribution of $O(k)$ terms in the non-relativistic pion-nucleon vertex (this includes the Galilean correction term) is fixed by the soft pion theorem.

5. Conclusion

For theories with local gauge invariance (e.g., electrodynamics) the requirement that the non-relativistic and relativistic Lagrangians have the same transformation properties uniquely fixes the Foldy-Wouthuysen transformation. This is not the case for two models of the pion-nucleon interaction which have an approximate global symmetry and satisfy PCAC. With exception of the pion mass term the pseudovector coupling pion-nucleon Lagrangian is invariant under addition of a constant to the pion field. This invariance is preserved by $\alpha\beta\beta$ unitarily

equivalent FW transformations. On the other hand there is no FW transformation which preserves the approximate *chiral* symmetry of the sigma model. Using *only* the symmetry properties of the Lagrangian it seems therefore that no restriction on the form of the effective non-relativistic pion-nucleon vertex is possible.

Using PCAC it is possible to derive a relation between the soft pion value $\mathcal{M}_\pi(0)$ of the pion emission amplitude and the axial-vector current of the nucleon, at least for those reactions in which the pion can be emitted by an external nucleon leg. We calculate the axial current for the FW transformed pseudovector and sigma models and show that PCAC is still satisfied in the non-relativistic theories. Furthermore, the nucleon's axial-vector current and hence $\mathcal{M}_\pi(0)$ are the same in both models non-relativistically as well as relativistically. Finally the terms in the axial current generated by unitarily equivalent FW transformations are examined. It is found that these terms are $O(k)$ and do not contribute to $\mathcal{M}_\pi(0)$. Therefore in processes where a pion can be emitted or absorbed on an external nucleon line the contribution of $O(k)$ terms in the effective non-relativistic pion-nucleon vertex (including the Galilean correction term) is fixed by the soft pion theorem.

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