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SOME COMMENTS ON THE SOFT-PHOTON APPROACH TO
PROTON-PROTON BREMSSTRAHLUNG*

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ABSTRACT

General features of proton-proton bremsstrahlung as obtained from soft-photon calculations are discussed, and numerical results of such calculations are compared with data from the TRIUMF, Manitoba, and UCLA experiments.

The soft-photon, or Low, theorem is well known and has been used for a variety of radiative processes. Here we discuss its application to proton-proton bremsstrahlung (ppv). The soft-photon approach (SPA) has a number of advantages and of course some compensating disadvantages. It is simple, trivially gauge invariant, and trivially relativistically invariant. This latter property may be important as relativistic corrections could be significant at higher energies and are very hard to include in potential model approaches. The SPA also directly uses known information about the elastic process. It is a limiting case of any other correct calculation and so serves as a valuable check on more detailed models. Finally, since it is simple it is very useful for preliminary exploration of new kinematic regions, etc. The main disadvantage of the SPA is that it depends only on on-shell information and thus can make no direct statement about the off-shell interaction, which is the primary motivation for studying ppv.

One's philosophy toward the SPA is thus ambivalent. It contains a core of what must be present and correct and yet one hopes it fails, in fact fails miserably, thus increasing the possibility of learning interesting off-shell information.

In any case, one can use the SPA to learn about a number of general features and results in ppv. Here we will discuss these general results and then see how numerical results in this approach compare with potential models and with data.

Recall first how the Low theorem is derived.¹⁻⁴ One begins with the amplitudes for the external emission graphs, which contain all of the pieces of $O(1/k)$ in the photon momentum k , and expands the off-shell nucleon-nucleon (NN) amplitude and the electromagnetic vertex functions about an on-shell point. Some $O(k^0)$ terms are then added to make the result gauge invariant. The choice of such terms is, as usual, not unique but is made in the simplest way and has the effect of including some contributions from the off-shell interaction, from internal radiation, etc., but contributions which are fixed by gauge invariance in terms of known on-shell quantities.

The resulting amplitude can be written schematically as $M_{NN\gamma} = A/k + B + Ck$. Here, A and B contain the elastic NN amplitudes and, in the case of B, also derivatives of these amplitudes, evaluated at an

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on-shell point. They also contain known, simple functions of the radiative variables arising from the kinematics and thus implicitly depend on k . A contains only charge terms whereas B contains anomalous magnetic moment terms as well. A and B depend on the choice of the on-shell point about which the expansion is made (taken as an average energy and momentum transfer in the Low prescription), i.e., on the choice of variables for the elastic amplitude. They may individually vary significantly for different choices, but it is a general result that the sum $A/k + B$ changes for different choices of the on-shell point only by amounts of $O(k)$, i.e., by terms which contribute only to $C \cdot 5^{-7}$ for larger k , when initial and final proton energies may differ significantly, or particularly when the final nucleons are left with low relative momentum, it may be preferable to use the Feshbach-Yennie^{8,9} prescription, which essentially avoids the expansion about an average energy.

The Ck term (we have lumped all higher-order terms into C so that C is an explicit function of k), contains a number of contributions, including in particular the unknown off-shell terms coming from the NN and γN interactions. However, it also contains higher-order on-shell terms, contributions from internal radiation, the ambiguity in the choice of on-shell point, etc. It therefore should be very clear that a measurement of C is not a direct measurement of the off-shell parts of the NN interaction. Thus C large allows, but does not require, sizable off-shell contributions. Conversely $C \approx 0$ does not, strictly speaking, mean off-shell effects are small, since in principle there could be cancellations among various large contributions. Such a situation seems unlikely over any finite region, however, as the kinematic dependences of the various terms are different. Thus one presumably should look for regions where C is large and then resort to more detailed models to decide whether off-shell effects are large in those regions or whether the large terms are coming from some other piece of C .

The cross section can be obtained by squaring $M_{NN\gamma}$ and is of the form $da \sim k |M_{NN\gamma}|^2 \sim A^2/k + 2\text{Re}AB^* + (B^2 + 2\text{Re}AC^*k + 2\text{Re}BC^*k^2 + C^2k^3)$ where the A^2 and AB terms and the B^2 part of the $O(k)$ term are given in terms of on-shell information by the SP theorem.

The first two terms in the cross section, $A^2/k + 2\text{Re}AB^*$, can be expressed in a rather neat way via the Burnett-Kroll theorem⁵ as an operator function of the kinematics acting on the unpolarized elastic cross section. Thus a knowledge of the elastic NN cross section is sufficient to determine the first two terms of the NN cross section. More detailed knowledge, i.e. phase shifts, is needed first for the B^2 part. This operator is independent of the anomalous magnetic moments so we have the general result that anomalous magnetic moment terms do not contribute through the leading two orders in k , but appear first in the B^2 piece.

So far we have discussed unpolarized cross sections. It is an important question whether or not measuring spins gives additional information. Observe first that the Low theorem is an amplitude relation. Hence knowledge of the elastic NN amplitudes, i.e., of the phase shifts, allows calculation of A/k and B for any spin combination and thus calculation of the leading cross section terms $A^2/k + 2\text{Re}AB^* + B^2k$ for any spins.

One can also generalize the Burnett-Kroll theorem to obtain the leading two terms in the polarized NN cross section as an operator acting on the polarized NN cross section.⁷ Again, it is thus sufficient to know the elastic cross section, albeit for all polarization states, to obtain the polarized bremsstrahlung cross section through the first two terms. As before, to calculate B^2 one needs the phase shifts.

Thus one cannot escape the SP theorem by measuring spins, i.e., one still wants regions where C is large. However, one does investigate different combinations of amplitudes when spins are measured. For example, now radiation from anomalous magnetic moments enters in $O(k^0)$, i.e., in the ReAB^* term. Thus the sensitivity may be different. Furthermore, there is the intriguing possibility of finding specific spin correlations for which the leading terms vanish, thus allowing a direct measurement of the higher-order pieces.¹⁰

Next we investigate the relative sizes of the various terms contributing to the SP cross section. Such a separation into individual terms proves rather instructive and, although there have been several previous SP calculations,^{2,3,11} the separation seems not to have been made before. In Fig. 1(a) we show the various contributions for a particular symmetric coplanar case, one of those investigated in the

Fig. 1. Contributions of individual terms in the SP expansion to the square of invariant ppy amplitude.

42 MeV Manitoba experiment.¹² At this energy the $A^2 + 2\text{ReAB}^*$ and the B^2 terms are comparable over much of the region except at 0° , 180° , and $\sim 80^\circ$ where the $A^2 + 2\text{ReAB}^*$ term vanishes. The B^2 term is relatively smooth and contains a sizable magnetic contribution. For the TRIUMF experiment at 200 MeV [see Fig. 1(b)] results are qualitatively similar with the very important exception that now the $A^2 + 2\text{ReAB}^*$ term is quite small and the cross section is given almost entirely by the B^2 contribution.

The vanishing of the $A^2 + 2\text{ReAB}^*$ term at certain points is an interesting general feature and has important consequences. The combination vanishes not because of cancellations but because the amplitude A vanishes. The proof of this for 0° and 180° is simple. Note that A is proportional to the kinematic quantity $\vec{e} \cdot (\vec{p}_1/k \cdot \vec{p}_1 + \vec{p}_2/k \cdot \vec{p}_2 - \vec{p}_3/k \cdot \vec{p}_3 - \vec{p}_4/k \cdot \vec{p}_4)$, with $\vec{e}$ the photon polarization vector and the $\vec{p}$'s nucleon momenta. In the lab system for $\theta_\gamma = 0^\circ$, 180° we have $\vec{p}_2 = 0$ and $\vec{p}_1 \sim \vec{k}$. Here also for symmetric geometry the four-vector combinations $k \cdot p_3$ and $k \cdot p_4$ are equal and $\vec{p}_3 + \vec{p}_4 \sim \vec{k}$. Thus at these points the entire term is proportional to $\vec{e} \cdot \vec{k}$ which is zero so that $A \equiv 0$.

This result has the important implication that at these points the remaining $O(k)$ term, i.e., that proportional to ReAB^* , vanishes as well and thus here the entire $O(k)$ contribution is given by the known B^2 term. Thus for this particular geometrical situation the SP theorem is good to one higher order in k than usual. By continuity arguments there will be regions in the vicinity of these points, and also a region involving asymmetrical angles where the result is approximately true and where one must look to the fourth term, which is $O(k^2)$, to find a deviation from the soft-photon theorem.

For the UCLA geometry,¹³ where one proton and the gamma angle and energy are measured, one gets a sampling of both previous energy regions (cf. Fig. 2). At low k the $A^2 + 2\text{ReAB}^*$ term dominates but as k increases this term falls rapidly until eventually the relatively constant B^2 term becomes most important. Again B^2 contains a large component of magnetic radiation.

Another interesting insight can be obtained by looking at the A^2 and 2ReAB^* terms individually. Recall that we have mentioned two ways of calculating these terms. One can construct the amplitudes A and B separately using on-shell variables for the invariant functions in A and B and radiative variables for the explicit kinematic factors and spinor wave functions, or one can use the Burnett-Kroll theorem which requires the cross section evaluated at on-shell variables. These two approaches amount to slightly different choices of the on-shell variables and hence may give different A^2 and ReAB^* terms, but the combination must be the same up to terms which contribute to $C, 5-7$. In Fig. 3 we show the A^2 , 2ReAB^* , and $A^2 + 2\text{ReAB}^*$ terms for both choices for the UCLA geometry.¹³ The individual terms do vary somewhat but the sum is essentially identical. Note also that the ReAB^* term is very small. Thus for this case keeping just the A^2 term in the Burnett-Kroll approach—which is just the external emission dominance (EED) model of the UCLA group¹³—gives a good approximation to the cross section which will fail only when the B^2 term becomes important. Note also that in the Burnett-Kroll approach a portion of the " ReAB^* " term is put into the " A^2 " term, as compared with the alternative

Fig. 2. Contributions of individual SP terms as in Fig. 1, plotted versus laboratory photon energy.

Fig. 3. A^2 and AB contributions for the two different methods of calculation described in the text plotted versus photon energy.

approach, so that the A^2 term is a better approximation to the sum. These effects are more pronounced for the TRIUMF experiment but unimportant there since the B^2 term dominates.

It is now of interest to see how the predictions of the SPA compare with model calculations and with the data. We should re-emphasize that the differences between data or model calculations and the SP prediction reflect the size of C (and perhaps, when comparing with potential models, relativistic corrections to the leading terms) but are not a direct measure of off-shell effects since C contains contributions from a number of sources. Thus a region of large C may or may not indicate large off-shell effects. However, once found, such regions should be explored using models which allow one to vary off-shell input independently. On the other hand regions of small C are probably less interesting, as then either off-shell effects are small or there has been some elaborate cancellation which, while possible, seems unlikely.

Figure 4 shows a comparison between the SP calculation and a Hamada-Johnston (HJ) potential calculation¹⁴ at 42 MeV. At 0° , 180° , and $\sim 80^\circ$ where $A \approx 0$ the results differ very little thus indicating that at these points the higher-order $2\text{Re}B^2K^2 + C^2K^3$ piece of the cross section must be small. This, together with our earlier observation that $A \gtrsim B$, probably means that the difference at the peaks is due mostly to the $2\text{Re}A^2K^2$ term.

In comparing with data, some caution must be exercised as the data are averaged over sizable non-coplanarity while the theory is not.

Fig. 4. Comparison of SP calculation with the data and a HJ potential calculation of the Manitoba group at 42 MeV.

Fig. 5. Comparison of SP calculation with HJ calculation of Bohannon and a OBE calculation of Kamal and Szyjwicz at 200 MeV.

When this is taken into account the integrated cross section tends to fall below the HJ prediction particularly at small and/or asymmetric angles.¹² One can conclude from the graph, however, that the difference between SP and HJ predictions is of the same order of magnitude as the size of the error bars.

At 200 MeV the situation is completely different (see Fig. 5). Here differences between HJ,¹⁵ a one-boson exchange (OBE) calculation,¹⁶ and SP are most pronounced at 0° and 180° . Since at these points $A \approx 0$ (recall as well that $A \ll B$ over the whole region) the difference must be due mainly to the $2\text{Re}B^2K^2$ and C^2K^3 terms. Thus at this energy one is measuring a different set of unknown terms than at 42 MeV. Compared with the preliminary data of the TRIUMF group¹⁷ as shown in Fig. 6 the SP calculation seems a bit low and the HJ calculation a bit high, though again the differences are not too much greater than the size of the error bars.

Finally, consider the UCLA experiment¹³ (Fig. 7). Here the geometry is entirely different and the energy is much higher, 730 MeV. Since for these kinematics generally $A \gg B$ and since the $\text{Re}A^2K^2$ term

Fig. 6. Comparison of the TRIUMF 200 MeV data with the SP calculation, HJ calculation of Bohannon (Ref. 15) and OBE calculation of Kamal and Szyjwicz (Ref. 16).

is small (cf. Figs. 2 and 3), it is perhaps not a surprise that the EED model¹³ describes the cross section well over a sizable region. At larger k , however, the B2 term dominates and it, together with a rising phase space, produces an increase in the cross section. Thus the SP result, although generally a bit low, differs from the data, at least for the cases plotted, by amounts again of the order of the size of the error bars. Again one must be a bit cautious in interpreting this as the theory has not been integrated over the rather large acceptance of counters, and in addition the phase shifts used in constructing the elastic amplitudes are not as well known here as at the lower energies.

Thus to summarize, we have examined the soft-photon approach to ppy and some of the general results one can obtain from it, looking in particular at the contributions of individual terms. We find that the particular geometrical situation and energy determine to some extent which of the unknown terms are likely to be most important. Thus for the symmetric coplanar geometry at low energies one apparently probes the ReAC* term and should find effects in the middle regions of θ_γ , whereas by 200 MeV the unknown contributions come predominately from the ReBC* and C2 terms and are most pronounced at $\theta_\gamma = 0^\circ$ and 180° . In the UCLA geometry one is looking primarily at the leading terms, with the B2 term becoming important only at larger k . Finally, in all cases considered here the SPA reproduces the qualitative features of the data and differs quantitatively by amounts not too much greater than the sizes of the error bars.

Fig. 7. Comparison of the SP calculation with the UCLA data at 730 MeV.

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