Rapporteur's talk at 8th International Conference
on Few Body Systems and Nuclear Forces
Graz, August 24-30

TRI-PP-78-16
Aug 1978 *C2*

05 DEC. 1978

RECENT WORK IN THREE-BODY SYSTEMS

A.W. Thomas^{*}
TRIUMF at U.B.C.

Vancouver, B.C., Canada V6T 1W5

CERN LIBRARIES, GENEVA

CM-P00067103

1. Introduction

In this report I shall briefly outline those contributions to Discussion Session 5 (DS5) which seem most important to me. As far as possible an attempt will also be made to place these contributions in the context of current developments in the appropriate field. Of course, it will be only too painfully obvious that given the wide range of topics included in DS5—ranging from low energy n - d scattering to hypernuclear physics—the emphasis given here is highly subjective. While I have attempted as far as possible to remove personal biases, I must apologise in advance to those whose work is undeservedly given inadequate treatment.

Very briefly the plan of this review is as follows. Section 2 deals with the most familiar three-body system namely n - d . In subsect. 2.1 we examine the case of low energy N - d (nucleon-deuteron) scattering where rigorous Faddeev treatments, including all necessary N - N partial waves, are still possible. We shall see that there is now a tremendous quantity of high quality data waiting to be analysed. The main theoretical advance made recently seems to be a rigorous theory for including Coulomb effects when two of the three nucleons are charged. Subsection 2.2 deals with p - d backward scattering at intermediate energies—now accessible to the meson factories. We shall see that here too experimental results seem to be coming too fast for the theorists!

Section 3 deals with pion scattering and absorption by two nucleons (usually a deuteron). This is a three-body system which has attracted great interest in the last two or three years. On the experimental side we have new data (and promises of much more) from LAMPF, Saclay and SIN. The credibility of the theoretical calculations has been very much strengthened in the last year by independent confirmation of a 'model' problem by groups at Lyon and TRIUMF - Weizmann Institute. In addition, a great deal of work has been performed which relates the usual semi-phenomenological three-body treatments to an underlying field theory. Specifically, the question of how to incorporate the possibility of real pion absorption is now very well understood in the πd system. We shall briefly mention the recently proposed di-baryon resonances in the context of the $NN\pi$ problem.

In subsect. 3.2 we shall also see that the application of three-body ideas to the pion-nucleus problem has also led to considerable progress recently.

^{*}Research supported by a Grant from the National Research Council of Canada (#A3233).

We mention particularly the consequences of the three-body model of the first order optical potential, and also the controversial question of the range of the πN interaction.

Finally in sect. 4 we mention applications of the Faddeev equations to $K^- d$ scattering, as well as the question of exchange currents in electromagnetic interactions with few nucleon systems.

2. Nucleon-Deuteron Scattering

2.1 Low Energy

A tremendous amount of effort has been spent on the low energy n-d problem since the first three-body calculations of Aaron *et al.*¹ It now seems possible, with a great deal of computational effort, to reproduce the major features of the elastic nucleon-deuteron differential cross-section and polarisation measurements below 50 MeV. There have been a number of reviews of the experimental and theoretical situation at recent conferences,^{2,3} to which we refer for more detail. In addition discussion session 6 (DS6) at this conference contains several new polarisation measurements.

One new measurement of the elastic n-d differential cross section (below break-up threshold) has been reported in this session (#5.1)*. It is unfortunate that while it does seem to be a very accurate measurement over a complete angular range, no comparison has been made yet with the best modern calculations. Nevertheless this data should be of considerable interest vis à vis p-d data at the same energy, in the light of the recent exact inclusion of Coulomb effects in three-body systems^{4,5}—as discussed at the end of this subsection.

As an example of the success the theory has had explaining low energy elastic data, in Fig. 1 we show the results of Benayoun *et al.*⁶ at 14.1 MeV. They used the (super soft core) local potential of de Tournell and Sprung (type C), including the 3S_1 - 3D_1 , 1S_0 and P-waves exactly (solid curve), and 1D_2 , 3D_2 perturbatively (dashed curve). Clearly the addition of D-waves helps resolve the small remaining discrepancy in the forward differential cross section apparent in an earlier calculation using the Reid soft core potential.⁷ Figure 1(b) is also a representative example of the excellent fits to polarisation variables which can now be obtained.⁶⁻⁹ (For comparison, the dash-dot curve shows the results of Stolk and Tjon,⁷ and suggests that their treatment of all waves $L \geq 1$ by perturbation theory was not adequate.) The lone exception in this success story is the deuteron vector polarisation, especially in the region of 100° .⁶ There has been some speculation that this failure may eventually constrain the N-N tensor interaction [$\delta(^3D_1)$ and ϵ_1] at low energy,^{3,8}

*Throughout this summary we refer to contributions in DS5 of volume 1 of the conference proceedings as #5.X. Hopefully this will reduce the length of our reference list.

Fig. 1. Calculations of the nd differential cross section and neutron polarisation at 14.1 MeV (from Benayoun *et al.*⁶), using the de Tourreil-Sprung potential. For details see the text.

but much more work is required on this point.

In any case, the main conclusions in this energy region are generally agreed upon. There is little or no evidence for drastic differences between n-d and p-d scattering from (14-50) MeV. The vector polarisations are strongly dependent on the P-wave N-N interaction, while the tensor force is primarily responsible for tensor polarisation.⁶⁻⁹ Quantitatively however, each observable requires both P- and D-waves and tensor coupling. It seems unlikely that the enigmatic deuteron d-state probability¹⁰ will be obtained from tensor polarisation measurements in this system.¹¹

There has recently been some investigation of vector analysing powers in n-d break-up. This seems to have revealed an intriguing discrepancy between theory¹² and experiment¹³ in the n-p final state interaction (FSI) region. Indeed, as we see in Fig. 2, the data looks closer to the elastic data at the same energy than to the calculations. Part of the reason for this discrepancy is probably that the theory restricts the n-p relative energy to 0 keV, whereas it should be averaged over the interval (0, 1) MeV. For further discussion and more recent results we refer to DS6 and its rapporteur's talk.

Perhaps the main motivation originally for studying N-d break-up was the hope that it would yield some information about the off-shell behaviour of the N-N interaction. It is fair to say that much of this enthusiasm faded after the results of Brayshaw's boundary condition model (BCM) were published.¹⁴ It is worthwhile to review exactly what Brayshaw was able to show, and particularly how this relates to contributions #5.4, #5.5 and #5.6 and their ilk.

Fig. 2. Comparison of the deuteron vector analysing power (iT_{11}), and neutron analysing power (A_y), calculated by Bruinsma and van Wageningen¹² (solid curve) with data¹³ in the n-p FSI region. The dash-dot curve is the appropriate elastic n-d parameter.

The BCM enables one to write a set of unitary three-body equations whose kernel has two parts.¹⁵ The first (or minimal) part is determined entirely by the two-body phase shifts and two- and three-body unitarity. What remains is a pair of real functions ($\hat{B}$ and $\hat{C}$) which can be varied (essentially) arbitrarily without affecting either the on-shell two-body input or unitarity. In principle one can include all possible two-body off-shell behaviour and three-body forces through variations of $\hat{B}$ and $\hat{C}$. By making such arbitrary variations in these functions (for the case of s-wave N-N interactions only) Brayshaw found empirically that the n-d break-up cross sections at 14.1 MeV were essentially identical provided the value of the doublet scattering length (2a) was kept fixed. This is a powerful result which at face value could be summarised in the following proposition, which we shall label proposition 13: 'n-d break-up contains no more information about the off-energy-shell behaviour of the N-N interaction, or about three-body forces, than is conveyed by the knowledge of 2a '.

Powerful as the BCM may be, proposition 13 is too strong. The calculation on which it is based did not include the N-N tensor force. Extensive tests were only performed at one energy (14.1 MeV). Most importantly, the question of whether all possible variations in $\hat{B}$ and $\hat{C}$ were tested is highly subjective. This is particularly significant within a formalism which is not widely used, and therefore for which we have little accumulated wisdom. (From this perspective it is obvious that independent repetitions of the Brayshaw calculation at somewhat higher energy should have a fairly high priority.) Thus, it is certainly possible that well chosen n-d break-up experiments may still reveal interesting off-shell information.

The BOL system which could simultaneously detect coincidences over essentially 4π geometry has yielded an enormous amount of N-d break-up data, much of which still

awaits analysis. Two contributed papers (#5.5 and #5.6) deal with the problems of displaying this data at 26.5 (E_d) and 50 MeV (E_p), and most important, finding regions which might defeat proposition 13! In Fig. 1 of #5.6 Blommesteijn *et al.* have roped off regions of phase space according to the difference (S) between theoretical predictions (in that region) using the Malfliet-Tjon I-III and I-IV potentials.¹⁶ (The former has a sum of one repulsive and one attractive Yukawa terms for the 3S_1 interaction, the latter only one attractive term—see also Ref. [17].) When S lies between $\pm 20\%$ one expects the theoretical predictions to be quite firm. However, in the sub-region where the observed cross-sections are small, experiment and theory were found to differ by more than a factor of two on over 20% of the grid points! [Even greater systematic differences were found in the low cross section region with $S \in (20, 60)\%$.] While this is encouraging, it is clear that much more realistic calculations including the tensor force and higher N-N partial waves must be included before one can take these differences seriously. In short, the ball is well and truly in the theorists' court!

A more positive view of proposition 13 says that if off-shell effects are negligible n-d break-up can be used to extract on-shell N-N parameters. For example the n-n effective range (r_{nn}) is relatively poorly known, and this introduces quite a large uncertainty in some 3N observables such as the triton binding energy. (Malfliet and Tjon,¹⁶ for example, found that E_T decreased by about 0.5 MeV when r_s was changed from 2.6 to 2.8 fm.) Contribution #5.4 by Cameron *et al.* is an attempt to determine r_{nn} from quasi-free neutron scattering on deuterium at 21.5 MeV. Their final value of $r_{nn} = 2.8 \pm 0.4$ fm (assuming $a_{nn} = -16.4$ fm) is based on an analysis of the data using separable interactions.¹⁸ Unfortunately this is not yet accurate enough to clarify the situation with respect to E_T . Finally we must observe that it is not strictly correct to use proposition 13 to justify the use of separable N-N interactions without keeping 2a fixed at the experimental value throughout. (Indeed the argument can be taken full circle because 2a and E_T are linearly dependent within a given potential model—the Phillips' line.¹⁹)

To conclude this section we note that one of the most recent successes in the low energy region has been the exact inclusion of Coulomb effects in the p-d system using the formalism of Alt *et al.*^{4,5} So far the formalism is restricted to only two charged particles—the three-body problem with Coulomb forces being still unsolved! Furthermore, all the applications so far have been for elastic p-d scattering. Contributions #5.19 and #5.20 deal with just this problem.

In #5.19 Ziegelmann first establishes that the limiting procedure proposed by Alt *et al.* works for a model two-body problem—by comparing with the Vincent-Phatak procedure.²⁰ A cut-off radius of about 30 fm seems to give quite accurate answers at all but the lowest energies. The exact n-d and p-d phase shifts are then compared. In the doublet case there is almost no Coulomb effect. Both this fact, and

the actual values of the Coulomb corrections for the quartet case (as low as 1 MeV), are in rather good agreement with the predictions of the simple formula of Plessas *et al.* (see Table 2 of #5.19).²¹

$$\delta_{l,sc}(p-d) - \delta_{l,s}(n-d) = \frac{2u\ell^2}{(2\ell+1)} \frac{\partial}{\partial k} (\delta_{l,s}(n-d)) . \quad (2.1)$$

Some explanation of the reason for the inordinate success of this simple theory (derived for two elementary particles) is called for. Finally, we note that as already observed experimentally the effect of Coulomb corrections on the final differential cross section (c.f. #5.20) is quite small.

2.2 Intermediate Energy

The problem of large angle N-d scattering at intermediate energy has considerable intrinsic interest because of the large momentum transfers involved. However the discovery of unexpected structure in the excitation function for 180° scattering—namely a shoulder between 250 and 550 MeV,^{22,23} possibly associated with the pion production threshold—has produced much excitement.²⁴⁻²⁹ Two papers reporting detailed n-d angular distributions at backward angles (roughly 140°-180°), have been contributed to this conference by groups from SIN (#5.2) and LAMPF (#5.3). A measurement of the analysing power in backward $\vec{p}d$ scattering from TRIUMF (#6.7) is also relevant to this discussion.

The SIN data in the region 350-550 MeV has been fitted rather well by the form

$$d\sigma/d\Omega = \alpha \exp \left\{ \beta(u_{\max} - u) \right\} \quad (2.2)$$

(with u the usual Mandelstam variable), at all but the extreme backward angles ($\theta > 177^\circ$). However the justification for this Regge form is not at all clear. Indeed, in the one nucleon exchange (ONE) model one would write schematically [see Fig. 3(a)]

$$d\sigma/d\Omega_{\text{ONE}} \sim |\psi(\vec{Q}) \psi(\vec{Q}') / (u - m_N^2)|^2 . \quad (2.3)$$

(Such a model readily explains the larger cross section observed for $p\alpha \rightarrow dt$ than $p\alpha$ elastic scattering at large angles,³⁰ in terms of the lower relative momenta, $\vec{Q}$ and $\vec{Q}'$, involved for deuteron than triton exchange in the process analogous to Fig. 3(a).)

The LAMPF n-d data covers the range (200-800) MeV on a fine grid of 27 energies! Below 350 MeV and above 650 MeV the simple exponential form

$$d\sigma/d\Omega = A \exp(-Bq) , \quad (2.4)$$

fits very well—with $q = [Q(\theta^*) - Q(180^\circ)]$, and Q the relativistic momentum variable defining the momentum of the exchanged nucleon in the c.m. of either deuteron. (Note that B is slightly energy dependent however.) For the region (350, 650) MeV,

Fig. 3. Two contributions to p-d backward scattering. Case (a) is the standard one nucleon exchange mechanism, while (b) involves virtual pion production.

however, their data falls below this exponential at $\theta > 165^\circ$. It will be very useful to check the consistency of these two sets of data in the region of overlap (when plotted against the same variable!). In any case the message seems to be that some new mechanism sets in about 300 MeV.

Figure 3(b) shows one of several possible graphs which would be expected to have their largest effect in this region, and which have therefore been suggested to explain the anomaly.²⁸ The top vertex is imagined to behave in a similar manner to the on-shell $pp \rightarrow \pi^+d$ amplitude, which peaks at about 2160 MeV (invariant mass). In the limit where the mechanism of Fig. 3(b) is completely dominant, Kolybasov and Smordinskaya²⁸ have been able to give an elegant proof [based only on factorising the $pp \rightarrow \pi^+d$ amplitude out of Fig. 3(b)], that the asymmetry in the $\vec{p}d$ reaction, in the laboratory frame, is equal to the asymmetry in $\vec{p}p \rightarrow \pi^+d$.

Anderson *et al.* (#6.7 and Ref. [31]) recently looked for this signature at 316 and 516 MeV at TRIUMF. (These energies were chosen to fill a gap in earlier measurements at 198, 425 and 630 MeV - see Ref. [31].) The 516 MeV data is crucial because the free $pp \rightarrow \pi^+d$ cross section peaks at 600 MeV, and Fermi motion should lower this to about 500 MeV in the pd case. Thus if the pion exchange mechanism dominates anywhere, 516 MeV is a good initial guess. Figure 4 compares the $\vec{p}d$ results with measured asymmetries in $\vec{p}p \rightarrow \pi^+d$.³² No quantitative agreement can be claimed, but the qualitative tendency is correct, and confirms the similarity noted a couple of years ago at 630 MeV.³³ Although no firm conclusion is possible, it now seems quite likely that the mechanism shown in Fig. 3(b) does play a significant role in backward N-d scattering at intermediate energy.

To conclude this section we mention briefly papers #5.7(a) and (b), which deal with the question of whether there is an enhancement in p-d break-up in the case where the three final particles are collinear. Such a phenomenon was first reported by Lambert *et al.* at 23 MeV.³⁴ Recently Yuasa *et al.*³⁵ reported a similar

enhancement at 156 MeV with even less impressive statistics. The calculation of Oryu [#5.7(b)] for the 156 MeV case, includes the effective three-body force (involving a Δ in the intermediate state) which is supposed to be responsible for the 'collinearity effect'. However, the calculation does not produce the fine structure indicated by Yuasa *et al.* Also on the negative side, a search for such an enhancement at the Manitoba cyclotron at 28.5 MeV [#5.7(a)] revealed no structure. With the present contradictory experimental evidence, based on relatively poor statistics, we can draw no conclusion. Nevertheless, further investigation seems worthwhile—possibly in the energy region near 500 MeV where a real Δ could take part.

Fig. 4. A comparison of the $\vec{p}d$ asymmetry measured at TRIUMF at 316 and 516 MeV,³¹ with the corresponding asymmetry²⁸ in $\vec{p}p \rightarrow \pi^+d$.³²

3. Pion Scattering and Absorption

3.1 The πNN System

The present experimental situation with respect to πd elastic scattering is somewhat like the final ascent of one of Austria's peaks. We have already had enough glimpses of the terrain below to want to see more, and there is a promise that the sun will soon break through, revealing the Alps in all their glory! Until now there has been only one modern πd experiment below 100 MeV,³⁶ and a handful of experiments with quite large statistical errors through the (3,3) resonance region.³⁷⁻³⁹ On the other hand, there is a tremendous promise of new high quality data immediately ahead. The total cross section data from SIN (which is too good for present theory!) is an example of what is possible.⁴⁰

At this conference the only new data available are the first low energy $\pi^\pm d$ results from Saclay (#5.15), which will be discussed below. In the near future we

can expect excellent data from (100-300) MeV from the SUSI spectrometer at SIN.⁴¹ Having quickly scanned the region from (230-500) MeV,⁴² LAMPF can now be expected to produce high quality data above the resonance. Groups at both SIN⁴³ and LAMPF⁴⁴ are considering the possibility of measuring tensor and vector polarisations.

To fully appreciate the advances made on the theoretical side, we can only suggest that the reader look through the reviews in the 5th and 6th conferences in this series.^{45,46} Four years ago, with the exception of the πd scattering length, there was only one three-body calculation available. It was based on a minimal relativistic extension of the non-relativistic Faddeev equations designed for energies below 100 MeV.⁴⁷ Furthermore, as such calculations are quite complicated there was a need for an independent check on the results—as urged at the Pittsburgh conference.⁴⁸ Already such a test has been carried out, and the completely independent results of the Lyon⁴⁹ and Weizmann-TRIUMF groups^{50,51} agree to better than 1%, for the semi-relativistic model⁴⁷ restricted to the deuteron (3S_1 - 3D_1) and P_{33} interactions. We strongly urge any group intending to calculate πd scattering within a three-body formalism to check their code on this 'homework problem' first. In addition to this advance, fully relativistic three-body calculations,⁵²⁻⁵⁴ including all spin and isospin complications, and πN channels (the 'small' πN waves) are now available. Many other technical advances, such as the use of more realistic deuteron wave functions, N-N P-wave interactions and so on, will be discussed. Finally we must mention that quite successful efforts have been made to relate the three-body calculations to an underlying field theory,⁵⁵⁻⁵⁸ as we shall discuss below.

3.1.1 Low Energy πd Scattering

In Fig. 5 we show the new $\pi^\pm d$ data of Balestri *et al.* (#5.15 taken at Saclay) at 47 MeV. We see that the π^+ data [Fig. 5(a)] is consistent with the earlier data of Axen *et al.*,³⁶ and possibly lies slightly above the theory in the (50°-65°) region. The π^- data at the same energy [Fig. 5(b)] constitutes a real advance, and will be very helpful in sorting out the Coulomb-nuclear interference effects which are crucial at this energy.

It is difficult to draw conclusions from the comparison with theory until the data at all angles is available. [Note that the calculation labelled Thomas in #5.15 appears to be taken from the Laval conference, whereas the best curve available (shown in Fig. 5(a)) comes from Ref. [47]]. Both calculations use the semi-relativistic theory of Ref. [47], mentioned above in connection with the 'homework problem'. One difference which is relevant at very backward angles is that Thomas⁴⁷ (and later Rinat and Thomas⁵⁰) deleted the coupling of πd channels with $\ell \neq \ell'$ in order to save computer space and time. This will be called the diagonal approximation. Fortunately this has been shown (#5.9 and Ref. [49]) to lead to very small errors in the differential cross section. This error is greatest near 180°, where

including $\ell \neq \ell'$ gives about a 7% increase at 142 MeV. At 48 MeV the error from this should be even smaller, and the small differences between the two calculations, particularly near 40°, probably have another source.

Fig. 5. Comparison of the calculations of Thomas⁴⁷ and Giraud *et al.*⁴⁹ at 47.7 MeV (a) with π^+d data at 47 MeV (#5.15), and (b) π^-d data at 47 MeV. (N.B. The theory in case (b) had $P_D = 7\%$, and the vertical scale is micro-barns/sr.)

One very interesting possibility is related to the nature of the πN interaction itself—that is a difference in treatment of the P_{11} interaction. Assuming that the nucleon pole term [Fig. 6(a)] is a dominant piece of the P_{11} interaction, Thomas excluded it from any channel (e.g. $L=1, J=1^+$) where the pole term would violate the Pauli principle (i.e. two nucleons with $L_{NN}=0, S=1, I=1$) in an intermediate state—see particularly the discussion in sect. 4.1 of Ref. [46]. This effect is related to the question of true pion absorption which we shall take up again in subsect. 3.1.3. Both of these aspects of low energy πd scattering merit more work.

Fig. 6. The Born approximation diagrams for πN scattering: (a) the nucleon pole term and (b) the crossed Born graph which generates the (3,3) resonance in the Chew-Low model.

3.1.2 πd Scattering in the (3,3) Resonance Region

Contributions #5.9 and #5.11 report some aspects of πd calculations in the resonance region. The calculations of Giraud *et al.* within the RPK formalism,^{47,50} constitute a real tour de force in terms of computational effort. Their use of the method of Padé approximants (i.e. iterative solution) enables them to include all s- and p-wave πN channels exactly. (This leads to up to 24 coupled channels, with of the order of 30 quadrature points per channel—i.e. a 700×700 matrix!) In view of the weakness of the πN interaction in all but the P_{33} channel, this should not be necessary. Hammel *et al.*^{51,59} have included these channels using perturbative methods which appear to converge fast. Figure 7 shows a comparison of the results of Giraud *et al.* at 142 MeV⁴⁹ (using the RPK theory), with those of Hammel *et al.*^{51,59} (using the fully relativistic (FR) theory of Rinat and Thomas⁵⁰), in the case of a simple deuteron wave function with $P_D = 4\%$ (without the diagonal approximation).

It is interesting that while the RPK and FR calculations were quite different when only the P_{33} interaction was used, the results were very close after all other πN s- and p-waves were introduced. Figure 8(a) shows the results of Giraud *et al.* (#5.9) using the rank-2 3S_1 - 3D_1 interaction of Pieper⁶⁰ (which gives the Reid soft core wave function exactly). Strangely the effect of the small waves in this case is almost identical to that found by Hammel *et al.*—c.f. Fig. 7. Note also that while the use of a realistic wave function does not qualitatively change the final cross section, there are significant quantitative changes near 180° (of order 20%—see #5.9). Although more calculations remain to be done, the final theoretical results seem to be quite firm, and much more accurate experiments are eagerly awaited.

Fig. 7. Comparison of the results of Giraud *et al.*,⁴⁹ and Hammel *et al.*,^{51,59} with data³⁷—showing the effects of πN channels other than P_{33} in the RPK and FR formalisms.

The original interest in πd polarisation measurements was spurred by Gibbs, who suggested that one might be able to determine P_D from a measurement of $t_{20}(180^\circ)$.⁶¹ Several groups are now considering this very difficult measurement.^{43,44} With the removal of the diagonal approximation from the work of Thomas and collaborators (which was the reason for the large difference between the three-body calculations and those of Gibbs, reported in Ref. 10), the theory seems quite settled. The small πN waves have little effect on this parameter. All three groups (#5.9 and Refs. 51, 59, 61) now agree within 10% or so, which is a more believable amount of model dependence. Unfortunately from the point of view of using this to measure P_D the situation is not promising. In fact, the curve of $t_{20}(180^\circ)$ versus P_D is quite flat beyond 4%, which is the interesting region. (Giraud *et al.* (#5.9) report that $t_{20}(180^\circ)$ is 0.02 with $P_D = 0\%$ and -0.56 with $P_D = 4\%$, but only -0.67 with $P_D = 7\%$.) However information about the angle and energy dependence of t_{20} would be extremely valuable in sorting out the theory. Finally we note that the vector polarisation [shown in Fig. 8(b) at 142 MeV] is strongly dependent on the small πN waves. This too is well worth measuring—particularly because recent (as yet) unpublished work by Hammel *et al.*⁵⁹ suggests that $i t_{11}$ at 256 MeV is strongly dependent on the deuteron wave function.

Fig. 8. Results of the calculations of Giraud *et al.* (#5.9) at 142 MeV using the rank-2 3S_1 - 3D_1 interaction of Pieper.⁶⁰ (a) The πd differential cross section, and (b) the πd vector polarisation.

To conclude this section, we turn from the successes of three-body theory at 142 and 180 MeV to the qualitative failure at 256 MeV.^{50,51} Contribution #5.9 has shown by using a rank-2 3S_1 - 3D_1 interaction (i.e. P2 versus P1) that the D-wave phase shift, which is badly wrong in a rank-1 model, has little effect on the

differential cross section, and is therefore not responsible for the problem. Myhrer and Thomas (#5.11) have calculated the triple scattering graph involving the 1P_1 N-N interaction to see if that could introduce a dip near 90° . While that is not the case, they do find significant effects on both $d\sigma/d\Omega$ and t_{20} throughout the resonance region when certain off-shell models of the π N and N-N interactions are used. In particular, the P_{33} interaction of Thomas⁴⁷ when used with the Mongan⁶² 1P_1 potential gave quite large effects. We shall mention #5.11 again in sect. 3.2 in relation to models of the π N t-matrix off-shell.⁶³

3.1.3 Field Theory of the π NN System and the Effect of Absorption

In the past couple of years a refreshing number of papers have looked at the π NN scattering problem from a less phenomenological point of view.^{46,55-58} Extreme among these is the work of Stingl and Stelbovics (#5.12) who start with the static Chew-Wick field theory^{64,65} for pions and nucleons, in which all states $(N + N + k\pi)$ with $k \geq 3$ are neglected. While any such truncation (Tamm-Dancoff approximation) leads to inconsistencies at some point, it does provide a closed solvable model including renormalisation effects. The authors have shown that the mathematical formulation of such a model for $\pi NN \rightarrow \pi NN$ is more complicated than the usual three-body problem, in that one finds a new type of disconnectedness—associated with renormalisation and pion absorption. These problems can be overcome in the restricted model and a set of equations with a 5×5 connected kernel written down.

On the other hand, Mizutani and Koltun⁵⁵ and Rinat⁵⁶ start from the same field theory, with the aim of seeing how pion absorption fits into a theory of elastic scattering. After manipulating the theory into a suitable form, the difficulties raised in #5.12 are hidden by introducing phenomenological vertex functions and propagators. One then has at least an internally consistent set of equations (obeying unitarity) within which Mizutani and Koltun, for example, calculated the effect of the coupling $\pi d \rightarrow NN \rightarrow \pi d$ on the πd scattering length ($a_{\pi d}$).

At this conference Avishai *et al.* (#5.10) have also reported results for $a_{\pi d}$. The equations which they used were essentially guessed from the Afnan-Thomas (A-T) equations.⁶⁶ (More recently it has been shown⁶⁷ that the equations of Avishai *et al.*, which couple $\pi d \rightarrow \pi d$, $\pi d \rightarrow NN$ and $\pi d \rightarrow N\beta$ —with β a π N interaction—in one unitary set of equations, can in fact be derived from the Mizutani-Koltun equations.⁵⁵) The latter equations were an attempt to model the effects of absorption on elastic scattering within a non-relativistic potential theory, in which the pion and nucleon had a bound state (N') at $-m_\pi$. Pion production was then described as $NN' \rightarrow \pi d$. The problem is of course that pion production must be described by a field theory, and as originally proposed the A-T equations treat the two nucleons in an unsymmetrical way—such that the Pauli principle is not automatic, but must be introduced by hand. Furthermore, because only one nucleon (the N') could emit a pion at any stage, the model undercounted the number of diagrams for elastic

scattering which involved virtual absorption and remission. Fortunately the P_{11} scattering length used in the A-T work was too large (by recent determinations) and very nearly compensated for this undercounting! Indeed the A-T results were quite close to those presented by Avishai *et al.* in #5.10. Once again the old result that $\text{Re } a_{\pi d}^{(\text{abs})} \simeq -\text{Im } a_{\pi d}^{(\text{abs})}$ has been confirmed.^{66,68,69} Finally we note that Avishai *et al.* seem to have obtained quite a reasonable value for p-wave pion production (coefficient β).

Although there were no contributions dealing with this question submitted to the present conference, we must also mention that in two recent preprints the first calculations of the effect of absorption on πd scattering at low energy,⁷⁰ and in the resonance region,⁷¹ have been reported. Hopefully the calculations near the (3,3) resonance region will result in a better understanding of the SIN total cross section data.⁴⁰

3.1.4 Dibaryon Resonances

The question of the existence of resonances with baryon number two, coupled to the N-N elastic channel, has been seriously raised by the recent Argonne polarisation measurements.⁷² A number of calculations have been made in an attempt to explain the data in terms of the opening of the $NN \rightarrow N\Delta$ channel⁷³—thus far with little quantitative success. Simonov and van der Velde⁷⁴ have tested a simple criterion for eliminating pseudo-resonances in the πD system — namely comparing the radii of forward and backward loops in an Argand plot. They find no evidence for real B=2 resonances coupled to the πD channel, either in available data,^{37,42} or in the three-body calculations.^{50,51,59}

In contrast with the calculations mentioned above, potential models of the N-N, N- Δ and Δ - Δ systems (with the Δ treated as a stable particle) have yielded a number of resonances. For example, Kamae and Fujita⁷⁵ have reported a state bound by as much as 100 MeV in several spin-isospin channels in the Δ - Δ system. The implications of this result for deuteron photo-disintegration have also been examined.⁷⁶ At this conference, contribution #5.14 by Ueda has extended this work, and allowed some of the freedom of the real πNN and $\pi\pi NN$ systems using the Heitler-London formalism. (For comparison, Ref. [75] treated the Δ - Δ system as a two-body problem with one boson exchange potentials.) He reports a number of resonances in the region of (2.2-2.3) GeV, most of which are quite broad. Direct experimental confirmation of such states would be very interesting.

3.2 Pion-Nucleus Scattering

It has been shown by Tandy *et al.*⁷⁷⁻⁷⁹ that the operator $\hat{t}(E)$, which defines the first-order optical potential ($U_{00}^{(1)} = (A-1) \langle 0 | \hat{t} | 0 \rangle$), can most naturally be written as the solution of a simple three-body problem (pion-nucleon-core) in which only the πN and N-core interactions are non-zero. This picture has led to a number

of advances in our understanding of low energy π -nucleus scattering.⁸⁰⁻⁸⁴ Even a crude inclusion of three-body effects, such as replacing $\hat{t}$ in impulse approximation by $t_{\pi N}(E_{3\text{-body}})$ can change the differential cross section by a factor of two at 50 MeV—as shown in Fig. 9 for $\pi^\pm$ ^4He scattering.⁸² More recently the three-body results⁷⁷⁻⁷⁹ have led to a clearer understanding of the reaction content of the first order optical potential.^{83,84}

Fig. 9. The differential cross section for $\pi^\pm$ ^4He scattering at 51 MeV (T_π^{lab}).⁸² The short and long dashed curves use a conventional two-body energy,⁸⁵ with πN phase shifts of CERN⁸⁶ and Salomon⁸⁷ respectively. The solid curve shows the more recent Salomon phases, with the three-body energy in the first order optical potential.

Afnan and Stelbovics (#5.13) have used the non-relativistic πd system⁶⁶ as a model where one knows the exact solution, and can test the accuracy of various optical model approximations. They find quite large differences between the usual impulse approximation, in either the Watson⁸⁸ or KMT⁸⁹ formulations, and the exact solution. Unfortunately order $(1/A)$ corrections are very large for the deuteron, and it is not an ideal test case. For example, the double scattering term involving charge exchange on both nucleons plays a very important role at all energies, but is omitted in their optical model. (In a real nucleus such processes are suppressed by the Pauli exclusion principle.)

Perhaps the major point of dissension in the theory of pion-nucleus scattering is the question of the range of the fundamental πN interaction. One side argues that it is essentially a point interaction (leading to the Kisslinger⁹⁰ or Ericson-Ericson⁹¹ potential), and the other insists that, as shown by solutions of the inverse scattering problem,^{92,93} the πN range is of order 0.6 fm. While both camps have enough freedom to fit the available elastic scattering data, the physics is

quite different. In particular, N-N correlations play a crucial role in the former case.

One of the implications of the Myhrer-Thomas paper (#5.11 and Ref. [63]) concerned this problem. That is, one can show on the grounds of three-body unitarity that there should be no pole in the energy variable of the off-shell P_{33} t-matrix for energy below $[m_N + m_\pi]$. However, Chew-Low theory, which does not distinguish between on and off-shell amplitudes, has a pole at $\omega=0$ (static case).⁹⁴ The answer is, of course, that in a multiple scattering theory one must distinguish between left and right hand singularities. The former (potential) singularities—including the $\omega=0$ pole in the P_{33} channel due to Fig. 6(b)—determine the range of the interaction. This point of view strongly suggests that the fairly long range πN interaction is the better of the two pictures mentioned above.⁶³

To conclude this section, it must be remarked that one of the crucial roles for the approach described in subsect. 3.1.3 dealing with effects of absorption in the πNN system, is to help clarify the effects of absorption in a real nucleus. Particularly at low energy, this seems (phenomenologically) to be a very big effect.⁸¹

4. Conclusion

The two main subjects with which we have dealt have been three-body calculations in the three-nucleon and pion-two-nucleon systems. There were four additional papers in DS5 which did not fit into either of these categories. Perhaps the paper by Bajzer (#5.8) dealing with the investigation of off-shell effects in the N-N system using μ^+d break-up, could eventually use the techniques developed for handling the Coulomb force in low energy p-d scattering. However, it is difficult to see why the muon reaction should be more useful than electro-disintegration. Barroso and Rej (#5.17) have calculated the exchange current contribution to the low energy (γ, π^+) reaction on ^3He , and found quite small effects ($\leq 10\%$ for $q^2 < 6 \text{ fm}^{-2}$)—mainly involving the symmetric s-state.

Dreschel *et al.* (#5.18) have calculated the enhancement to the dipole sum rule for E1 absorption using exact wave functions for d, ^3H and ^3He , and a correlated oscillator model for ^4He . They found enhancements κ of order 1 for ^4He , in agreement with experiment. More general sum rules for electron scattering from these nuclei were also evaluated by including explicit isobar components. Lastly we mention the rather ambitious three-body calculation of K^-d scattering (#5.16), including explicitly the coupling to the $\pi^- \Lambda p$ channel, by Toker *et al.* This is a fascinating system because of the variety of channels which are open. This is also a reason to expect that a theory which strictly guarantees unitarity should be essential. The agreement with the Λp effective mass spectrum which they obtain is certainly promising.

Let us now return to the two major topics. Low energy N-d elastic scattering

seems to be well understood, with only the deuteron vector polarisation being troublesome. The break-up region is in desperate need of more sophisticated calculations to match the quality of data available. In addition, there seems to be room for more work with Brayshaw's BCM formalism, to independently test 'proposition 13'. The main thrust towards understanding backward N-d scattering at intermediate energies seems to concern the role played by virtual pion production. Indeed recent experimental evidence does suggest it is important.

Our understanding of the interaction of pions with the deuteron, and nuclei in general, is presently undergoing rapid expansion. The long awaited promise of comprehensive, good quality data is just being fulfilled. In parallel there have been tremendous advances in the theoretical treatment, both from the field theoretic point of view, and in terms of the accuracy and content of the semi-phenomenological relativistic three-body calculations. It is certain that the task of the rapporteur in this field at the 9th Few Body Conference will be a pleasant one!

Acknowledgements

A great number of people have assisted me in the preparation of this review—either by private communication of their results, by discussions on one or many of the topics raised, or by their valued contributions as collaborator, wife or typist. I would particularly like to thank, E. Alt, Y. Avishai, T. Diaz, C. Fayard, H.W. Fearing, J. Källne, W. Kloet, D. Koltun, F. Myhrer, A.S. Rinat, B. Saghai, W. Sandhas, J. Thomas and R. Woloshyn.

References

1. R. Aaron, R.D. Amado and Y. Yam, Phys. Rev. B140, 1291 (1965).
2. H.E. Conzett, in Few Body Dynamics, ed. A.N. Mitra, I. Slaus, V.H. Bhasin and V.K. Gupta (North Holland, Amsterdam, 1976) p. 611; plus contributed papers.
H.E. Conzett, in Few Body Problems in Nuclear and Particle Physics, ed. R.J. Slobodrian, B. Cujes and K. Ramavataram (Les Presses de l'Université Laval, Québec, 1975) p. 566; plus contributed papers.
3. W.M. Kloet, in Proc. International Conf. on Nucleon-Nucleon Interactions, ed. D.F. Measday, H.W. Fearing and A. Strathdee (AIP Conf. Proc. #41, 1978) p. 392
4. E.O. Alt, W. Sandhas, H. Zankel and H. Ziegelmann, Phys. Rev. Lett. 37, 1537 (1976).
5. E.O. Alt, W. Sandhas and H. Ziegelmann, Phys. Rev. C17, 1981 (1978).
6. J.J. Benayoun, J. Chauvin, C. Gignoux and A. Laverne, Phys. Rev. Lett. 36, 1438 (1976).
7. C. Stolk and J.A. Tjon, Phys. Rev. Lett. 35, 985 (1975).
8. P. Doleschall, Nucl. Phys. A201, 264 (1973).
P. Doleschall, Nucl. Phys. A220, 491 (1974).
9. S.C. Pieper, Nucl. Phys. A193, 529 (1972).
S.C. Pieper, Phys. Rev. C8, 1702 (1973).
10. A.W. Thomas, in Ref. [3] p. 373.
11. C. Fayard, G.-H. Lamot and E. Elbaz, Lett. al Nuovo Cim. 7, 423 (1973).
12. J. Bruinsma and R. van Wageningen, Nucl. Phys. A282, 1 (1977).
13. F.N. Rad, H. Conzett, R. Roy and F. Seilen, Phys. Rev. Lett. 35, 1134 (1975).
14. D.D. Brayshaw, Phys. Rev. Lett. 32, 382 (1974).
15. D.D. Brayshaw, Phys. Rev. D8, 752 (1973).
D.D. Brayshaw, The Boundary Condition Method, chapt. 4 in Modern Three Hadron Physics ed. A.W. Thomas (Springer-Verlag, Berlin, 1977).

16. R.A. Malfliet and J.A. Tjon, Ann. Phys. (N.Y.) 61, 425 (1970).
17. W.M. Kloet and J.A. Tjon, Nucl. Phys. A210, 380 (1973).
18. W. Ebenhöh, Nucl. Phys. A191, 97 (1972).
19. A.C. Phillips, Nucl. Phys. A107, 209 (1968).
20. C.M. Vincent and S.C. Phatak, Phys. Rev. C10, 391 (1974).
21. W. Plessas, L. Streit and H. Zingl, Acta Physica Austriaca 40, 272 (1974).
22. W.T.H. van Oers, in Few Body Problems in Nuclear and Particle Physics (see Ref. [2]) p. 307; TRIUMF report TRI-77-3.
23. B.E. Bonner *et al.*, Phys. Rev. Lett. 39, 1253 (1977).
24. A.K. Kerman and L.S. Kisslinger, Phys. Rev. 180, 1483 (1969).
25. N.S. Craigie and C. Wilkin, Nucl. Phys. B14, 477 (1969).
26. G.W. Barry, Ann. Phys. (N.Y.) 73, 482 (1972).
27. J.V. Noble and H.J. Weber, Phys. Lett. 50B, 233 (1974).
28. V.M. Kolybasov and N.Ya. Smorodinskaya, Sov. J. Nucl. Phys. 17, 630 (1973).
29. S.A. Gurvitz, Y. Alexander and A.S. Rinat, Ann. Phys. (N.Y.) 94, 223 (1975); *ibid.* 98, 346 (1976).
30. J.M. Cameron *et al.*, 'Large Angle Measurements of $p + {}^4\text{He} \rightarrow {}^3\text{He} + d \dots$ ': TRIUMF report TRI-UAE-5005.
31. A.N. Anderson *et al.*, Phys. Rev. Lett. 40, 1553 (1978).
32. G. Jones, in Ref. [3], p. 292; M.G. Albrow *et al.*, Phys. Lett. 34B, 337 (1971).
33. L.I. Lapidus, Proc. 6th International Conf. on High Energy Physics and Nuclear Structure, ed. D.E. Nagle *et al.* (AIP Conf. Proc., #26, 1975) p. 129.
34. J.M. Lambert *et al.*, Phys. Rev. C13, 43 (1976).
35. T. Yuasa, H. Nakamura-Yokota and N. Fujiwara, Suppl. Prog. Theor. Phys. (Kyoto) 61, 161 (1977).
36. D. Axen *et al.*, Nucl. Phys. A256, 387 (1976).
37. E.G. Pewitt *et al.*, Phys. Rev. 131, 1826 (1963).
38. J.H. Norem, Nucl. Phys. B33, 512 (1971).
39. K. Gabathuler *et al.*, Nucl. Phys. B55, 397 (1973).
40. J. Domingo *et al.*, SIN Newsletter (March, 1977) 42, and to be published.
41. J. Arvieux, priv. communication.
42. R.C. Minehart *et al.*, Phys. Rev. C17, 681 (1978).
43. W. Gruebler, SIN proposal #R-73-01.1, and priv. communication.
44. R. Holt, priv. communication.
45. J. Eisenberg, Proc. International Conference on Few Particle Problems in the Nuclear Interaction, ed. I. Slaus *et al.* (North Holland, 1973) p. 796.
46. A.W. Thomas, in Ref. [2b] (Laval), p. 287.
47. A.W. Thomas, Nucl. Phys. A258, 417 (1976).
48. A.W. Thomas, in Proc. International Conf. on Meson-Nucleus Physics (AIP Conf. Series, #33, 1977) p. 375.
49. N. Giraud, C. Fayard and G.H. Lamot, Phys. Rev. Lett. 40, 438 (1978); and priv. communication.
50. A.S. Rinat and A.W. Thomas, Nucl. Phys. A282, 365 (1977).
51. A.W. Thomas, A.S. Rinat, Y. Starkand and E. Hammel, contribution E1 to the 7th Int. Conf. on High Energy Physics and Nuclear Structure (Zurich, 1977).
52. R. Aaron, R.D. Amado and J.E. Young, Phys. Rev. 174, 2022 (1968).
53. R.M. Woloshyn, E.J. Moniz and R. Aaron, Phys. Rev. C12, 909 (1975).
54. R. Aaron, chapt. 5 in Modern Three-Hadron Physics (see Ref. [15]).
55. D.S. Koltun and T. Mizutani, Ann. Phys. (N.Y.) 109, 1 (1978).
56. A.S. Rinat, Nucl. Phys. A287, 399 (1977).
57. A.T. Stelbovics and M. Stingl, Nucl. Phys. A294, 391 (1978).
58. M. Stingl and A.T. Stelbovics, J. Phys. G. (to be published).
59. E. Hammel *et al.* (to be published).
60. S.C. Pieper, Phys. Rev. C9, 883 (1973).
61. W.R. Gibbs, Phys. Rev. C3, 1127 (1971); and in Proc. 4th International Conf. on Polarisation Phenomena (Birkhauser, Zurich, 1976) p. 61.
62. T.R. Mongan, Phys. Rev. 175, 1260 (1968).
63. A.W. Thomas and F. Myhrer, TRIUMF preprint: TRI-PP-78-7 (1978).
64. G.C. Wick, Rev. Mod. Phys. 27, 339 (1955).
65. G. Chew, Phys. Rev. 95, 1669 (1954).
66. I.R. Afnan and A.W. Thomas, Phys. Rev. C10, 109 (1974).

67. Y. Avishai, priv. communication.
68. K. Brueckner, Phys. Rev. 98, 769 (1955).
D.J. Thouless, Proc. Phys. Soc. (London) 69, 280 (1956).
69. A.W. Thomas, in Proc. 7th International Conference on High Energy Physics and Nuclear Structure, ed. M. Locher (Birkhäuser Verlag, Basel, 1977) p. 109.
70. N. Giraud, Y. Avishai, C. Fayard and G.H. Lamot, preprint Univ. Lyon (1978).
71. A.S. Rinat *et al.*, Weizmann Institute preprint: WIS-78/17-Ph.
72. W. De Boer *et al.*, Phys. Rev. Lett. 34, 558 (1975); I.P. Auer *et al.*, Phys. Lett. 67B, 113 (1977).
73. G.H. Thomas, in Ref. [3], p. 269; G. Epstein and D.O. Riska, Z. für Phys. A283, 193 (1977).
74. Yu. A. Simonov and M. van der Velde, Phys. Lett. 76B, 277 (1978).
75. T. Kamae and T. Fujita, Phys. Rev. Lett. 38, 471 (1977).
76. T. Fujita *et al.*, Heidelberg preprint (1977).
77. P.C. Tandy, E.F. Redish and D. Bollé, Phys. Rev. Lett. 35, 921 (1975).
78. P.C. Tandy, E.F. Redish and D. Bollé, Phys. Rev. C16, 1924 (1977).
79. E.F. Redish, chapt. 6 in Modern Three-Hadron Physics, Ref. [15].
80. R.H. Landau and A.W. Thomas, Phys. Lett. 61B, 361 (1976).
81. R.R. Johnson *et al.*, Nucl. Phys. A296, 444 (1978).
82. R.H. Landau and A.W. Thomas, Nucl. Phys. A302, 461 (1978).
83. A.W. Thomas, lectures presented at the NATO/NSF school on Theoretical Methods in Medium Energy and Heavy Ion Physics, Madison, 1978. (Plenum Press, to be published.)
84. A.W. Thomas and R.H. Landau, Phys. Lett. (accepted for publication, 1978).
85. R.H. Landau, S.C. Phatak and F. Tabakin, Ann. Phys. (N.Y.) 78, 299 (1973).
86. S. Almeded and C. Lovelace, CERN report TH 1408 (1971).
87. M. Salomon, TRIUMF report TRI-74-2 (1974).
88. K.M. Watson, Phys. Rev. 89, 575 (1953).
89. A.K. Kerman, H. McManus and R.M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).
90. L.S. Kisslinger, Phys. Rev. 98, 761 (1955).
91. M. Ericson and T.E.O. Ericson, Ann. Phys. (N.Y.) 36, 323 (1966).
92. R.H. Landau and F. Tabakin, Phys. Rev. D5, 2746 (1972).
93. J. Londergan, K. McVoy and E. Moniz, Ann. Phys. (N.Y.) 86, 147 (1974).
94. G.F. Chew and F.E. Low, Phys. Rev. 101, 1570 (1956).